

Speculating on Higher Order Beliefs*

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February 15, 2024

Abstract

Higher order beliefs – beliefs about others’ beliefs – may be important for trading behavior and asset prices, but have received little systematic empirical examination. We study more than twenty years of evidence from the Robert Shiller Investor Confidence surveys, which directly elicit details on investors’ higher order beliefs about the U.S. stock market. We find that investors’ higher order beliefs provide substantial motivations for non-fundamental speculation, e.g., to buy into a stock market perceived to be overvalued. To explore the general equilibrium implications, we construct a model of level k thinking that matches the evidence, where investors believe that asset price movements are driven by other, less sophisticated investors. The model reveals that investors’ higher order beliefs amplify stock market overreaction and excess volatility. These phenomena persist in equilibrium due to investors’ limited strategic reasoning.

*This is a substantially revised version of “Asset Pricing with Higher Order Beliefs.” We thank Federica Ambrosio, Natee Amornsiripanitch, Marios Angeletos, Snehal Banerjee, Nick Barberis, Paul Fontanier, Stefano Giglio, Will Goetzmann, Leandro Gomes, Paul Goldsmith-Pinkham, Gary Gorton, Peter Hansen, Zhen Huo, Jon Ingersoll, Toomas Laarits, Yukun Liu, Ben Matthies, Toby Moskowitz, Milad Nozari, Cameron Peng, Julia Selgrad, Kelly Shue, Alp Simsek, Sam Slocum, Aleh Tsyvinski, Joao Paulo Valente, and seminar participants at Yale SOM, Purdue Daniels, Chicago Booth, Caltech, UNC Kenan-Flagler, Harvard Business School, Emory Goizueta, the New York Fed, NYU Stern, Columbia Business School, the Treasury OFR, BC Carroll, and Stanford GSB for helpful comments and conversations. We are grateful to Bob Shiller for sharing data, and the Yale ICF for data support.

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Higher order beliefs – beliefs about others’ beliefs – are important in many equilibrium models in economics and finance. As emphasized by a strand of primarily theoretical literature starting with [Keynes \(1936\)](#), higher order beliefs may be particularly important for understanding investor behavior and financial market fluctuations. This is because investors have strong profit incentives to trade based on predictions of others’ beliefs rather than their own valuations, potentially leading asset prices to deviate substantially from fundamental values.¹ Despite their ubiquity and potential importance, higher order beliefs have received little systematic empirical examination, primarily due to challenges in measurement.

In this paper, we examine survey data on investors’ higher order beliefs, and consider the corresponding asset pricing implications. Our data come from the Robert Shiller Investor Confidence survey, which directly elicits investors’ perceptions of other investors’ beliefs. The survey also reports investors’ return expectations, which embed additional information about higher order beliefs. This is because an investor’s return expectations reflect her forecasts of other investors’ future stock market demand, which, in turn, depend on her forecasts of their beliefs. Guided by the evidence, we construct a theoretical model that illustrates the potential importance of higher order beliefs in driving stock market fluctuations. Below, we summarize our empirical and theoretical findings.

Empirical Results. While previous work has documented episodes of non-fundamental speculation – e.g., buying into risky assets perceived to be overvalued – our headline finding is that for the U.S. stock market, *non-fundamental speculation is the rule, not the exception*.² Higher order beliefs are important for giving rise to non-fundamental speculation.

We examine the Shiller survey, which, for more than 20 years, has asked individual and institutional investors a number of questions regarding their stock market expectations. Particularly relevant for our exploration, the survey asks investors if they perceive other investors to be overly optimistic (or pessimistic) about the U.S. stock market’s prospects, as well as if they perceive the stock market to be over- or under-valued. The majority of survey respondents report that others have mistaken beliefs, with the direction of their responses aligned with their perception of the under- or over-valuation of the stock market.

We find that when investors report that others are overly optimistic, they also report expectations of higher than average returns for the short term (1-to-3 months ahead), before expecting reversion in subsequent periods. A natural, higher order belief-based interpretation is that investors forecast other investors will become even more optimistic in the near term, fueling short horizon returns, before optimism and prices revert. These expectations provide

¹For example, [De Long et al. \(1990\)](#) note that George Soros “has been generally successful... by betting not on fundamentals but, he claims, on future crowd behavior.” [Brunnermeier and Nagel \(2004\)](#) argue the evidence from the dot-com bubble is “consistent with the view that hedge fund managers were able to predict some of the investor sentiment that was arguably behind the wild fluctuations in valuations of technology stocks.”

²This is speculation in the sense of [Keynes \(1936\)](#) and [Harrison and Kreps \(1978\)](#). We use the prefix ‘non-fundamental’ to distinguish from another common use of the term, where investors buy or sell an asset based on their relative optimism and pessimism about the asset’s fundamental value, e.g., [Simsek \(2021\)](#).

investors with strong incentives to buy into a stock market they see as overvalued. Consistent with such speculation, surveyed investors recommend holding positions aligned with their short term expectations, and against their fundamentals-based valuations. Moreover, investors' short term return expectations have strong explanatory power for the stock market positioning of asset managers. The speculative motives accompanying differently signed short and long term return expectations may help explain the insensitivity of investors' stock allocations to their long term return expectations (Giglio et al. (2021)).

Exploring the source of investors' higher order beliefs, we find that these beliefs and the accompanying non-fundamental speculation arise in response to macroeconomic news. For example, following positive macroeconomic news, investors report beliefs that others have become increasingly optimistic and markets have become overvalued. At the same time, they report expectations of higher short term returns followed by lower long term returns, which are also consistent with a belief in momentum and reversal. The evidence indicates that investors believe that others overreact to fundamental news upon its arrival and will continue to overreact in subsequent periods. This belief induces investors to engage in non-fundamental speculation.

Evaluating investors' short term return expectations, we find that non-fundamental speculation is unprofitable on average for the investors in our sample. A monthly-rebalanced market timing strategy that takes long and short positions in proportion to the average investor's reported one-month ahead return expectations earns a Sharpe ratio of -0.29. This poor performance aligns with previous work documenting a negative relationship between survey-based measures of return expectations and realized returns (e.g., Greenwood and Shleifer (2014)).

Model. Our empirical results pose a challenge for existing models. Most notably, models without an explicit focus on higher order beliefs cannot simultaneously explain investors' return expectations, valuations, and perceptions of other investors' beliefs.

To interpret the evidence and understand its general equilibrium implications, we construct a theoretical model that reveals that higher order beliefs induce asset price overreaction and excess volatility. The model features a riskless asset and a risky asset (the stock market) that pays a publicly observed dividend each period, which is drawn from persistent but unobserved fundamentals. The economy is populated by two types of investors: speculators, whose beliefs match features of the survey data, and who trade based on their expected one-period ahead returns; and fundamental traders, who buy when they perceive the market to be undervalued, and sell when they perceived it to be overvalued. Each investor receives a noisy but unbiased private signal about fundamentals. In the spirit of differences-of-opinion models, each investor believes that others' signals are uninformative conditional on their own. The average investor's belief about fundamentals is consistent with rational expectations, so all excess price movements in the model come from higher order beliefs.

In our baseline specification, speculators (incorrectly) believe that all other investors are

non-strategic, i.e., that all other investors are fundamental traders, and they also believe that others overestimate the persistence of the fundamentals process. This specification is sufficient to match the survey evidence on non-fundamental speculation. A belief that the economy is comprised of fundamental investors that overestimate the persistence of fundamentals leads speculators to believe that asset prices overreact to news. However, given that the economy is composed of a combination of speculators and fundamental investors with correct beliefs on average, asset prices overreact less than speculators expect. Accordingly, speculators infer that other investors received an attenuated signal, and forecast continued overreaction in subsequent periods as other investors fully internalize past news.³ Based on their forecasts, speculators willingly buy into a market they perceive has overreacted, in the process *causing* it to overreact. In the periods following the arrival of fundamental news, prices revert, as speculators' forecasts of continued overreaction do not manifest.

We next generalize our model to allow speculators to recognize the presence of other speculators by embedding our baseline specification into a model of level k thinking. We find that unsophisticated equilibrium reasoning is crucial for explaining the evidence. Level k thinking is an idea developed in the experimental literature where agents believe other agents to be less strategically sophisticated than themselves (see [Crawford, Costa-Gomes and Iriberri \(2013\)](#) for a review). In our setting, we define our baseline specification as a level 1 equilibrium, where speculators believe that all other speculators are strategically unsophisticated level 0 thinkers (fundamental traders). A level k equilibrium is defined recursively as one featuring speculators who perceive prices as arising from a level $k - 1$ equilibrium where all speculators are level $k - 1$ thinkers. A higher k captures more sophisticated thinking, in the sense of incorporating more rounds of strategic reasoning.

We find that every level k equilibrium qualitatively matches the empirical evidence. That is, the model matches the patterns in the data as long as there is a seed of a belief that other speculators (believe that other speculators believe that other speculators...) believe that investors overestimate the persistence of fundamentals. Importantly, however, for a given set of model parameters, as we increase strategic sophistication, asset price overreaction and excess volatility fall. In the limit, as speculators' sophistication becomes unbounded ($k \rightarrow \infty$), as assumed in traditional models, asset prices converge to the rational expectations value, and non-fundamental speculation disappears. In order to rationalize non-fundamental speculation, investors must lack a sophisticated understanding of general equilibrium typically assumed in traditional models.

We can understand this point as follows: level k speculators recognize that non-fundamental speculation induces asset prices to overreact to news. They still buy into the market, because their perception that others are less sophisticated than themselves means that they

³The features that investors attribute to other investors' beliefs – that they overestimate the persistence of fundamentals and persistently update their beliefs in the direction of past news – are the same frictions that can explain forecast errors of macroeconomic forecasters ([Angeletos, Huo and Sastry \(2021\)](#)).

observe less overreaction than they expect, leading them to infer that others received an attenuated signal and that prices will continue to overreact in the near term. However, as we increase speculators' strategic sophistication, they increasingly recognize that the current price is driven by speculators buying into the market in anticipation of continued overreaction that does not reliably arrive. This recognition leads speculators to decrease their expected returns from non-fundamental speculation, reducing contemporaneous overreaction to news. As they engage in multiple iterations of such reasoning, speculators fully internalize non-fundamental speculation as the cause of asset price overreaction and as unprofitable, and revert to becoming fundamental value investors; prices converge to the rational expectations value. The fact that we observe evidence consistent with non-fundamental speculation indicates the lack of such reasoning by investors.

In addition to highlighting that higher order beliefs and imperfect strategic sophistication give rise to overreaction and excess volatility, we also use our model to understand the interaction between higher order beliefs and fundamental beliefs. In isolating the role of higher order beliefs, our model presents a straightforward way to understand how higher order beliefs may interact with fundamental belief biases documented in the literature. For example, if investors' fundamental beliefs overreact to news, as suggested by recent work, then higher order beliefs and fundamental beliefs reinforce one another to produce even stronger equilibrium overreaction of asset prices. Additionally, we discuss a variant of our model where investors learn about fundamentals from prices. With learning about fundamentals from prices, investors' higher order beliefs induce excess volatility in their fundamental beliefs, via a mis-specified learning channel, related to recent work by [Bastianello and Fontanier \(2022a\)](#). In turn, higher order beliefs may introduce further excess volatility into prices.

Related Literature. Our paper relates to work on higher order beliefs in asset pricing, to which we bring empirical discipline using survey data.⁴ Previous work can be partitioned into two traditions: noisy rational expectations models, where rational investors face frictions that prevent them from observing others' beliefs and fundamentals ([Singleton \(1987\)](#), [Allen, Morris and Shin \(2006\)](#), [Bacchetta and van Wincoop \(2006, 2008\)](#), [Makarov and Rytchkov \(2012\)](#), [Kasa, Walker and Whiteman \(2014\)](#), [Cespa and Vives \(2015\)](#), and [Nimark \(2017\)](#)); and differences-of-opinion models, where investors know and disagree with other investors' private valuations ([Harrison and Kreps \(1978\)](#), [Harris and Raviv \(1993\)](#), [Kandel and Pearson \(1995\)](#), [Scheinkman and Xiong \(2003\)](#), and [Banerjee and Kremer \(2010\)](#)).

Relative to both, our paper presents empirical evidence on higher order beliefs, and seeks to model higher order beliefs in a manner consistent with the evidence. In doing so, we find relaxations of the underlying assumptions of both types of models to be important. Noisy rational expectations models typically assume full depth of reasoning ($k = \infty$) and suggest

⁴There are two notable exceptions that empirically examine higher order beliefs. [Egan, Merkle and Weber \(2014\)](#) survey private investors and find that beliefs about others' return expectations affect investment decisions. [Coibion et al. \(2021\)](#) survey firm managers in New Zealand, with a focus on testing noisy information models.

that investors only speculate on information that is orthogonal to public information; we find that both are inconsistent with the evidence. Differences-of-opinion models typically assume investors' beliefs to be common knowledge and do not give rise to non-fundamental speculation. In contrast, we find that incorrect higher order beliefs may be important for rationalizing the empirical evidence (see also [Banerjee, Kaniel and Kremer \(2009\)](#) and [Han and Kyle \(2018\)](#) for models featuring incorrect higher order beliefs). Our paper also highlights survey evidence that may be useful for future work – namely direct questions about investors' higher order beliefs, as well as data on the term structure of expected returns, which help pin down investors' beliefs about the dynamics of other investors' beliefs.

Our modeling approach contributes to a small, but growing literature in finance and macroeconomics on level k thinking and errors in strategic reasoning. In finance, previous models study investors neglecting the information content of prices ([Eyster, Rabin and Vayanos \(2019\)](#)) and neglecting that other investors may learn about fundamentals from prices ([Bastianello and Fontanier \(2022a,b\)](#)), with implications for trading volume and asset price fluctuations. We find that level k thinking helps explain investors' higher order beliefs and return expectations, and can rationalize unprofitable non-fundamental speculation that drives excess volatility.⁵ Our results also comport with a nascent empirical literature in finance documenting investors' neglect of general equilibrium effects in formulating their return expectations ([Andre, Schirmer and Wohlfart \(2023\)](#), [Bybee \(2023\)](#)).

Our paper also relates to a literature on non-fundamental speculation in financial markets, where investors willingly buy into markets they see as overvalued. The literature documents a number of instances of non-fundamental speculation in bubble-like episodes (e.g. [McKay \(1841\)](#), [Kindleberger \(1978\)](#), [Temin and Voth \(2004\)](#), [Brunnermeier and Nagel \(2004\)](#), [Soros \(2015\)](#)) and prominent theoretical work on the topic includes [De Long et al. \(1990\)](#), [Harrison and Kreps \(1978\)](#), [Scheinkman and Xiong \(2003\)](#), [Abreu and Brunnermeier \(2002, 2003\)](#), and [Martin and Papadimitriou \(2022\)](#). Our paper illustrates that non-fundamental speculation is a pervasive feature of the U.S. stock market, and illuminates how higher order beliefs give rise to such speculation. Moreover, while the literature focuses on episodes of non-fundamental speculation where informed investors (e.g., hedge funds) may have profited, we find evidence that non-fundamental speculation is unprofitable for the investors in our sample.

Finally, our paper is related to a literature in finance using survey data to understand market participants' beliefs ([Adam and Nagel \(2022\)](#) provide a survey). A sizeable literature has studied the importance of fundamental and return expectations for investor behavior and stock market returns. Work on return expectations includes [Vissing-Jorgensen \(2003\)](#), [Bacchetta, Mertens and van Wincoop \(2009\)](#), [Greenwood and Shleifer \(2014\)](#), [Amromin and](#)

⁵Theoretical work on level k thinking in macroeconomics includes [Farhi and Werning \(2019\)](#) and [García-Schmidt and Woodford \(2019\)](#), who argue that level k thinking leads to underreaction of macroeconomic aggregates in response to shocks. [Angeletos and Lian \(2022\)](#) discuss the relationship between macroeconomic models of level k thinking and models of incomplete information.

Sharpe (2014), Barberis et al. (2015, 2018), Adam, Marcet and Beutel (2017), and Nagel and Xu (2022b), while work on fundamental expectations includes Chen, Da and Zhao (2013), Bordalo et al. (2020), De La O and Myers (2021), and Nagel and Xu (2022a).⁶ Our results bridge together fundamental and return expectations via higher order beliefs. See also Giglio et al. (2021), Jin and Sui (2022), and McCarthy and Hillenbrand (2021) for work relating fundamental and return expectations.

The rest of the paper proceeds as follows. In Section 1, we discuss the survey data and present empirical evidence on investors' short horizon return expectations. In Section 2, we present a theoretical asset pricing model that interprets the empirical evidence and explores its general equilibrium implications. Section 3 concludes.

1 Empirical Evidence from Survey Data

We study expectations of U.S. equity market returns reported by retail and institutional investors in the Robert Shiller Investor Confidence survey. Whereas other available investor surveys mostly ask a single or a few questions to investors about their return expectations at a fixed horizon, the Shiller survey is unique in providing a long time-series where investors are simultaneously asked about their higher order beliefs, their stock market valuations, and their return expectations over multiple horizons. This makes it particularly well-suited for studying the questions of interest.

We find that investors often believe that other investors hold incorrect stock market valuations, but find it profitable to speculate in the direction of these incorrect valuations. We also find that investors report a belief that stock markets overreact to news upon its arrival, and report return expectations that are consistent with the stock market exhibiting momentum and reversal. We discuss the ingredients required of higher order beliefs to match the empirical evidence, and the predictions made by existing models, which are able to match some, but not all of the evidence.

1.1 Data Description

The main data used in our empirical analysis come from the Shiller surveys, which are the microdata underlying the Robert Shiller Stock Market Confidence indices. The survey data have been collected continuously since 1989 – semi-annually for a decade, and then monthly by the International Center for Finance at the Yale School of Management since July 2001.

⁶Prior work largely focuses on expectations of returns at a fixed future horizon (e.g., one-year ahead). We focus on the future path of expected returns. In contemporaneous work, Gandhi, Gormsen and Lazarus (2023) also study the term structure of return expectations. Our evidence complements and provides a potential explanation for their finding that investors overestimate the persistence of returns.

Shiller (2000) discusses the survey questions in more detail. For our analysis, we focus on the continuous sample from July 2001 through April 2023.

1.1.1 Survey Respondents

The surveys are conducted by a market survey firm, which mails 500 surveys to high net-worth individual investors, and 500 surveys to institutional investors each month, with a sampling goal of 20 to 50 responses by each of the two types - individual and institutional. For both institutional and individual investors, the investor mailing lists are purchased from Data Axle (previously known as InfoUSA).

The micro data do not provide detailed demographic information on survey respondents (or non-respondents). There is likely to be selection into responding to the survey, as in other work surveying investors. For example, Giglio et al. (2021) find in a survey of Vanguard investors that their survey respondents tend to be older, wealthier, more likely to be male, and tend to trade more often than nonrespondents. The selection criteria for both individual and institutional investors in the Shiller survey, and the data that are available on investor characteristics, indicate that individual investor respondents are likely to have high income and be wealthy, and that institutional respondents manage large portfolios. While likely not fully representative of the investor population, survey respondents are a substantial and important class of investors. In Appendix C.1, we test and find no evidence of systematic business cycle variation in the number of responses to the survey.

For individual investors, the mailing list for the surveys is constructed by sampling households with a household income of greater than \$150,000 per year from the Infogroup Consumer Database. We have no additional demographic information on the respondents.

For institutional investors, the mailing list is constructed by sampling companies from the Infogroup Business Database with the SIC codes 628202 (Investment Management), 628203 (Financial Advisory Services), 628204 (Financing Consultants), and 628205 (Financial Planning Consultants). Survey respondents are asked to provide the ‘Size of the common stock portfolio(s) you make decisions about.’ In the sample, the 25th, 50th, and 75th percentiles of responses are \$1.8 million, \$25 million, and \$100 million. Summing across respondents by month, the 25th, 50th, and 75th percentiles of the sum of responses are \$642 million, \$1.57 billion, and \$9.00 billion.

1.1.2 Questions of Interest

Especially relevant to us, the survey data contain investor responses to a series of questions regarding investors’ beliefs about other investors’ beliefs, their beliefs about stock market valuations, and their expectations of returns over different time horizons. These questions include (with potential responses in parentheses):

(i) Questions regarding higher order beliefs

- (a) *Many people are showing a great deal of excitement and optimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them. (True; False; No opinion)*
- (b) *Many people are showing a great deal of pessimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them. (True; False; No opinion)*

(ii) Questions regarding stock market returns, valuations, and behavior

- (a) *How much of a change in percentage terms do you expect [for the Dow Jones index] in the following 1 month? 3 months? 6 months? 1 year? 10 years?*
- (b) *Stock prices in the United States, when compared with measures of true fundamental value or sensible investment value are (Too low; Too high; About right; Do not know).*
- (c) *Although I expect a substantial drop in stock prices in the U.S. ultimately, I advise being relatively heavily invested in stocks for the time being because I think that prices are likely to rise for a while (True, False, No opinion; if True, indicate best guess for date of peak).*
- (d) *Although I expect a substantial rise in stock prices in the U.S. ultimately, I advise being less invested in stocks for the time being because I think that prices are likely to drop for a while (True, False, No opinion; if True, indicate best guess for date of bottom).*
- (e) *If the Dow dropped 25% over the next six months, I would guess that the succeeding six months, the Dow would: (Increase (Give percent), Decrease (Give percent), Stay the same, No opinion).*

(iii) Questions regarding drivers of higher order beliefs

- (a) *What do you think is the cause of the trend of stock prices in the United States in the past six months? (It properly reflects the fundamentals of the U.S. economy and firms; It is based on speculative thinking among investors or overreaction to current news; Other (write-in); No opinion)*

Answers to question (i.a) and (i.b) directly provide information regarding investors' higher order beliefs. The questions themselves may submit multiple interpretations, especially given the multi-part nature of the questions, and the fact that respondents may answer 'True' to both questions. We present cross-sectional and time-series evidence in this section that when investors respond that they believe many others are overly optimistic and they must be careful not to be influenced by them, they also report beliefs that stock markets are overvalued, and that they expect long term stock market returns to be low. Based on this evidence, investors appear to interpret these questions as asking:

- (i.a) *I believe that many other investors hold overly optimistic stock market valuations.*

(i.b) *I believe that many other investors hold overly pessimistic stock market valuations.*⁷

While there may be some noise associated with assigning this interpretation to the question – it is likely that some respondents may have differing interpretations – the interpretation is consistent with responses to other questions on average and is informative about investors' views. This is especially the case in light of the long time series of evidence relative to other surveys that may ask similar questions.

Answers to questions (ii.a-e) provide information about investors' return expectations at different horizons, and the behavior associated with those expectations. These questions provide additional validation for our interpretation of the questions regarding investors' higher order beliefs. Moreover, the questions on investors' return expectations at different horizons encode additional information about investors' higher order beliefs, in particular capturing how investors expect others' beliefs to evolve over time. Questions (ii.c-e) also link investors' higher order beliefs and return expectations with potential speculative trading behavior, as we discuss in more detail.

While questions (ii.a) and (ii.e) ask investors to explicitly report numerical values for their expected stock market returns, the other questions are provided in multiple choice format. For empirical analysis, we map answers for most of the questions to numerical values. For questions (i.a-b) and (ii.c-d), we map the responses (True; False; No opinion) to (1, -1, 0), so that higher numbers indicate increasing agreement with the questions. For question (ii.b), we map the responses (Too low; Too high; About right; Do not know) to (-1; 1; 0; 0), so that lower numbers correspond with perceived undervaluation and higher numbers correspond with perceived overvaluation; we find our results are robust to dropping observations where investors report 'Do not know.'

1.2 Summary Statistics

We begin by summarizing responses in the Shiller survey. Table 1 reports summary statistics over the full sample in response to the relevant questions. For all questions except for questions (ii.a) and (ii.e), the table reports the proportion of respondents in the sample that gave a specific answer in response to that question; for question (ii.a) and (ii.e), the table reports expected returns averaged across survey respondents. The table reports statistics separately for individual and institutional investor respondents; the results are qualitatively and quantitatively similar for the two groups.

Focusing on the first two rows, 59% of individual investors report that they believe many others to be overly optimistic and 62% report that they believe many others to be overly

⁷There is particular ambiguity about the meaning of the second part of the questions – 'I must be careful not to be influenced by them.' Based on the evidence that we present in this section comparing investors' responses across questions, particularly questions (ii.c) and (ii.d), investors seem to indicate that they believe others' valuations may be overly- optimistic or pessimistic, but nevertheless that others' valuations still enter into their investment decisions given the influence that others have on short term market returns.

pessimistic (for institutional investors, the proportions are 52% in response to both questions). These results indicate that several respondents simultaneously indicate a belief that many others are overly optimistic and overly pessimistic.⁸ They also suggest the presence of substantial disagreement about the stock market; the majority of respondents in the sample report that other investors have incorrect beliefs.

Looking to the third and fourth rows, 35% of individual investors report that they expect markets to drop but recommend being overweight, while 35% report that they expect markets to eventually rise, but recommend being underweight (the numbers are 33% and 32% for institutional investors). Given that the questions are logically mutually exclusive, the responses suggest that a substantial majority of investors expect short and long term returns to be differently signed.⁹ This result is notable, because it suggests the presence of non-fundamental speculative motivations on the part of investors, e.g., many investors that believe markets to be presently overvalued (and that market returns will be low in the long run) still recommend being overweight stocks due to potential short term profits from doing so.

Turning to the last four rows, individual and institutional investors report small return expectations for the next month, with an expectation of more positive returns for 3- to 6-months ahead. Return expectations for 12-months ahead are 3.7% on average for individual investors and 4.9% on average for institutional investors. Regarding prices vis-a-vis fundamentals, 11% of individual investor respondents indicate that stock valuations are low relative to fundamentals, 37% say they are high, 44% say they are about right, and 8% express no opinion (these numbers are 19%, 30%, 49% and 2% for institutional investors). Regarding recent stock market trends, of individual investor responses, 24% indicate that market movements properly reflect fundamental news, while 52% indicate that the movements reflect speculative thinking and overreaction by other investors; these numbers are 28% and 37% among institutional investors. And lastly, given the hypothetical situation where the stock market drops 25% in the next 6 months, individual investors expect returns in the subsequent 6 months to be +13.6% on average, and institutional investors expect returns to be +16.9%. This is consistent with investors believing that stock market declines reflect overreaction that will revert in the intermediate term.

⁸The simultaneity can be seen by the fact that the sum of the proportions of investors reporting that others are overly optimistic and pessimistic is greater than 100%. This simultaneity does not affect our analysis, but does suggest that investors generally perceive that others have more extreme beliefs than they do.

⁹By-and-large, survey respondents recognize that the questions are mutually exclusive, though some do not. Only 7.2% of institutional investors and 7.8% of institutional responses are *True* to both questions (ii.c) and (ii.d) at the same time. Among individual investors, 71% that respond *True* to (ii.c) response *False* to (ii.d) and 73% that report *True* to (ii.d) report *False* to (ii.c); these numbers are 70% and 72% for institutional investors.

	Individual				Institutional			
	True	False	No Opin.		True	False	No Opin.	
<i>(i.a) Others overly optimistic about stocks</i>	59%	28%	13%		52%	32%	16%	
<i>(i.b) Others overly pessimistic about stocks</i>	62%	23%	15%		52%	20%	14%	
<i>(ii.c) Expect eventual drop but overweight</i>	35%	54%	11%		33%	56%	12%	
<i>(ii.d) Expect eventual rise but underweight</i>	35%	52%	12%		32%	55%	13%	
	1M	3M	6M	12M	1M	3M	6M	12M
<i>(ii.a) Percent expected return</i>	-0.2%	0.5%	1.4%	3.7%	0.1%	0.6%	2.0%	4.9%
	Low	High	Right	No Op.	Low	High	Right	No Op.
<i>(ii.b) Stock prices vs. fundamental value are...</i>	11%	37%	44%	8%	19%	30%	49%	2%
	Value	Overreac.	Other	No Op.	Value	Overreac.	Other	No Op.
<i>(iii.a) Cause of 6-month market trend</i>	24%	52%	22%	2%	28%	37%	34%	1%
<i>(ii.e) Expected 6M return after 25% drop</i>	13.6%				16.9%			
<i>Total Number of Responses</i>	6688				5591			

TABLE 1: SHILLER SURVEY SUMMARY STATISTICS

Note: The table reports summary statistics of survey responses to the Shiller surveys, reporting statistics separately for individual and institutional investor respondents. For questions (i.a,b), (ii.b,c,d) and (iii.a), the table reports the proportion of survey respondents in the sample that gave a particular answer in response to a given question. For question (ii.a) and (ii.e), the table reports the average k -month ahead expected return reported by respondents. Questions and potential responses are presented in abbreviated form in the table; details on the questions and responses are provided in Section 1.1.

1.3 Higher Order Beliefs and Perceived Market Valuations

We next examine responses to questions (i.a) and (i.b), regarding other investors' optimism and pessimism. We provide evidence consistent with respondents generally interpreting the questions to indicate that other investors are *overly* optimistic or *overly* pessimistic in their stock market valuations.

We construct two variables: *Higher Order Optimism* and *HO Pessimism*, which map the responses to questions (i.a) and (i.b), (True; False; No Opinion), to the values (1; -1; 0). The *HO Optimism* variable is increasing in agreement with the statement that other investors' are overly optimistic, while the *HO Pessimism* variable is increasing in agreement with the statement that other investors are overly pessimistic. We regress the *HO* variables on an *Overvaluation* variable, which we compute by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). Higher values of *Overvaluation* correspond with higher stock market prices relative to fundamentals.

Table 2 reports the regression results. Columns 1 to 3, on which we focus in our discussion, report regression results pooling together individual and institutional investor responses. Columns 1 and 2 report results from regressions using survey response level observations. With month fixed effects (column 2), the regressions capture cross-sectional comparisons, for example, whether an investor that believes others are more optimistic also is more likely to believe the stock market is overvalued. We also run the regressions as time-series regressions (column 3), by using cross-sectional monthly averages of the variables as the observations in our regressions. The time-series regressions capture whether, for example, in time periods where investors believe others to be more optimistic, they are also more likely to believe that markets are overvalued, consistent with what we would expect.

Panel A reports results where *HO Optimism* is the dependent variable, Panel B reports results where *HO Pessimism* is the dependent variable, and Panel C reports results where the dependent variable is *HO Belief*, which we define as $HO\ Optimism - HO\ Pessimism$. We find consistent evidence of a strong relationship in the expected direction for each of the regression specifications. We focus our discussion on Panel C, where the measure captures a given respondent's higher order belief, incorporating information from their responses about others' optimism and pessimism. The coefficients are 0.48 in the response-level regression with no fixed effects, 0.41 in the response-level with time fixed effects, and 0.85 in the regression using monthly averages as observations, with R^2 values of 0.08, 0.14, and 0.34.

For the rest of our empirical analysis in the main text, we focus on the *HO Belief* measure. In each of the panels in Table 2, columns 4-6 report regression results using the sample of individual investors and columns 7-9 report regression results using the sample of institutional investors. The results are qualitatively the same across the individual and institutional investor samples, with some slight quantitative variation. For brevity, we report results in the

pooled sample for the rest of the paper and provide additional breakdowns of the results for the individual versus institutional investor samples in the appendix. We note that our main conclusions are largely similar for individual and institutional investors.

The results indicate a strong relationship between the *HO Belief* and the *Overvaluation* measures; when investors report that they think markets are overvalued, they are substantially more likely to report many other investors are optimistic, and less likely to report that other investors are pessimistic.

1.4 Higher Order Beliefs and Return Expectations

We next turn to studying the relationship between higher order beliefs and return expectations of different horizons. Responses to questions (i.a) and (i.b) capture investors' beliefs about other investors' *present* beliefs. Return expectations capture how stock prices are expected to evolve in the future, embedding beliefs about other investors' *future* stock market demand and beliefs. That is, return expectations capture an additional dimension of higher order beliefs that is important for understanding investor behavior.

We regress investors' reported return expectations of different horizons (multiplied by 100) on the *HO Belief* variable. Panel A of Table 3 reports the regression results.

The first four columns report regression results for time-series regressions, where the unit of observation is the cross-sectional average of survey responses in a given month. The coefficient on *HO Belief* for 1-month return expectations is 1.57, indicating that a unit increase in the *HO Belief* variable corresponds with a 1.57% higher expected return for the following month. The coefficients decline with horizon; the coefficients on 3-, 6-, and 12-month return expectations are 0.59, -0.32, and -2.46. Appendix Table C.4 verifies that these results hold for both *HO Optimism* and *HO Pessimism* as independent variables.

The results reveal an interesting, hump-shaped pattern of cumulative return expectations corresponding with higher order beliefs, plotted in Figure 1. On average, in periods where investors report a belief that other investors are overly optimistic (and that stock markets are overvalued), they expect stock markets to rise over the next month – they expect stock market demand to rise. Though other factors may contribute, a natural, higher order belief-based interpretation is that investors expect others to become even more optimistic in the short term, fueling the increasing demand.

These expectations provide motivations for non-fundamental speculative trade, whereby investors may choose to take long (or overweight) positions in the stock market even when they perceive it to be overvalued, because they perceive the market may continue to rise before valuations correct and returns are lower.

The last four columns in Panel A of Table 3 report regression results for cross-sectional regressions, which include month fixed-effects, and where the unit of observation is a survey response. The coefficients for 1-, 3-, 6-, and 12-month ahead returns in the cross-sectional

Panel A: $y=HO\ Optimism$									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
<i>Overvaluation</i>	0.34 (0.02)	0.29 (0.02)	0.63 (0.07)	0.26 (0.02)	0.20 (0.02)	0.61 (0.07)	0.41 (0.02)	0.37 (0.02)	0.59 (0.07)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
<i>N</i>	12135	12135	259	6596	6596	259	5539	5539	259
<i>R</i> ²	0.06	0.11	0.37	0.04	0.11	0.30	0.10	0.16	0.24
Panel B: $y=HO\ Pessimism$									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
<i>Overvaluation</i>	-0.14 (0.01)	-0.13 (0.01)	-0.23 (0.06)	-0.13 (0.02)	-0.11 (0.02)	-0.12 (0.09)	-0.16 (0.02)	-0.14 (0.02)	-0.20 (0.07)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
<i>N</i>	12135	12135	259	6596	6596	259	5539	5539	259
<i>R</i> ²	0.01	0.05	0.08	0.01	0.06	0.02	0.02	0.08	0.04
Panel C: $y=HO\ Belief:=HO\ Optimism - HO\ Pessimism$									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
<i>Overvaluation</i>	0.48 (0.02)	0.41 (0.02)	0.85 (0.12)	0.39 (0.03)	0.31 (0.02)	0.73 (0.14)	0.57 (0.03)	0.51 (0.03)	0.80 (0.09)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
<i>N</i>	12135	12135	259	6596	6596	259	5539	5539	259
<i>R</i> ²	0.08	0.14	0.34	0.05	0.14	0.23	0.12	0.19	0.28

TABLE 2: HIGHER ORDER OPTIMISM AND PERCEIVED VALUATIONS

Note: The table reports results from regressions of *Higher Order Belief* variables constructed from the Shiller surveys on a perceived *Overvaluation* measure constructed from the surveys. The *HO Optimism* and *HO Pessimism* measures are constructed by mapping the responses to questions (i.a) and (i.b.) regarding other investors' optimism and pessimism, (True; False; No Opinion), to the values (1; -1; 0). The *Overvaluation* measure is constructed by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals, (Too low; Too high; About right; Do not know), to the values (-1; 1; 0; 0). Columns 1-3 pool together observations across the individual and institutional investor samples, and columns 4-6 and 7-9 separately report results for the two samples. The unit of observation for Columns 3, 6, and 9 is the monthly cross-sectional average of the variables; Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observations for columns 1, 2, 4, 5, 7, and 8 are survey responses; Driscoll-Kraay standard errors (12 lags) for coefficients are reported in parentheses.



FIGURE 1: RETURN EXPECTATIONS AND HIGHER ORDER BELIEFS

Note: The figure plots coefficients from regressions of average return expectations on monthly cross-sectional averages of *HO Belief*. Dependent variables are multiplied by 100, so that coefficients can be interpreted as expected returns with a unit change in *HO Belief*. Standard errors are Newey-West standard errors (12 lags). The figure also plots plus and minus two standard errors for the estimated coefficients for return expectations.

regressions are 0.05, -0.31, -1.03, and -1.79. The results indicate that in cross-sectional comparisons, an investor that holds a stronger belief that others are overly optimistic does not necessarily believe that short term returns will be higher than an investor that holds a weaker belief that others are optimistic, though they do expect worse long term market performance.

We provide additional validation for the time-series pattern of investors' return expectations by using investors' responses to questions (ii.c) and (ii.d), which ask investors whether they expect the stock market to reach a peak (trough) in the short run though they expect it to decline (rise) in the long run. We construct *Short-Term Peak* and *ST Trough* variables by mapping responses to questions (ii.c) and (ii.d), (True; False; No Opinion), to the values (1, -1, 0). We run time-series regressions of *ST Peak* and *ST Trough* on *HO Belief* and *Overvaluation*, with cross-sectional monthly averages as the unit of observation.

Panel B of Table 3 reports the regression results. The first two columns report results where the independent variable is *HO Belief*. The coefficient on *HO Belief* is 0.41 for *ST Peak* (R^2 of 0.22) and -0.28 for *ST Trough* (R^2 of 0.16). These results provide important validation of the relationship between investors' beliefs regarding others' optimism and their belief that markets will continue to rise before eventually declining, using a qualitative elicitation method. This evidence also helps assuage potential concerns with respondents struggling in providing quantitative responses.

Affirmative responses to (ii.c) and (ii.d), used to construct the *ST Peak* and *ST Trough* mea-

asures, also indicate investors' recommendations to be overweight stocks even though they expect an eventual decline in stocks, or underweight despite expecting an eventual rise in stocks. The relationship between *HO Belief*, *ST Peak*, and *ST Trough* provide further evidence in support of non-fundamental speculation induced by higher order beliefs.

The last two columns in Panel B of Table 3 reports results where the independent variable in the regression is *Overvaluation*, capturing investors' beliefs that the market is overvalued. We observe a similarly strong relationship between *ST Peak* and *Overvaluation* (coefficient of 0.65, R^2 of 0.26), though a weaker relationship between *ST Trough* and *Overvaluation* (coefficient of -0.06, R^2 of 0.00).

1.5 Non-Fundamental Speculation

The evidence in this section indicates that investors have an incentive to engage in non-fundamental speculation, e.g., to 'ride the bubble' and buy into a stock market they perceived as overvalued due to the expectation of short term positive returns. Indeed, survey respondents' affirmative responses to questions (ii.c) and (ii.d), advising taking positions in the stock market that are opposite their long term stock market return expectations, provide support for non-fundamental speculation motives. An important question is whether the expectations data actually capture investors' trading behavior, and in particular, whether investors speculate based on their short term return expectations.

We provide additional evidence of non-fundamental speculation using data on futures positions. We find that the trading behavior of buy-side investors (asset managers, hedge funds, etc.) tracks short horizon return expectations reported in the Shiller survey. Investors increase their market exposure corresponding with higher short term return expectations, and reduce their market exposure corresponding with lower short term expectations.

We obtain weekly data on the positions of investors in S&P500 equity index futures from the Traders in Financial Futures report from the Commodity Futures Trading Commission. The data have been published weekly since 2010, and have been backfilled to 2006. The report presents the number of long and short contracts held in aggregate by investors classified into one of four categories based on self-reported business purposes: futures dealers, levered funds (i.e., hedge funds), institutional asset managers, and other. [Hazelkorn, Moskowitz and Vasudevan \(2023\)](#) find that positions in futures contracts from the report capture demand for equity market exposure that is reflected in equity market valuations.¹⁰ Changes in futures positions reflect a combination of funds' rebalancing in response to flows (which may, for example, reflect individual investors' expected returns and demand for equity market expo-

¹⁰Other papers, for example [Brunnermeier, Nagel and Pedersen \(2008\)](#) and [Moskowitz, Ooi and Pedersen \(2012\)](#), use futures positioning data from the Commitment of Traders Report, a similar report also published by the Commodity Futures Trading Commission that groups traders into more coarse categories, and similarly find that the futures positions capture futures demand that is relevant for asset price behavior.

Panel A: Term Structure of Expected Cumulative Returns								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Belief</i>	1.57 (0.31)	0.59 (0.39)	-0.32 (0.47)	-2.46 (0.71)	0.05 (0.05)	-0.31 (0.06)	-1.03 (0.08)	-1.79 (0.11)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	259	259	259	259	10137	10137	10137	10137
<i>R</i> ²	0.14	0.01	0.01	0.09	0.00	0.00	0.02	0.04

Panel B: Short term Peaks and Troughs				
	<i>ST Peak</i>	<i>ST Trough</i>	<i>ST Peak</i>	<i>ST Trough</i>
<i>HO Belief</i>	0.41 (0.07)	-0.28 (0.04)		
<i>Overvaluation</i>			0.65 (0.12)	-0.06 (0.15)
Time FE	NA	NA	NA	NA
<i>N</i>	259	259	259	259
<i>R</i> ²	0.22	0.16	0.26	0.00

TABLE 3: HIGHER ORDER BELIEFS AND RETURN EXPECTATIONS

Note: Panel A of the table reports results from regressions of cumulative return expectations on the *Higher Order Belief* variable constructed from the Shiller surveys, pooling together observations across individual and institutional investors. Each column, labeled $\mathbb{E}_t(R_{t,t+k})$ represents cumulative return expectations in month t for returns from month t to month $t+k$. The unit of observation in the first four columns is the monthly cross-sectional average of survey responses. Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observation for the last four columns are individual survey responses. Driscoll-Kraay standard errors (12 lags) of coefficients are reported in parentheses. Panel B of the table reports regressions of *Short Term Peak* and *ST Trough* on the *HO Belief* and *Overvaluation* measures. The variable *ST Peak* is constructed from question (ii.c), which asks whether investors expect markets to eventually fall but reach a peak in the near term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The variable *ST Trough* is constructed from question (ii.d), which asks whether investors expect markets to eventually rise but reach a trough in the near term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The units of observation in the regressions are monthly cross-sectional averages of the variables. Newey-West standard errors (12 lags) of coefficients are reported in parentheses.

sure), as well as changes in the return expectations of fund managers. That is, both individual and institutional investors' expectations may be important for explaining changes in futures positions.

Building on the previous literature, we construct $\Delta Net Positioning$ as the change in the number of short contracts minus the number of long contracts held by dealers, normalized by lagged open interest. Futures contracts are in zero net supply, and dealers meet the futures demand of other investor types, so $\Delta Net Positioning$ is a measure of the change in net long demand for equity market exposure by buy-side investors (Hazelkorn, Moskowitz and Vasudevan (2023)).¹¹ We standardize $\Delta Net Positioning$ to have zero mean and unit standard deviation. We run contemporaneous regressions of quarterly changes in $Net Positioning$ on changes in the cross-sectional average of investors' return expectations in the same period as the independent variables.

Table 4 reports the regression results. The coefficient on 1-month return expectations is 0.27 (standard error of 0.08, R^2 of 0.16) in the univariate regression, and 0.20 (standard error of 0.09) in the multivariate regression. The coefficient on 3-month return expectations is 0.25 (standard error of 0.07, R^2 of 0.12) and 0.14 (standard error of 0.10) in the multivariate regression. The coefficients for 6-month return expectations are 0.22 (standard error of 0.12) and 0.02 (standard error of 0.13), while the coefficients for 12-month return expectations are 0.12 (standard error of 0.10) and 0.04 (standard error of 0.11). The evidence indicates that return expectations over the next 1-3 months are strongly related to investors' futures positions, while return expectations over the next 6-12 months are weakly related to investors' futures positions.¹²

We also examine the joint dynamics of return expectations and futures positions. Figure 2 plots the impulse response functions of net positioning and return expectations to return expectation shocks. The top two panels of the figure plot impulse responses of quarterly net positioning and 1-month return expectations to an AR(1) innovation in 1-month return expectations. The bottom two panels plot impulse responses of quarterly net positioning and 12-month return expectations to an AR(1) innovation in 12-month return expectations. The dynamics of investors' futures positions closely track their 1-month ahead return expectations. In comparison, there is little relationship between the dynamics of 12-month return expectations and investors' futures positions.¹³

One interpretation of the results is that the survey data reflect buy side investors' expectations, which are accordingly reflected in their positions. Under this interpretation, the

¹¹Dealers generally do not take on equity market exposure, but hedge their futures positions by trading in the spot market, as discussed in Hazelkorn, Moskowitz and Vasudevan (2023).

¹²One explanation for the stronger relationship using quarterly observations is that investors may trade on changes in their beliefs with a lag.

¹³Breaking down the results by investor types in the appendix, we find a particularly close relationship between 1-month return expectations and institutional asset managers' (e.g., mutual funds') futures positioning.

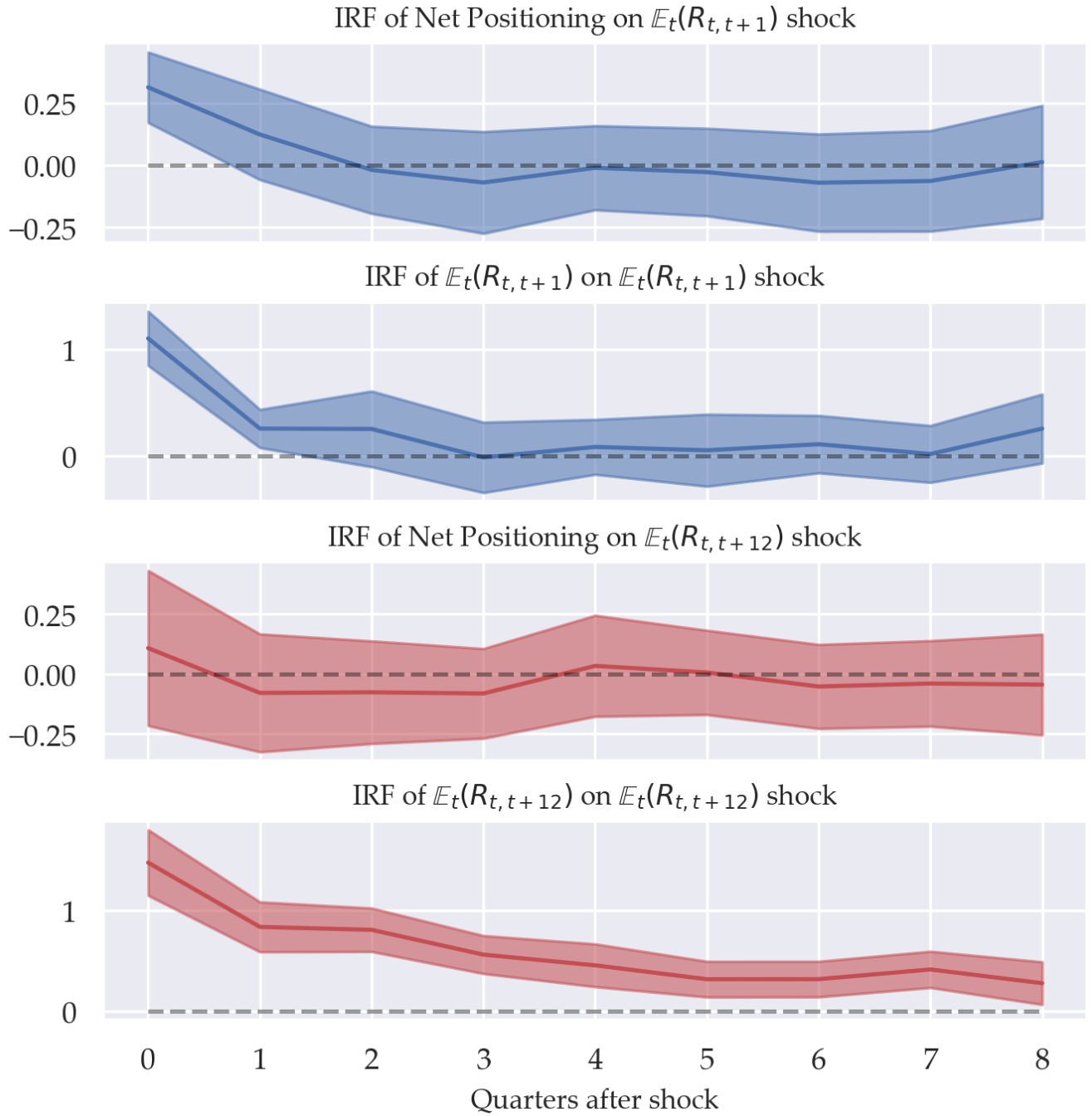


FIGURE 2: FUTURES POSITIONS AND RETURN EXPECTATIONS

Note: The figure plots impulse responses of Net Positioning, 1-month, and 12-month return expectations in response to AR(1) return expectation innovations. Innovations are standardized to have zero mean and unit standard deviation. Net positioning is the change in net positioning from the quarter before the shock to the given period, normalized by open interest, and standardized to have zero mean and unit standard deviation. Return expectations are the change in the cross-sectional average of investors' return expectations from the quarter before the shock to the given period, in percentage points. Standard errors are Newey-West standard errors (4 lags). The figure also plots plus and minus two standard errors for the estimated coefficients.

	(1)	(2)	(3)	(4)	(5)
$\mathbb{E}_t(R_{t,t+1})$	0.27 (0.08)				0.20 (0.09)
$\mathbb{E}_t(R_{t,t+3})$		0.25 (0.07)			0.14 (0.10)
$\mathbb{E}_t(R_{t,t+6})$			0.22 (0.12)		0.02 (0.13)
$\mathbb{E}_t(R_{t,t+12})$				0.12 (0.10)	0.04 (0.11)
R^2	0.16	0.12	0.10	0.04	0.21
N	67	67	67	67	67

TABLE 4: RETURN EXPECTATIONS AND INVESTOR FUTURES POSITIONS

Note: The table reports results from regressions of changes in investors' futures positions on changes in return expectations. Our measure of futures positions is $\Delta Net Positioning$, defined as the change in the number of short minus long S&P500 futures contracts held by futures dealers in aggregate, normalized by lagged open interest. Data are from the Traders in Financial Futures report. Return expectations are the average return expectations in a given period from the Shiller survey. Panel A corresponds with monthly changes and Panel B corresponds with quarterly changes. Newey-West standard errors (12 lags for monthly, 4 lags for quarterly observations) of coefficients are reported in parentheses.

results indicate that investors' short term return expectations lead them to engage in non-fundamental speculation, and they increase their positions because of perceived short term profits. While our evidence is consistent with such an interpretation, we are also cautious, in that we cannot link the identity of survey respondents with their trades.

Our results also may help explain a puzzle posed in the literature, that investors' trading is surprisingly insensitive to changes in their 1-year expected returns (Giglio et al. (2021)).¹⁴ Our results similarly indicate a weak relationship between changes in investor positioning and 1-year ahead return expectations, which may be driven by the fact that investors often expect high returns at the one year horizon but low returns at shorter horizons, or vice-versa. In the presence of such misalignment of short and long horizon return expectations, investors' speculative trading motivations weaken the contemporaneous relationship between changes in long term return expectations and trading, consistent with our results.

¹⁴In an experimental setting, Beutel and Weber (2022) find much stronger evidence of pass through of beliefs into risky asset shares. They attribute the wedge between their results and previous results as coming from endogeneity concerns in measuring the relationship between beliefs and portfolios in non-experimental settings.

1.6 What Drives Higher Order Beliefs and Return Expectations?

Given the observed time-series relationship between investors' reported higher order beliefs and return expectations, we next seek to better understand the causes and drivers of these beliefs. We find macroeconomic news to be a key driver, with positive macroeconomic news increasing *HO Belief* and short term return expectations while decreasing long term return expectations.

We use two measures of macroeconomic news in our analysis. The first is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, which is a composite index of 10 leading macroeconomic indicators.¹⁵ The second is quarterly AR(1) innovations in discussion of recessions in the Wall Street Journal, from [Bybee et al. \(2021\)](#).¹⁶ The first measure corresponds with positive macroeconomic news, while the second corresponds with negative macroeconomic news.

We regress quarterly changes in the cross-sectional averages of *HO Belief*, *Overvaluation*, and return expectations of different horizons on the measures of macroeconomic news. We standardize the independent variables, and changes in *HO Belief* and *Overvaluation*, to have zero mean and unit standard deviation. The coefficients for return expectations can be interpreted as the change in expected returns (in percentage points) corresponding with a one standard deviation innovation to the independent variable, and the coefficients for the other dependent variables can be interpreted as correlation coefficients.

Figure 3 plots the regression coefficients. With innovations to leading economic indicators as the independent variable, the coefficient on 1-month return expectations is 0.49, indicating that a one standard deviation innovation corresponds with a 49 basis point higher return expectation for the next month. The coefficients for 3-, 6-, and 12-month return expectations are 0.09, -0.21, and -0.44, indicating that investors lower their return expectations for the next year contemporaneous with the arrival of positive macroeconomic news. Innovations to the leading economic indicators index are 0.45 correlated with changes in *HO Belief* and *Overvaluation*, indicating that substantial variation in investors' reports that others are overly optimistic and that markets are overvalued occurs in quarters with positive macroeconomic news. With innovations to recession attention as the independent variable, the coefficients on 1-, 3-, 6-, and 12-month return expectations are -0.73, -0.21, -0.08, and 0.18, indicating expectations of strong negative short term performance that will revert in the future. Innovations to recession attention are -0.40 and -0.24 correlated to *HO Belief* and *Overvaluation*.

We can interpret the evidence as follows: in quarters with positive macroeconomic news,

¹⁵The data are provided as an index – we construct innovations in the percent change in the index. We lag observations by one month to ensure that innovations are in investors' information sets. We also report results in the appendix using coincident business cycle indicators, and find similar results.

¹⁶[Bybee et al. \(2021\)](#) find that discussion of recessions has substantive predictive power for macroeconomic outcomes. [Bybee, Kelly and Su \(2023\)](#) find that the recession topic has substantial power to explain risk premia as an asset pricing factor. We use an updated series from the authors, which contains data through January 2021.

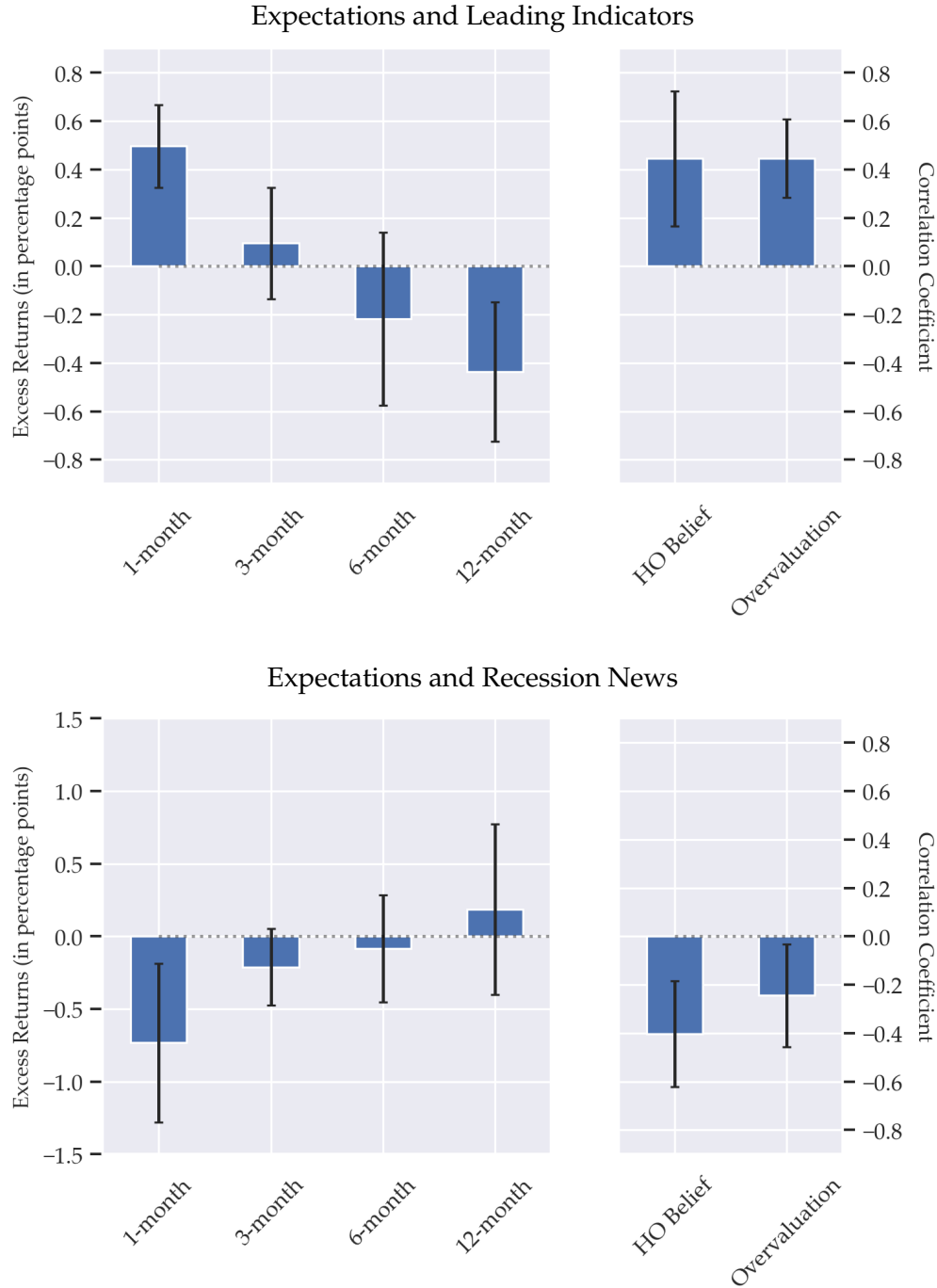


FIGURE 3: MACROECONOMIC NEWS, RETURN EXPECTATIONS, AND HIGHER ORDER BELIEFS

Note: The figure plots coefficients from contemporaneous regressions of changes in quarterly average 1-,3-,6-, and 12-month return expectations, *HO Belief*, and *Overvaluation* on measures of macroeconomic news. The measure of macroeconomic news in the first panel is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, which is a composite index of 10 leading macroeconomic indicators. The measure of macroeconomic news in the second panel is AR(1) innovations in attention paid to recession news in the Wall Street Journal from [Bybee et al. \(2021\)](#). The independent variables, *HO Belief* and *Overvaluation*, are scaled to have zero mean and unit standard deviation, and return expectations are multiplied by 100. Standard errors are Newey-West standard errors (4 lags). The figure also plots plus and minus two standard errors for the estimated coefficients.

the stock market tends to appreciate. The contemporaneous quarterly return associated with a one standard deviation shock to leading indicators is 2.09%. Investors perceive that in the month following macroeconomic news, returns will be 49 basis points higher, but that in the subsequent 11 months, this short term return will entirely revert, and further, that returns will be lower by nearly a quarter of the contemporaneous response to the news. That is, the evidence is consistent with the 2.09% return reflecting overreaction to news, with investors having a sense that the initial reaction is about 25% larger than justified by fundamentals.

The evidence from these time-series regressions and the accompanying interpretation are further augmented by responses in the survey that do not rely on regressions from a single time-series. In particular, the evidence is consistent with investors' direct responses when asked about the drivers of stock markets in question (iii.a). Unconditionally, 52% of individual investors and 37% of institutional investors answer that the cause of the 6-month market trend is overreaction and speculative thinking by other investors.¹⁷ Additionally, the interpretation of investors' perception of overreaction is also consistent with investors' direct responses to a hypothetical situation in question (ii.e) – when asked how they expect the market to perform following a 25% drop in the next 6 months, investors report expectations of reversals of 13.6% to 16.9%.

1.7 Return Expectations and Realizations

Our evidence suggests that investors' decisions are driven by their short horizon return expectations. Table 5 illustrates the performance associated with investors' 1-month return expectations. The first row displays coefficients from regressions of realized returns on 1-month return expectations; coefficients are negative for the pooled, individual, and institutional sample. The coefficient in the pooled regression is statistically significant at the 10% level. The table displays the Sharpe ratio of a market-timing strategy that takes long and short positions in the market in proportion to the average return expectation of investors in the survey. The Sharpe ratios are -0.29, -0.32, and -0.15. While the sample is somewhat limited in length, the evidence suggests that investors' short horizon return expectations are often wrong and that short term speculation is unprofitable. Moreover, the evidence is in line with results found in survey data of longer time samples, e.g., [Greenwood and Shleifer \(2014\)](#), that investors' return expectations tend to be negatively predictive of future returns.

1.8 Summarizing the Evidence and Implications for Theory

We can summarize the evidence presented in this section as follows:

¹⁷The true proportions are a bit higher; several respondents select 'other,' and choose to fill in custom responses that indicate a view that stock prices are driven by others' overreaction or speculation acting in conjunction with additional forces, such as monetary policy.

	Pooled	Individual	Institutional
Coefficient	-0.32 (0.19)	-0.26 (0.16)	-0.15 (0.17)
Mkt Timing Sharpe	-0.29	-0.32	-0.15

TABLE 5: RETURN EXPECTATIONS AND REALIZED RETURNS

Note: The top row of the table displays coefficients from a regression of 1-month realized returns on investors' return expectations for the same period. Newey-West standard errors (12 lags) of coefficients are reported in parentheses. The last row of the table reports the Sharpe ratio of a market timing strategy that takes long and short positions in the stock market in proportion to the average respondent's return expectations for the next month.

- (i) **(Non-fundamental speculation).** In time periods when investors perceive others to be overly optimistic and markets to be overvalued, they forecast short term returns to be high and long term returns to be low. Speculators seek to 'ride-the-bubble' and buy into an overvalued stock market.
- (ii) **(Overreaction).** Investors perceive that other investors become overly optimistic, and stock markets become overvalued with the arrival of fundamental news. That is, the stock market displays initial overreaction to fundamental news.
- (iii) **(Time-series momentum and reversal).** Investors forecast that the stock market exhibits momentum and reversal in response to fundamental news. In periods following positive fundamental news, investors forecast positive returns in the short term and negative returns multiple periods into the future.

The results provide systematic evidence from survey data that for the U.S. stock market, non-fundamental speculation is the rule not the exception, where previous work documents such behavior in particular episodes (e.g., [Brunnermeier and Nagel \(2004\)](#), [Temin and Voth \(2004\)](#)). Investors believe in patterns such as overreaction-driven momentum and reversal, but choose to buy into the stock market when they perceive it to be driven up by overly optimistic investors, because they see short term profits from doing so. Additionally, in episodes previously highlighted where informed investors profited from their non-fundamental speculation, our evidence suggests that non-fundamental speculation is unprofitable for the investors studied.

Below, we discuss mechanisms presented in the literature, and how they may help explain the results. A belief that other investors make systematic errors in forecasting fundamentals is sufficient to explain the results. Two other mechanisms highlighted in the literature – higher

order uncertainty, and beliefs that other investors may form their return expectations on the basis of past price changes – may also help explain the results.

Higher Order Uncertainty. Higher order uncertainty – uncertainty about whether other investors agree with one’s beliefs – is a form of higher order beliefs that has received particular attention in the literature. In the presence of higher order uncertainty, investors with short investment horizons may not trade fully towards their beliefs, because of uncertainty regarding whether prices will reflect their beliefs in the near future. In one strand of literature, higher order uncertainty leads asset prices to underreact to fundamental news upon its arrival, and monotonically drift towards fundamental values in subsequent periods. This can happen even when the average belief is an unbiased estimate of fundamentals ([Allen, Morris and Shin \(2006\)](#), [Banerjee, Kaniel and Kremer \(2009\)](#)).¹⁸ However, such uncertainty does not capture the patterns of expectations we document, namely that asset prices initially overreact to news, and that they continue to overreact before reversing.

A form of higher order uncertainty that generates synchronization risk can help explain our results. In [Abreu and Brunnermeier \(2002, 2003\)](#), arbitrageurs become aware of mispricing sequentially, and have higher order uncertainty about other arbitrageurs’ awareness of the mispricing. With short investment horizons and the need for coordination to correct mispricings, arbitrageurs may engage in non-fundamental speculation and hold long positions in a stock market they see as overvalued.

In and of itself, however, this form of uncertainty is not sufficient to explain the patterns we observe, and requires mispricing to arise and persist from an exogenous set of investors. In the model that we present, the source of mispricing is endogenized as the investors engaged in non-fundamental speculation; such an assumption is also consistent with the poor performance associated with non-fundamental speculation.

Self-fulfilling expectations. Another form of higher order beliefs that can produce asset price fluctuations comes in the form of self-fulfilling beliefs (e.g., [Khorrami and Mendo \(2021\)](#), [Gârleanu and Panageas \(2021\)](#) and [Zentefis \(2022\)](#)). Models of self-fulfilling expectations feature multiple equilibria, with fluctuations driven by investors’ coordination on a particular equilibrium. Here, we find that investors systematically report that other investors have mistaken beliefs and report return expectations that are systematically negatively correlated with realized returns, features that are not present in models of self-fulfilling expectations. While self-fulfilling expectations may play a role more generally, they are insufficient to explain the results for our sample of investors.

Return Extrapolation. Recent work emphasizes that return extrapolation – investors

¹⁸In [Allen, Morris and Shin \(2006\)](#), noisy signals about fundamentals lead to underreaction of prices to fundamental news, though [Banerjee, Kaniel and Kremer \(2009\)](#) note that if investors learn from prices, this effect may disappear. [Banerjee, Kaniel and Kremer \(2009\)](#) argue that price drift can reliably occur when investors agree-to-disagree about fundamental values, but have uncertainty about other investors’ higher order beliefs. [Banerjee, Kaniel and Kremer \(2009\)](#) also note that in noisy rational expectations models, speculation must be driven by information orthogonal to public information; we document speculation in response to public information.

forming their expected returns based on past returns – may explain investor behavior. Our empirical results provide some evidence consistent with return extrapolation: investors’ short term return expectations are correlated with past returns. However, without additional assumptions, return extrapolation cannot speak to investors’ multi-period return expectations and perceptions of others’ beliefs.

Interestingly, beliefs that other investors return extrapolate can lead to non-fundamental speculation. In an economy populated by non-speculative fundamental traders, return extrapolators, and rational speculators, the rational speculators can push prices to overreact to fundamental news due to a recognition that such speculation may trigger future excitement by return extrapolators (De Long et al. (1990)).¹⁹ While the rational speculators in the model of De Long et al. (1990) would report return expectations and valuations consistent with the survey responses we observe, we would not expect them to report that they increasingly find others to be overly optimistic contemporaneous with positive news shocks (as in the data). The speculators in De Long et al. (1990) expect excess optimism to arise from return extrapolators in the periods following news. Moreover, non-fundamental speculation is profitable for these investors. Beliefs in other investors’ return extrapolation may help explain the patterns, but existing models do not fully match the evidence.

Errors in Forecasting Fundamentals. A growing literature has suggested that investors may make systematic mistakes in forecasting asset price fundamentals (Chen, Da and Zhao (2013), Bordalo et al. (2020), De La O and Myers (2021), Nagel and Xu (2022a)). In the absence of assumptions about higher order beliefs, such theories can make vastly different predictions about return expectations. For example, if investors with mistaken fundamental beliefs believe all other investors share their beliefs, they always expect constant returns in the absence of time-varying risk premia, inconsistent with our evidence.

However, a belief in other investors making errors in forecasting fundamentals can explain the evidence. In the model we present in the next section, investors believe that other investors persistently update their beliefs in the direction of past news, and overestimate the persistence of fundamentals. These belief updating biases are effectively identical to the ones in Angeletos, Huo and Sastry (2021), who show that these frictions help match macroeconomic forecasters’ expectations.²⁰ As we further discuss in an extension of the model, however, this is not the only explanation consistent with the evidence, which can also be rationalized, for example, by investors believing that other investors believe that *other* investors make fundamental belief mistakes.

Martin and Papadimitriou (2022) propose a model where investors have differences-of-

¹⁹If fundamental traders and return extrapolators are the only investors in the economy, and there is no third type, as in Barberis et al. (2018), the fundamental traders would believe that stock markets exhibit initial underreaction rather than believing in initial overreaction (as the investors in the Shiller surveys do).

²⁰Valente, Vasudevan and Wu (2021) use a model based on Angeletos, Huo and Sastry (2021), to match survey data on investors’ interest rate expectations in order to understand puzzles in foreign exchange markets.

opinion about the (fixed) fundamental value of an asset. Investors that are correct in hindsight become wealthier, and so the belief and identity of the representative agent becomes more optimistic following good news and more pessimistic following bad news. Investors internalize the role of this shifting ‘sentiment,’ and may engage in non-fundamental speculation. Investors’ beliefs regarding the shifting relative wealth of optimists and pessimists in response to news may also help explain the facts we document.

Investor Sentiment. More broadly, a voluminous body of work studies *investor sentiment*, defined in a literature review of the topic by [Baker and Wurgler \(2007\)](#) as “a belief about future cash flows and investment risks that is not justified by the facts.” Investor sentiment may encompass both errors in forecasting fundamentals and errors in forecasting returns.²¹ Work on investor sentiment, while related to our results, cannot directly speak to the reason that investors buy into overvalued markets – is it because they are overly optimistic about the asset’s fundamentals, or because of their forecasts of other investors’ future behavior? In our model, we discuss the roles that each of these might play and their potential interactions, which can help decompose the drivers of sentiment.

Other Models. The previously discussed forms of higher order beliefs – higher order uncertainty and beliefs about other investors’ fundamental and return expectations – may help explain the results. We briefly discuss other common models in the literature.

Expectations of time-varying returns are commonly attributed to time-varying risk premia ([Campbell and Cochrane \(1999\)](#), [Bansal and Yaron \(2004\)](#), [Gabaix \(2008\)](#), [Wachter \(2013\)](#)). Such models typically assume a representative agent with correct beliefs, and accordingly struggle to confront the evidence that other investors hold incorrect expectations, and that the stock market is mispriced. They also generally predict that risk premia and return expectations decrease contemporaneously with positive news (which makes agents less risk-averse), whereas we find evidence that investors’ short term return expectations and equity market exposures increase contemporaneously with positive news.²² Moreover, these models predict that investors’ return expectations should be positively related to future returns, which we do not find to be the case.

A common explanation for non-fundamental speculation is that investors may face short sale costs and constraints ([Harrison and Kreps \(1978\)](#), [Scheinkman and Xiong \(2003\)](#), [Duffie, Garleanu and Pedersen \(2002\)](#)). The presence of short sale costs leads pessimistic investors to avoid taking short positions, and asset prices reflect the valuations of optimistic investors, plus the added benefit they receive from the possibility of reselling to more optimistic investors that arrive in the future. In our setting – the aggregate U.S. stock market – short sale constraints are unlikely to be binding for most investors.²³ Moreover, short sale constraints

²¹[Baker and Wurgler \(2007\)](#) cite examples of both in their literature review.

²²Though, with specific parametrization of preferences and the cash flow process, models of time-varying risk premia may be able to generate hump-shaped pattern of return expectations in response to fundamental news.

²³It is essentially costless to short the stock market via futures ([Hazelkorn, Moskowitz and Vasudevan \(2023\)](#)).

alone cannot explain the cyclical patterns of return expectations we document. Short sale constraints likely contribute to our results, but may not be the primary driver, and are not a sufficient explanation.

2 A Model of Non-Fundamental Speculation

In this section, we present a stylized asset pricing model. The model provides a framework for interpreting and understanding the empirical results, clarifying the relationship between return expectations and higher order beliefs, and illustrating the impact that investors' higher order beliefs have on asset prices in general equilibrium.

In our baseline specification, a group of speculative traders trade a risky asset (the stock market) against a mass of fundamental investors that 'lean against the wind' – they buy when the market is undervalued and sell when the market is overvalued. The average fundamental belief of both groups matches the rational expectations belief, so the model allows us to isolate the effect of higher order beliefs on asset prices. We capture the empirical evidence with speculators having higher order beliefs that other investors overestimate the persistence of fundamentals governing the payoffs of the risky asset. We find that speculators' higher order beliefs and the accompanying non-fundamental speculation amplify asset price overreaction and excess volatility.

As an expositional note, for convenience, we discuss the risky asset becoming overvalued and investors having positive return expectations following positive news. However, the model is symmetric, in the sense that it also produces undervaluation and lower than average return expectations following negative news, consistent with the survey evidence.

2.1 Baseline Model Setup

We begin by laying out the model environment and describing investors' beliefs.

2.1.1 Model Environment

There is a risky asset (the stock market) in zero net supply, and a riskless asset in perfectly elastic supply. The payoff of the riskless asset is normalized to zero. The asset pays a dividend D_t each period, where D_t evolves according to the process

$$\begin{aligned} D_t &= \mathbf{d}_t + v_t, \text{ where } v_t \sim N(0, \sigma_v^2), \text{ and} \\ \mathbf{d}_t &= \rho \mathbf{d}_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma_\epsilon^2). \end{aligned} \tag{1}$$

The term \mathbf{d}_t captures the persistent component of dividends, which we refer to as the asset's fundamentals, while v_t captures a transitory component of dividends. While dividends are observed each period, the underlying fundamentals are never revealed to investors.

The model follows an overlapping generations structure. Each period, a unit mass of individually infinitesimal investors is born, indexed by $i \in [0, 1]$. Investors born in period t make an investment decision in that period. In period $t + 1$ they liquidate their investments, consume the proceeds, and pass their beliefs onto the newly born investor i . The assumption of overlapping generations is common in work on higher order beliefs (e.g., [Allen, Morris and Shin \(2006\)](#)), which serves to accentuate the importance of short term price movements for traders.

In period t , in addition to observing the publicly announced dividend, D_t , each investor i also receives a private signal,

$$\begin{aligned} s_t^i &= s_t + \phi_t^i, \text{ where} \\ s_t &= \mathbf{d}_t + \eta_t, \\ \eta_t &\sim N(0, \sigma_\eta^2), \text{ and } \phi_t^i \sim N(0, \sigma_\phi^2). \end{aligned}$$

Each investor's private signal contains a common component that is informative about fundamentals, s_t , as well as idiosyncratic noise, ϕ_t^i . We later provide additional structure on how investors treat these signals in forming their higher order beliefs.

There are two types of investors: Speculators, with mass θ , indexed by $i \in [0, \theta]$; and Fundamental investors, with mass $(1 - \theta)$, indexed by $i \in [\theta, 1]$. Investors of each type share beliefs about the parameters governing the economy with other investors of the same type.

Speculators, the primary focus of our study, trade based on their one-period ahead subjective expected return. Their return expectations are determined by their beliefs about fundamentals and their higher order beliefs, as we detail further. Speculator i 's demand is given by

$$Q_t^s = \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t)}{\gamma \sigma^2}, \quad (2)$$

where γ is the coefficient of risk-aversion, σ^2 is the volatility of the asset return, which we treat as exogenous, and $\mathbb{E}_t^i(\cdot)$ is the subjective expectations operator for investor i .²⁴

Fundamental investors are boundedly rational. They correctly perceive the parameters of the economy in period t when forming their expectations, but they trade as if all investors born in period $t + 1$ will share their beliefs. This form of bounded rationality is standard in other work (for example, see [De Long et al. \(1990\)](#); [Xiong \(2001\)](#); [Barberis et al. \(2018\)](#)), and leads fundamental investors to purchase the risky asset in period t when the price is below their fundamental valuation and they sell when the price is above their fundamental

²⁴We do not lose generality with exogenous treatment of volatility. Differences in speculators' and fundamental investors' perceived volatility can simply be represented as shifts in the proportion of speculators, θ .

valuation.²⁵ Fundamental investor i 's demand in period t can be expressed as

$$Q_t^i = \frac{\left(\frac{\rho}{1-\rho} \mathbb{E}_t^i(\mathbf{d}_t) - P_t\right)}{\gamma \sigma^2}. \quad (3)$$

The market clearing condition is given by $0 = \int_0^1 Q_t^i di$.

2.1.2 Investors' Beliefs

Investors are Bayesian in forming their beliefs about fundamentals, \mathbf{d}_t . Using their beliefs about the dividend process and their observations of past dividends, they form their expectations of \mathbf{d}_t by Kalman filtering. We follow the common assumption that a sufficient number of periods have passed such that investors are in a learning steady state. This means that investors' Kalman gain – the weight they place on new information that arrives in period t versus their prior in their fundamental beliefs – is constant each period.

Before presenting the exact formulation of investors' belief updating, we make an assumption about how investors process their own and other investors' signals.

Assumption 1 (Differences-of-opinion) *The noise term in investor i 's private signal, ϕ_t^i , is an idiosyncratic interpretation that i imputes to the informative component of s_t . Investors treat other investors' signals as being uninformative about fundamentals conditional on their own private signals. When updating their beliefs about \mathbf{d}_t , investor i treats their private signal s_t^i as if it has variance σ_η^2 .*

The assumption that investors treat others' signals as uninformative follows in the spirit of 'differences-of-opinion' models (e.g., [Harris and Raviv \(1993\)](#), [Kandel and Pearson \(1995\)](#), [Banerjee and Kremer \(2010\)](#)).²⁶ Given Assumption 1, investor i perceives that the average signal received by other investors, $s_t^{-i} \equiv \mathbb{E}_t^i \int s_t^j dj$, is a biased signal about fundamentals. In a variant of the model, we relax this assumption, and allow investors to treat others' signals as informative.²⁷ With this assumption in hand, Lemma 1 outlines how investors' fundamental beliefs evolve.

²⁵In principle, these investors could be considered analogous to large institutional asset managers that make equity allocations based on capital market assumptions of equity returns that appear close to correct on average (as studied in [Dahlquist and Ibert \(2022\)](#)).

²⁶Differences-of-opinion means that our model also match the cross-sectional evidence on investors' return expectations, and helps rationalize the substantial belief heterogeneity in return expectations found in other work (e.g., [Giglio et al. \(2021\)](#)), though these are not our main focus. Microfoundations for differences of opinion may include overconfidence ([Odean \(1998\)](#), [Daniel and Hirshleifer \(2015\)](#)), or motivated reasoning ([Banerjee, Davis and Gondhi \(2021\)](#)).

²⁷The treatment of s_t^i as having variance σ_η^2 , rather than $\sigma_\eta^2 + \sigma_\phi^2$, means that the average fundamental belief is an unbiased signal of \mathbf{d}_t . In the alternative case, the average fundamental belief underreacts to news, as in models of noisy rational expectations ([Woodford \(2001\)](#) and [Sims \(2003\)](#)). This would not meaningfully affect our results, but we shut down this channel to clearly isolate the role of higher order beliefs in driving asset price fluctuations.

Lemma 1 (Fundamental Beliefs) *Investor i 's beliefs about fundamentals, \mathbf{d}_t , evolve according to the updating process*

$$\mathbf{d}_t^i \equiv \mathbb{E}_t^i(\mathbf{d}_t) = (1 - \kappa_1 - \kappa_2)\rho\mathbf{d}_{t-1}^i + \kappa_1 D_t + \kappa_2 s_t^i, \quad (4)$$

where

$$\begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} = \Sigma H (H \Sigma H^T + R)^{-1}, H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}, \text{ and} \\ \Sigma = \rho^2 \Sigma - \rho^2 \Sigma H^T (H \Sigma H^T + R)^{-1} H \Sigma + \sigma_\epsilon^2.$$

Proof. Given the linear-Gaussian environment, belief updating follows the standard Kalman filter recursion method derived in [Hamilton \(2020\)](#). \square

Investors update their beliefs in response to new information based on the signal-to-noise ratio of D_t and s_t . When these signals are informative about dividends, investors give them additional weight (higher κ_1 and κ_2), whereas they rely more on their priors when these signals are less informative. For notational convenience, we denote investor i 's beliefs about fundamentals as $\mathbf{d}_t^i = \mathbb{E}_t^i(\mathbf{d}_t)$.

We next make two assumptions about investors' higher order beliefs that form the core frictions in our model.

Assumption 2 (Second Order Beliefs) *Speculators believe that all other investors misperceive the persistence of fundamentals. That is, speculators' second order belief is that others perceive the persistence of fundamentals, ρ , as ρ_{sf} .*

Assumption 3 (Additional Higher Order Beliefs) *Speculators believe that all other investors are fundamental (non-strategic) traders. That is, they believe that others perceive that all other investors born in period t forecast that investors born in period $t + 1$ will share their beliefs about \mathbf{d}_t and their beliefs about the parameters governing the economy.*

Jointly, assumptions 2 and 3 characterize speculators' higher order beliefs. Assumption 2 pins down how speculators believe that other investors update their beliefs. In particular, speculator i 's second order belief is that the average fundamental belief in the economy evolves according to

$$\mathbf{d}_t^{i,sf} \equiv \mathbb{E}_t^i \int \mathbf{d}_t^j = (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_{t-1}^{i,sf} + \alpha\kappa_1 D_t + \alpha\kappa_2 s_t^{-i}, \quad (5)$$

where $\alpha > 0$, and $\begin{bmatrix} \alpha\kappa_1 & \alpha\kappa_2 \end{bmatrix}^T$ are speculators' second-order beliefs about Kalman gains, constructed using the expression in Lemma 1.²⁸ Assumption 3 specifies all higher than second

²⁸In our main specification, α does not impact the results, and can be simply thought of as reflecting the fact that perceived persistence is different than true persistence, which influences the speed with which other investors are perceived to learn. The term α is pinned down by ρ_{sf} , and governs the difference in the speed with which speculators perceive other investors learn from data due to misperceiving persistence.

order beliefs, with speculators treating other investors as non-strategic. This assumption can be thought to coincide with a form of level k thinking – where investors believe that other investors are less strategically sophisticated than they are. We later relax assumption 3 to capture a more general form of level k thinking.

2.1.3 Equilibrium

We next define equilibrium, and then derive expressions for beliefs and the risky asset price in equilibrium.

Definition 2.1 (Equilibrium) *An equilibrium in period t is a combination of a price, P_t , and beliefs, such that*

- (i) *Investor i 's demand, Q_t^i , maximizes their subjective expected utility;*
- (ii) *Markets clear ($\int_0^1 Q_t^i di = 0$); and*
- (iii) *Investors' (potentially incorrect) beliefs about fundamentals and higher order beliefs are consistent with the price they observe.*

Given its definition, we derive equilibrium in three steps:

- (i) We derive speculators' perceived pricing function for the risky asset, given their higher order beliefs.
- (ii) We derive the true pricing function for the risky asset.
- (iii) We specify investors' beliefs at the equilibrium that equate (i) and (ii).

Lemma 2 (Speculators' Perceived Pricing Function) *Speculators perceive the period t price as*

$$P_t = \frac{\rho_{sf}}{1 - \rho_{sf}} \mathbf{d}_t^{i,sf}. \quad (6)$$

Proof. Given their higher order beliefs, speculator i perceives each other investor j 's demand as $Q_t^j = \frac{\frac{\rho_{sf}}{1 - \rho_{sf}} \mathbb{E}_t^i(\mathbf{d}_t^j) - P}{\gamma \sigma_2}$. The market clearing condition is $\int Q_t^j = 0$. The perceived pricing function follows from plugging perceived demand into market clearing and solving for P_t . \square

Given how speculators perceive prices, we can derive the true pricing function for the economy.

Lemma 3 (Equilibrium Pricing Function) *The equilibrium pricing rule for the economy is given by*

$$P_t = a_1 \mathbf{d}_t^s + b_1 \mathbf{d}_t^{sf} + c_1 \mathbf{d}_t^f,$$

where

$$\begin{aligned} a_1 &\equiv \theta \frac{\rho(1 - (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})}{1 - \rho_{sf}}, \\ b_1 &\equiv \theta \frac{(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}^2}{1 - \rho_{sf}}, \\ c_1 &\equiv (1 - \theta) \frac{\rho}{1 - \rho}, \end{aligned}$$

$\mathbf{d}_t^s \equiv \frac{1}{\theta} \int_0^\theta \mathbf{d}_t^i$ is the average of speculators' first order beliefs, $\mathbf{d}_t^{sf} \equiv \frac{1}{\theta} \int_0^\theta \mathbf{d}_t^{i,sf}$ is the average of speculators' second order beliefs, and $\mathbf{d}_t^f \equiv \frac{1}{1-\theta} \int_\theta^1 \mathbf{d}_t^i$ is the average of fundamental investors' first order beliefs.

Proof. To forecast the price in period $t + 1$, speculator i forecasts the average belief in period $t + 1$, based on their forecast of \mathbf{d}_{t+1} and s_{t+1} :

$$\begin{aligned} \mathbb{E}_t^i(\mathbf{d}_{t+1}^{i,sf}) &= \mathbb{E}_t^i \int \mathbf{d}_{t+1}^j = (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_t^{i,sf} + \alpha\kappa_1 \underbrace{\mathbb{E}_t(D_{t+1})}_{=\rho\mathbf{d}_t^i} + \alpha\kappa_2 \underbrace{\mathbb{E}_t(s_{t+1}^{-i})}_{=\rho\mathbf{d}_t^i} \\ &= (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_t^{i,sf} + \alpha(\kappa_1 + \kappa_2)\rho\mathbf{d}_t^i. \end{aligned}$$

Speculator i 's expected period $t + 1$ payoff is

$$\begin{aligned} \mathbb{E}_t^i(D_{t+1} + P_{t+1}) &= \underbrace{\rho\mathbf{d}_t^i}_{=\mathbb{E}_t^i(D_{t+1})} + \underbrace{\frac{\rho_{sf}}{1 - \rho_{sf}}\mathbb{E}_t^i(\mathbf{d}_{t+1}^{i,sf})}_{\mathbb{E}_t^i(P_{t+1})} \\ &= \frac{\rho(1 - (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})}{1 - \rho_{sf}}\mathbf{d}_t^i + \frac{(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}^2}{1 - \rho_{sf}}\mathbf{d}_t^{i,sf}. \end{aligned} \quad (7)$$

Substituting speculators' expected payoffs into their demand function, $Q_t^i = \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t}{\gamma\sigma^2}$, plugging the speculator and fundamental traders' demand functions into the market clearing condition, and solving for P_t , we get that

$$\begin{aligned} P_t &= \int_0^\theta \left(\frac{\rho(1 - (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})}{1 - \rho_{sf}}\mathbf{d}_t^i + \frac{(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}^2}{1 - \rho_{sf}}\mathbf{d}_t^{i,sf} \right) + \int_\theta^1 \frac{\rho}{1 - \rho}\mathbf{d}_t^i \\ &= a_1\mathbf{d}_t^s + b_1\mathbf{d}_t^{sf} + c_1\mathbf{d}_t^f. \end{aligned}$$

□

Speculators' demand is based on their return forecasts. Their return forecasts, in turn, depend on their higher order beliefs and their forecasts of future dividends, which they believe will determine how prices will evolve. Alongside fundamental investors' first order beliefs,

these quantities feed into determining the market clearing price.

The final step in solving for equilibrium is determining beliefs such that the perceived price in Lemma 2 coincides with the true price in Lemma 3.

Lemma 4 (Investors' Equilibrium Beliefs) *Speculator i 's second order beliefs and extracted signals are given by*

$$\mathbf{d}_t^{i, sf} = \mathbf{d}_t^{sf} = \left(\frac{\rho_{sf}}{1 - \rho_{sf}} - b_1 \right)^{-1} \left(a_1 \mathbf{d}_t^s + c_1 \mathbf{d}_t^f \right), \text{ and}$$

$$s_t^{-i} = \frac{\mathbf{d}_t^{sf} - (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_{t-1}^{sf} + \alpha\kappa_1 D_t}{\alpha\kappa_2},$$

which are identical for each speculator.

Proof. Because they think others' signals are uninformative, all investors' fundamental beliefs are given by the updating equation in Lemma 1. Note that this means that $\mathbf{d}_t^s = \mathbf{d}_t^f = \mathbb{E}_t(\mathbf{d}_t)$, i.e., the average speculator's beliefs and fundamental investors' beliefs are equal, and are equal to the rational expectations belief about \mathbf{d}_t . This leaves investors' second order beliefs to solve for.

In equilibrium, prices from investors' perceived pricing rule must coincide with the observed price, so, from Lemmas 2 and 3,

$$\frac{\rho_{sf}}{1 - \rho_{sf}} \mathbf{d}_t^{i, sf} = a_1 \mathbf{d}_t^s + b_1 \mathbf{d}_t^{sf} + c_1 \mathbf{d}_t^f.$$

Since the right-hand side is fixed across investors, it follows that $\mathbf{d}_t^{i, sf} = \mathbf{d}_t^{sf}$ for all speculators i . Then, solving for \mathbf{d}_t^{sf} , we get the expression provided. The expression for s_t^{-i} follows from plugging the derived quantities into Equation (5). \square

Lemma 4 illustrates that in order to rationalize the price they observe with their higher order beliefs, investors use prices to infer what they believe other investors' beliefs to be given their perceived market equilibrium, even though they don't use prices to learn about fundamentals.²⁹

Lemmas 2, 3, and 4 are sufficient to characterize the equilibrium. We also present an alternative representation of the equilibrium pricing rule.

Lemma 5 (Fundamental Belief Representation of Equilibrium Prices) *We can write the equilibrium pricing function in period t as*

$$P_t = \frac{1 - \theta\rho - (1 - \theta\rho + \theta\alpha(1 - \rho)(\kappa_1 + \kappa_2))\rho_{sf}}{(1 - \rho)\rho_{sf}(1 - \theta(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})} \rho \mathbf{d}_t^s. \quad (8)$$

²⁹Second order beliefs are identical across speculators in our model. In models with a richer information structure, for example, where investors receive signals along different dimensions, announcements may result in polarized second order beliefs, with implications for trading volume (Kondor (2012)).

Proof. The expression follows from substituting $\mathbf{d}_t^f = \mathbf{d}_t^s$, and \mathbf{d}_t^{sf} from Lemma 4 into the equilibrium pricing function from Lemma 3, and simplifying. \square

Lemma 5 illustrates that the equilibrium price collapses to a scaled version of the average speculator's fundamental beliefs. This is a useful representation for understanding the behavior of equilibrium asset prices in the model, as discussed further.

2.2 Matching the Survey Evidence and Implications for Asset Prices

With the solved model in hand, we next turn to describing how the model can match the evidence on investors' expectations. We then proceed to exploring the equilibrium asset pricing implications of the model.

2.2.1 Matching the Evidence

In the context of the model, we can summarize the conditions required to match the empirical evidence:

- (i) **(Perceived overreaction)**: on average, given positive fundamental innovation $\epsilon_t > 0$, $P_t > \sum_{h=1}^{\infty} \rho^h \mathbf{d}_t^s = \frac{\rho}{1-\rho} \mathbf{d}_t^s$, i.e., P_t exceeds speculators' perceived sum of future dividends.
- (ii) **(Perceived time-series momentum and reversal)**: on average, given a positive fundamental innovation $\epsilon_t > 0$, $\mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) > 0$ (momentum); and $\lim_{h \rightarrow \infty} \mathbb{E}_t P_{t+h} + \sum_{j=1}^h \rho^j \mathbf{d}_t^s - P_t < 0$ (reversal).
- (iii) **(Non-fundamental speculation)**: speculators buy into a market they see as overvalued; this follows from the combination of (i) and (ii).

Based on these conditions, we can derive the parameter values under which the model can match the survey evidence:

Proposition 1 (Matching the Survey Evidence) *If speculators believe that other investors overestimate the persistence of fundamentals, $\rho_{sf} > \rho$, then the average speculator*

- (i) *perceives that the market overreacts to news;*
- (ii) *perceives that the market exhibits time-series momentum and reversal;*
- (iii) *and engages in non-fundamental speculation.*

Proof. See appendix. \square

In equilibrium, speculators expect other investors to overreact to the news they observe, as they believe that others overestimate the persistence of fundamentals. Since the economy is composed of speculators and fundamental investors with correct beliefs on average, the equilibrium price does not reflect as much overreaction as speculators expect for a given

piece of positive news. Hence, speculators infer that other investors must have received attenuated signals that led them to not fully incorporate the news.³⁰ This leads speculators to hold positive return expectations in spite of viewing the market as overvalued. Speculators willingly buy into a market they perceive has overreacted because they think it will overreact even more in the future, before subsequently reverting. Overvaluation and positive expected returns co-exist in equilibrium because speculators' positive expected returns are the cause of overvaluation.

Remark 1 (Fundamental Beliefs and Higher Order Beliefs) *Investors' higher order beliefs about others' belief mistakes in the model – that the average of fundamental expectations tend to underreact to shocks ($\mathbf{d}_t^{sf} < \mathbf{d}_t^s$ following a positive shock), and that other investors overestimate persistence, $\rho_{sf} > \rho$ – are consistent with the frictions that can explain the dynamics of forecast errors of macroeconomic fundamentals made by professional forecasters (Angeletos, Huo and Sastry (2021); Reis (2020)).*

2.2.2 Equilibrium Asset Pricing Implications

Having matched the survey evidence, we next turn to explore the equilibrium asset pricing implications.

Result 1 (Overreaction and Reversal in Equilibrium) *Whenever speculators engage in non-fundamental speculation, on average, asset prices overreact to news in period t and revert in subsequent periods.*

Proof. See appendix. □

Markets overreact in spite of the fact the average investor's belief is consistent with rational expectations, due to speculators' higher order beliefs. The logic of this result holds more generally than the particular setup where speculators believe all other investors to be fundamental investors, as higher return expectations lead to increased demand, which in turn amplifies overreaction and overvaluation. We further highlight this point when extending our model to more general level k thinking.

After they overreact contemporaneous with news, asset prices experience a gradual reversal, corresponding with speculators revising their beliefs about the average investor's valuation, $\frac{\rho_{sf}}{1-\rho_{sf}} \mathbf{d}_t^{sf}$. Speculators' initial excitement – that other investors would overreact even more, leading to short term profits – turns out to be incorrect, resulting in negative forecast errors of returns.

³⁰This attenuated signal can be understood as $\mathbf{d}_t^{sf} < \mathbf{d}_t^s$ following a positive shock. It follows immediately from the average speculator's positive expected returns. In particular, $E_t^s(P_{t+1} + D_{t+1} - P_t) > 0 \iff \rho d_t^s + \frac{\rho_{sf}}{1-\rho_{sf}} (\mathbb{E}_t^s(\mathbf{d}_{t+1}^{sf}) - \mathbf{d}_t^{sf}) > 0$, where $\mathbb{E}_t^s(\mathbf{d}_{t+1}^{sf}) = (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_t^{sf} + \alpha(\kappa_1 + \kappa_2)\rho\mathbf{d}_t^s$. In turn, this is true if and only if $\mathbf{d}_t^{sf} < \frac{\rho(1+\alpha\rho_{sf}(\kappa_1+\kappa_2)-\rho_{sf})}{\rho_{sf}(1+\alpha\rho_{sf}(\kappa_1+\kappa_2))}\mathbf{d}_t^s < \mathbf{d}_t^s$.

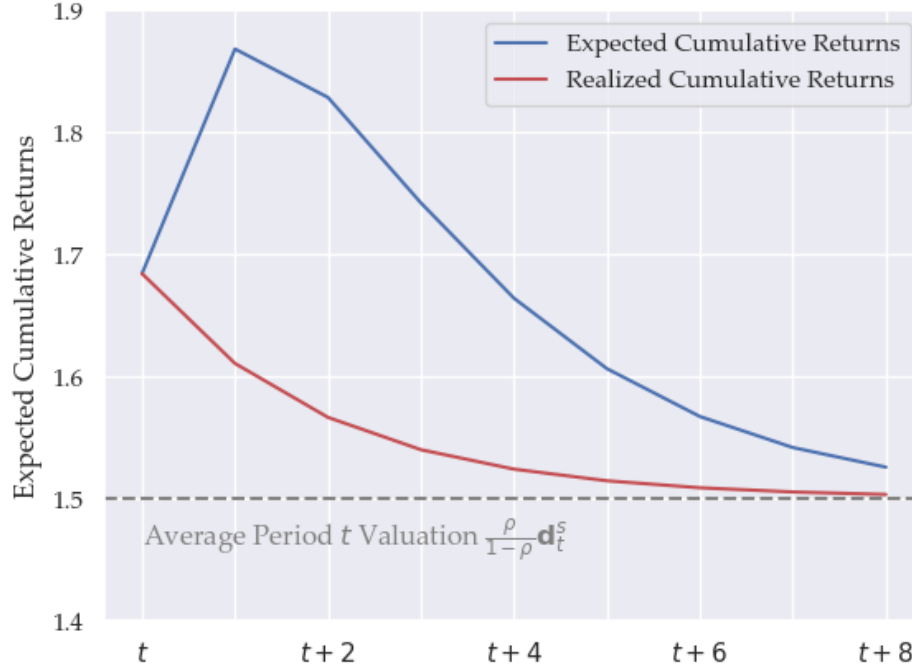


FIGURE 4: NON-FUNDAMENTAL SPECULATION, OVERREACTION, AND REVERSAL

Note: The figure plots the asset price in period t following a unit shock to \mathbf{d}_t^s starting from steady state. The blue line represents speculators' cumulative return expectations from period t to $t+h$. The red line represents the average realized cumulative returns from period t to $t+h$. The illustrative parameter values used are $(\theta, \rho, \rho_{sf}, \kappa_1, \kappa_2) = (0.5, 0.6, 0.7, 0.2, 0.2)$.

Figure 4 summarizes Proposition 1 and Result 1. The figure plots the price in period t following a unit shock to \mathbf{d}_t^s starting from steady state in $t-1$. The asset price exceeds the average speculator's valuation in period t . The figure displays the period t cumulative return expectations of the average speculator in blue. Though the asset price exceeds their valuation, speculators expect to earn even more positive returns in period $t+1$, though they expect the cumulative returns to revert to their buy-and-hold valuations in the long run. This pattern matches the analogous survey evidence. The figure plots the average realized returns in red. In the periods following the shock, the cumulative returns are negative, as asset prices revert.

We make two additional remarks to help interpret these results.

Remark 2 (Overreaction and Perceived Overreaction) *That asset prices overreact to news despite speculators' beliefs that other investors overreact is unique from the logic in static models (e.g., [Bastianello and Fontanier \(2022a\)](#)). In a static model, when investors believe that other investors overreact to news, they trade in the opposite direction of news, leading equilibrium asset prices to underreact. Here, in a dynamic model, speculators' forecasts of continued overreaction makes them seek to "ride the bubble" in spite of their belief in overreaction. In turn, this behavior gives rise to overreaction in equilibrium.*

Remark 3 (Interaction with Fundamental Beliefs) *Result 1 isolates the impact of higher order beliefs on equilibrium asset prices by assuming that the average belief about fundamentals \mathbf{d}_t^s matches rational expectations. Lemma 5 reveals the market clearing price can be written as a linear function of \mathbf{d}_t^s , and the proof of Proposition 1 similarly shows that the average speculator’s return expectations can be written as a linear function of \mathbf{d}_t^s . These facts suggest that return expectations (and investor demand) can be understood as driven by speculators’ higher order beliefs conditional on their fundamental expectations.*

Moreover, the behavior of equilibrium asset prices relies on the interaction of fundamental and higher order beliefs. While our focus is on higher order beliefs, empirical work using analysts’ expectations suggests that \mathbf{d}_t^s may depart from rational expectations, though there is disagreement about the exact nature of the departure (Chen, Da and Zhao (2013); Bordalo et al. (2020); De La O and Myers (2021); McCarthy and Hillenbrand (2021)). If \mathbf{d}_t^s sluggishly responds to dividend or earnings shocks, as suggested in models of sticky or noisy information (Mankiw and Reis (2002), Bouchaud et al. (2019)), then asset prices may display momentum in addition to overreaction and reversal. If fundamental beliefs overreact to news, as suggested in Bordalo et al. (2021), then asset price overreaction may be even stronger. Finally, we highlight that though empirical work often separately considers return expectations and fundamental expectations (e.g., McCarthy and Hillenbrand (2021); De La O and Myers (2021)), our analysis suggests the two are tightly linked, as empirically documented by (Giglio et al. (2021)), with higher order beliefs playing an important role.

2.3 Level k Thinking

We next generalize our model by embedding our baseline specification into a more general form of level k thinking, where investors may recognize that other speculators are strategic, but consider others to be less strategically sophisticated than themselves. This allows our model to more closely match the survey question on whether the cause of the trend in stock prices is ‘speculative thinking among other investors.’ In our more general specification, the main conclusion – that higher order beliefs lead to overreaction and reversals – still follows through under the same parameter restriction ($\rho_{sf} > \rho$). However, as we increase speculators’ depth of reasoning, asset prices converge towards the rational expectations fundamental value. Hence, the survival of non-fundamental speculation as an equilibrium outcome indicates limited strategic sophistication by investors.

Definition 2.2 (Level k Equilibrium) *A level 1 equilibrium is the equilibrium solved in the previous section, where the speculators are defined as level 1 speculators. For $k > 1$, a level k equilibrium features level k speculators who believe that the equilibrium structure is a level $k - 1$ equilibrium, i.e., they believe that all speculators are level $k - 1$ speculators.*

A level k speculator, for $k > 1$, correctly recognizes that a mass θ of investors are speculators, and believes that other speculators are level $k - 1$ thinkers that believe other speculators

are level $k - 2$ thinkers. Note that for $k > 1$, in a level k equilibrium, speculators believe that all investors' fundamental belief updating follows the same structure as their own (Lemma 1), but recognize that other investors have private interpretations of news that lead them to disagree about fundamental values. Each speculator i believes that the average fundamental belief in the economy evolves according to.

$$\mathbf{d}_t^{isf} = \mathbb{E}_t^i \int \mathbf{d}_t^j = (1 - \kappa_1 - \kappa_2) \mathbf{d}_{t-1}^{isf} + \kappa_1 D_t + \kappa_2 s_t^{-i}. \quad (9)$$

As before, speculators each seek to infer what other investors' signals are, which they use to reconcile their perceived pricing function with the true price they observe. We highlight that in the level k equilibrium, speculators' second order beliefs are that all investors perceive the persistence of the fundamental process as ρ , but they believe that level 1 speculators believe that all other investors perceive persistence as ρ_{sf} .

Proposition 2 (Level k Pricing Rule and Equilibrium Beliefs) *In the level k equilibrium, the equilibrium pricing function can be expressed as*

$$P_t = A_k \mathbf{d}_t^s, \text{ where}$$

$$A_1 \equiv \frac{1 - \theta\rho - (1 - \theta\rho + \theta\alpha(1 - \rho(\kappa_1 + \kappa_2))\rho_{sf})}{(1 - \rho)\rho_{sf}(1 - \theta(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})} \rho, \text{ and}$$

$$A_k \equiv \frac{\frac{1 - \theta}{1 - \rho} + \theta(1 + A_{k-1}(\kappa_1 + \kappa_2))}{1 - \theta\rho(1 - \kappa_1 - \kappa_2)} \rho, k > 1.$$

Proof. The proof follows by induction. For the level 1 equilibrium, the claim follows immediately from the fundamental belief representation from Lemma 5. For $k > 1$, assume that in the level k equilibrium, speculators perceive the period t price as governed by the level $k - 1$ pricing function, i.e., $\mathbb{E}_t^s(P_t) = A_{k-1} \mathbf{d}_t^{sf}$. Then the average speculator's expected period $t + 1$ payoff can be written as

$$\begin{aligned} \mathbb{E}_t^s(P_{t+1} + D_{t+1}) &= A_{k-1}(\mathbb{E}_t^s(\mathbf{d}_{t+1}^{sf}) - \mathbf{d}_t^{sf}) + \rho \mathbf{d}_t^s \\ &= \rho(1 + A_{k-1}(\kappa_1 + \kappa_2)) \mathbf{d}_t^s + \rho A_{k-1}(1 - \kappa_1 - \kappa_2) \mathbf{d}_t^{sf}. \end{aligned}$$

The market clearing price can be written as

$$\begin{aligned} P_t &= \underbrace{\theta \mathbb{E}_t^s(P_{t+1} + D_{t+1})}_{\text{Speculative Demand}} + \underbrace{(1 - \theta) \frac{\rho}{1 - \rho} \mathbf{d}_t^s}_{\text{Fundamental Demand}} \\ &= \frac{\rho}{1 - \rho} (1 - \theta\rho + A_{k-1}\theta(1 - \rho)(\kappa_1 + \kappa_2)) \mathbf{d}_t^s + A_{k-1}\theta\rho(1 - \kappa_1 - \kappa_2) \mathbf{d}_t^{sf}. \end{aligned} \quad (10)$$

Then, the equilibrium second order belief that sets the market clearing price from Equation

(10) equal to the perceived price, $A_{k-1}\mathbf{d}_t^{sf}$, is $\mathbf{d}_t^{sf} = \frac{\frac{1-\theta}{1-\rho} + \theta(1+A_{k-1}(\kappa_1+\kappa_2))}{A_{k-1}(1-\theta\rho(1-\kappa_1-\kappa_2))}\rho\mathbf{d}_t^s$. Substituting this into the perceived pricing function provides the expression in the proposition. \square

Proposition 2 provides a recursive representation for the asset price in the level k equilibrium which can always be expressed in terms of the level $k-1$ equilibrium pricing coefficient, and in turn, as a scaled version of the average speculator's fundamental beliefs. Although they are not explicitly provided, in the level k equilibrium, speculators form k higher order beliefs, e.g., level 3 speculators form beliefs about level 2 speculators' beliefs about level 1 speculators' beliefs and second order beliefs about the average fundamental belief. Each of these higher order beliefs is exactly pinned down by the fact that speculators' (higher order) beliefs have to be consistent with the prices they observe, and their beliefs about other investors' beliefs about equilibrium. Given the level k equilibrium's representation, we next turn to studying the impact of strategic sophistication.

Result 2 (Overreaction and Reversal in a Level k Equilibrium) *In the level k equilibrium, if $\rho_{sf} > \rho$:*

- (i) *Asset prices overreact to news and exhibit long term reversals, which the average speculator recognizes to be the case.*
- (ii) *Asset prices exhibit short term reversals.*
- (iii) *The average speculator has positive return expectations following a positive shock to fundamentals and engages in non-fundamental speculation.*

Proof. See appendix. \square

Result 2 clarifies that the results derived in the case of level 1 speculators extend to cases where speculators have higher depth of reasoning, and understand that not all investors are fundamental traders. Asset price overreaction and reversals persist for the exact same parameter values, $\rho_{sf} > \rho$, regardless of the level of investors' strategic sophistication. That is, the results obtain as long as there is a seed of a belief that other speculators (believe that other speculators believe that other speculators...) believe that investors overestimate the persistence of fundamentals.

We can understand the result as follows: a level 2 speculator understands that level 1 speculators react too strongly in response to the news they perceive. However, in a level 2 equilibrium, given that all speculators are level 2 speculators, prices overreact less strongly than speculators expect given their fundamental beliefs and their perception of equilibrium, so speculators infer that other investors received an attenuated signal, and will overreact more strongly in the next period. The level 2 speculators accordingly still have positive return expectations and buy into the overvalued market. Markets revert when speculators' second order beliefs are revealed to be incorrect, and investors do not revise their fundamental beliefs upwards in the subsequent period. The same logic holds for each level k of reasoning.

While the results qualitatively remain the same when we increase depth of reasoning, the equilibrium behavior of the model is not identical.

Result 3 (Equilibrium with Sophisticated Speculators)

- (i) For a given $\rho_{sf} > \rho$, asset price overreaction is lower for higher levels of strategic sophistication (higher k).
- (ii) In the limit, as speculators have infinite depth of reasoning ($k \rightarrow \infty$),
 - (a) The asset price converges to its rational expectations fundamental value—that is, $\lim_{k \rightarrow \infty} A_k = \frac{\rho}{1-\rho}$;
 - (b) Non-fundamental speculation disappears; speculators become fundamental value investors ($Q_t^i \propto \frac{\rho}{1-\rho} \mathbf{d}_t^i - P$).

Proof. See appendix. □

Result 3 indicates that as we increase speculators' depth of reasoning, prices overreact less, and converge to the rational expectations fundamental value in the limit. For each level k of reasoning, speculators' return expectations and the resulting overreaction are attenuated relative to the level $k - 1$ equilibrium, due to level k speculators' understanding of level $k - 1$ speculators' overreaction.

Iterating ad infinitum, non-fundamental speculation does not survive as an equilibrium outcome with sophisticated ($k = \infty$) speculators, who are able to correctly extract the average fundamental belief of other investors. Rather, sophisticated speculators bet on prices converging towards their subjective fundamental valuation, as in standard heterogeneous prior or differences-of-opinion models. Note that though they are fundamental value investors, sophisticated speculators do not completely neglect the presence of other investors – the aggressiveness with which they trade depends upon how quickly they believe other investors' beliefs will converge towards their own, which depends on the signal-to-noise ratio of the signals about fundamentals. Moreover, though each speculator individually has non-zero return expectations in response to news, the effect washes out in the aggregate; the average speculator (correctly) believes that prices match the rational expectations value.

The convergence of prices to the rational expectations fundamental value with higher levels of reasoning is rapid. Figure 5 plots the price in period t following a unit shock to \mathbf{d}_t^s starting from steady state in $t - 1$ for different levels of k . Overvaluation exponentially decays as we increase speculators' strategic sophistication. The figure displays the period t cumulative return expectations of the average speculator in blue for k . The declining overvaluation can be understood by the fact that speculators' one period ahead return expectations display similar decline as overvaluation. The figure plots the average realized returns in red. Consistent with overvaluation and return expectations being considerably smaller, reversals are considerably less sharp as we increase speculators' strategic sophistication.

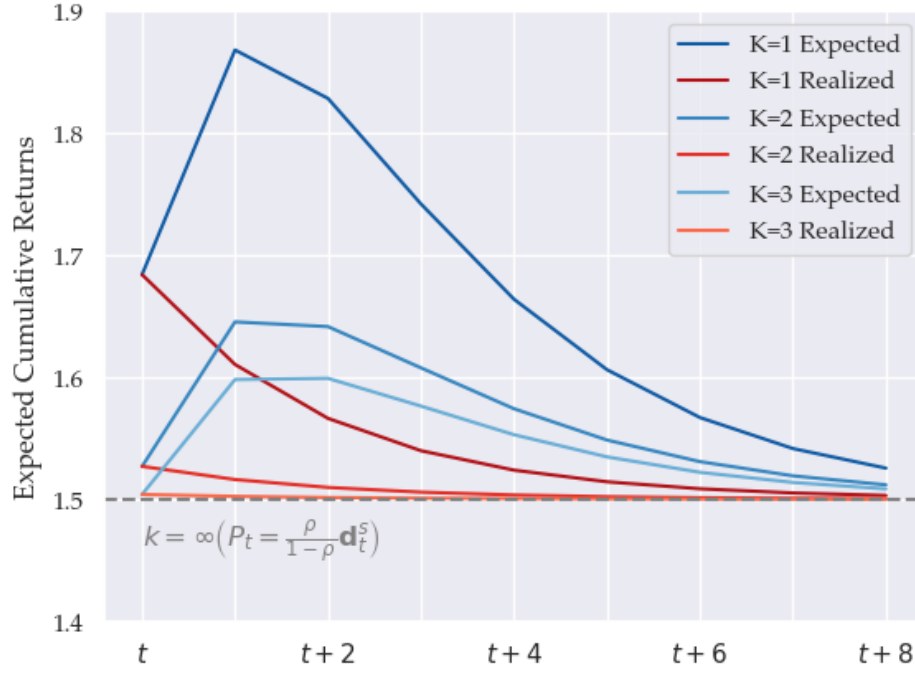


FIGURE 5: CUMULATIVE RETURN EXPECTATIONS AND REALIZED RETURNS FOR DIFFERENT k

Note: The figure plots the asset price in period t following a unit shock to \mathbf{d}_t^s starting from steady state for different levels of strategic sophistication, k . The blue lines represent speculators' cumulative return expectations from period t to $t+h$. The red lines represent the average realized cumulative returns from period t to $t+h$. The illustrative parameter values used are $(\theta, \rho, \rho_{sf}, \kappa_1, \kappa_2) = (0.5, 0.6, 0.7, 0.2, 0.2)$.

Result 3 highlights the importance of imperfect depth of reasoning for non-fundamental speculation, overreaction, and reversals as equilibrium outcomes. These phenomena each decline substantially with even limited amounts of higher order reasoning. If speculators engaged in higher order reasoning, then speculators should bet on returns moving in the direction of their perceived valuation.

Remark 4 (Models of Rational Non-Fundamental Speculation) *Our conclusion that non-fundamental speculation does not survive as we increase depth of reasoning differs from the conclusions drawn by models where rational investors may find it profitable to engage in non-fundamental speculation, such as De Long et al. (1990) and Abreu and Brunnermeier (2002, 2003). There, the underlying cause of mispricing and overvaluation is the behavior of non-strategic investors. Here, we endogenize the source of mispricing as coming from speculators that engage in non-fundamental speculation; and when these speculators become sophisticated, non-fundamental speculation disappears. That is, we can trace the differences in conclusions drawn to a focus on different types of investors.*

2.4 Learning about Fundamentals from Prices

In our previous analyses, investors treat other investors' signals as uninformative conditional on their own. Here, we briefly discuss a model that extends our baseline specification and relaxes this assumption. We find that allowing investors to learn about fundamentals from prices may distort investors' fundamental beliefs and introduce further excess volatility. We provide a more detailed exposition of the model in Appendix B.

With learning about fundamentals from prices, the equations that govern the perceived and true pricing functions for the economy (Lemmas 2 and 3) remain the same. However, the filtering equation that governs speculators' fundamental belief updating depends on higher order beliefs. In particular, \mathbf{d}_t^s and \mathbf{d}_t^{sf} must jointly satisfy

$$\begin{aligned}\mathbf{d}_t^s &= (1 - \kappa_1 - \kappa_2)\rho\mathbf{d}_{t-1}^s + \kappa_1 D_t + \kappa_2 \mathbb{E}_t^s(s_t), \text{ and} \\ \mathbf{d}_t^{sf} &= (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_{t-1}^{sf} + \alpha\kappa_1 D_t + \alpha\kappa_2 \mathbb{E}_t^s(s_t).\end{aligned}$$

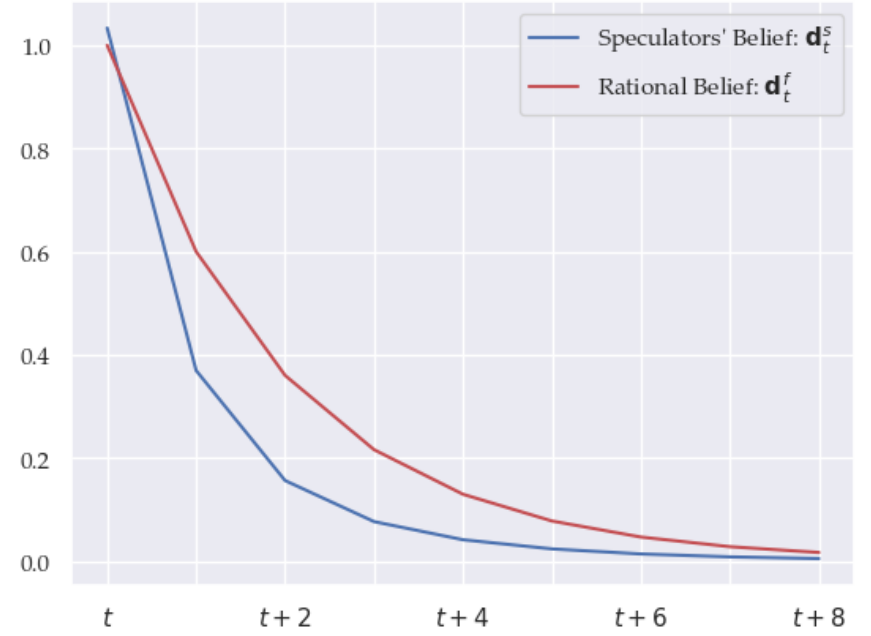
Because speculators misunderstand the equilibrium structure and belief formation of other investors, they extract a biased signal from prices about fundamentals in equilibrium, i.e., $\mathbb{E}_t^s(s_t) \neq s_t$.

With non-fundamental speculation, the same finding as in the main model – that asset prices overreact to news and exhibit reversals – still holds in the model with learning from prices. However, an additional feature is that asset prices exhibit additional excess volatility stemming from an additional source: excess volatility in speculators' subjective beliefs about fundamentals.

Figure 6 illustrates this excess volatility for a sample set of parameters. Panel (a) of Figure 6 plots cumulative expected returns in response to a positive fundamental shock in period



(a) Expected and Realized Returns



(b) Dynamics of Subjective Beliefs

FIGURE 6: NON-FUNDAMENTAL SPECULATION WITH LEARNING FROM PRICES

Note: Panel (a) of the figure plots the average asset price in period t following a unit shock to \mathbf{d}_t^f starting from steady state. The blue line represents speculators' cumulative return expectations from period t to $t+h$. The red line represents the average realized cumulative returns from period t to $t+h$. The illustrative parameter values used are $(\theta, \rho, \rho_{sf}, \kappa_1, \kappa_2, \alpha) = (0.5, 0.6, 0.7, 0.2, 0.2, 0.7)$. Panel (b) of the figure plots the evolution of speculators' subjective beliefs, \mathbf{d}_t^s following the same shock, in blue. For comparison, the figure also plots the rational expectations belief, \mathbf{d}_t^f , in red.

t . Speculators engage in non-fundamental speculation as before – they believe that markets have overreacted to news, but buy in based on the belief that returns will be positive in the near term. However, in period $t + 1$, the asset price crashes to *below* the fundamental value of the risky asset, before slowly recovering in subsequent periods.³¹

Panel (b) of Figure 6 illustrates the excess volatility in speculators’ subjective beliefs that arises from fundamental learning from prices. For the given parameters, speculators’ fundamental beliefs initially overreact to the shock. However, these beliefs crash to well below the rational expectations value in period $t + 1$. This is because speculators, disappointed by not seeing the asset price increase they expected, infer that fundamentals are actually worse than they are.

These results abstract away from potential biases in investors’ belief updating that don’t have to do with higher order beliefs, so only reflect one channel through which fundamental beliefs may affect asset prices. But we find the broader insight – that investors’ higher order beliefs can induce excess volatility in their beliefs via a learning channel, and in turn, in asset prices – to be realistic.

3 Conclusion

We study investors’ higher order beliefs, using survey data from the Robert Shiller Investor Confidence surveys. While previous work has documented instances of non-fundamental speculation – investors taking positions in a risky asset in a direction that conflicts with their fundamental views – we find that such speculation is the norm for the U.S. stock market. The majority of investors in the Shiller surveys, who represent an important class of investors, report that other investors have mistaken beliefs, but nevertheless report positive return expectations from speculating in the direction of these mistaken beliefs. In addition, investors report that they believe that stock markets overreact and exhibit momentum and reversal in response to news. Investors’ non-fundamental speculation is unprofitable, however; investors’ short term return expectation tend to perform poorly in predicting subsequent market returns.

To explore the general equilibrium implications of the empirical evidence, we construct a theoretical model that can match the survey evidence, where investors believe that the patterns in prices are driven by other, less sophisticated investors. We find that investors’ higher order beliefs amplify stock market overreaction and drive excess volatility. Moreover, we find that for higher order belief-driven non-fundamental speculation to survive in equilibrium, investors must not engage in the types of sophisticated strategic reasoning typically assumed in traditional models.

³¹Asset prices in the model with learning about fundamentals from prices do not have the same sharp analytical characterization as in our main analysis. However, we find this result for all parameter values evaluated.

Our paper also provides direction for future work. The types of higher order beliefs we study may be at play in other asset markets. For example, in a previous version of this paper, [Schmidt-Engelbertz and Vasudevan \(2023\)](#), we examine foreign exchange markets, and find similar patterns of investors' return expectations that we document here. Additionally, quantitative work may be helpful for better understanding the extent to which stock market fluctuations can be attributed to speculation. We also do not provide micro-foundations for investors' higher order beliefs (and our approach to modeling them is admittedly 'backwards-engineered'). But exploring the source of investors' higher order beliefs, and how they interact with fundamental beliefs, may prove fruitful.

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A Omitted Proofs

Proof of Proposition 1

Proof. We can write the average speculator's expected returns as

$$\frac{1}{\theta} \int_0^\theta \left(\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t \right) = \frac{1}{\theta} \underbrace{(a_1 \mathbf{d}_t^s + b_1 \mathbf{d}_t^{sf})}_{=\int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1})} - \frac{1}{\theta} \underbrace{\frac{\rho_{sf}}{1 - \rho_{sf}} \mathbf{d}_t^{sf}}_{=P_t}.$$

Substituting in a_1 and b_1 from Lemma 3, \mathbf{d}_t^{sf} from Lemma 4, and $\mathbf{d}_t^f = \mathbf{d}_t^s$, and simplifying, we get that

$$\frac{1}{\theta} \int_0^\theta \left(\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t \right) = \frac{(1 - \theta)(1 - (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})}{(1 - \rho)(1 - \rho_{sf})(1 - \theta(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})} (\rho_{sf} - \rho) \rho \mathbf{d}_t^s. \quad (\text{A.1})$$

From Lemma 1, $\mathbb{E}_t(\mathbf{d}_t^s) > 0$ given a positive shock, so, for $\rho_{sf} \in [0, 1)$, Equation (A.1) is positive if and only if $\rho_{sf} > \rho$. For perceived overvaluation, note that from the market-clearing condition, we can re-write the market clearing price as

$$P_t = \frac{1}{1 - \theta} \int_0^\theta \mathbb{E}_t^i((P_{t+1} + D_{t+1}) - P_t) + \underbrace{\frac{\rho}{1 - \rho} \mathbf{d}_t^f}_{=\frac{\rho}{1 - \rho} \mathbf{d}_t^s},$$

which is greater than $\frac{\rho}{1 - \rho} \mathbf{d}_t^s$ if and only if the average speculator's expected returns, $\int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t$, are positive. As shown, this is guaranteed by $\rho_{sf} > 0$. \square

Proof of Result 1

Proof. Overreaction and long term reversal follow immediately from the proof of Proposition 1, since the average investor's fundamental beliefs coincide with rational expectations. Short term reversal is that, under rational expectations, given a positive fundamental shock in period t , $\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) < 0$. Given that \mathbf{d}_t^s is the rational expectations value of fundamentals, and substituting $\rho \mathbf{d}_t^s$ as the period $t + 1$ dividend and as $\mathbb{E}_t(\mathbf{d}_{t+1}^s)$, and after simple algebra, we get that

$$\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = \frac{\theta \rho (1 - (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})}{(1 - \rho_{sf})(1 - \theta(1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf})} (\rho - \rho_{sf}) \mathbf{d}_t^s. \quad (\text{A.2})$$

This is negative when $\rho_{sf} > \rho$ if $\mathbf{d}_t^s > 0$, which is true in expectation in all periods following a positive shock. \square

Proof of Result 2

Proof. For (i), we show overreaction by induction. It is sufficient to show that $A_k > \frac{\rho}{1-\rho}$. For $k = 1$, $A_1 > \frac{\rho}{1-\rho}$ by the proof of Proposition 1. Now assume that $A_{k-1} > \frac{\rho}{1-\rho}$. We observe that

$$\frac{A_k}{\frac{\rho}{1-\rho}} = \frac{1 - \theta\rho + A_{k-1}\theta(1-\rho)(\kappa_1 + \kappa_2)}{1 - \theta\rho(1 - \kappa_1 - \kappa_2)}.$$

Subtracting the denominator from the numerator, we get that the difference is positive (and hence $A_k > \frac{\rho}{1-\rho}$) if and only if $A_{k-1} > \frac{\rho}{1-\rho}$, which is true by assumption.

For (ii), note that objective expected returns can be written as $\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = A_k(\rho \mathbf{d}_t^s - \mathbf{d}_t^s) + \rho \mathbf{d}_t^s$. Since $A_k > \frac{\rho}{1-\rho}$, for $\mathbf{d}_t^s > 0$, $A_k(\rho \mathbf{d}_t^s - \mathbf{d}_t^s) < \frac{\rho}{1-\rho}(\rho \mathbf{d}_t^s - \mathbf{d}_t^s) = \rho \mathbf{d}_t^s$, so returns are negative.

For (iii), the same proof from Proposition 1 applies. □

Proof of Result 3

Proof. For (i), it is sufficient to show that $A_k < A_{k-1}$. We can observe that

$$\frac{A_k}{A_{k-1}} = \frac{\rho \left(\frac{1-\theta}{1-\rho} + \theta(1 + A_{k-1}(\kappa_1 + \kappa_2)) \right)}{A_{k-1}(1 - \theta\rho(1 - \kappa_1 - \kappa_2))}.$$

Subtracting the denominator from the numerator, and simplifying, we get that the difference is negative (and hence $A_k < A_{k-1}$) if and only if $\frac{\rho}{1-\rho} < A_{k-1}$. We have shown this to be true for all k in the proof of Result 2, claim (i).

For (ii.a), denoting $X \equiv \frac{\theta\rho(\kappa_1 + \kappa_2)}{(1-\theta\rho(1-\kappa_1-\kappa_2))}$, we can solve the recurrence relation for A_k as

$$A_k = X^{k-1} A_1 + \frac{\theta\rho(\kappa_1 + \kappa_2)(1 - X^k) - (1 - \theta\rho)X^k}{\theta(1 - \rho)(\kappa_1 + \kappa_2)}.$$

Observing that $\lim_{k \rightarrow \infty} X^k = 0$, we have that $\lim_{k \rightarrow \infty} A_k = \frac{\rho}{1-\rho}$.

For (ii.b), note that if speculators perceive the pricing function as $\frac{\rho}{1-\rho} \mathbf{d}_t^s$, as in the $k \rightarrow \infty$ equilibrium, their subjective expected return is

$$\begin{aligned} \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t) &= (1 - \kappa_1 - \kappa_2)\mathbf{d}_t^s + (\kappa_1 + \kappa_2)\mathbf{d}_t^i + \rho \mathbf{d}_t^i - \frac{\rho}{1-\rho} \mathbf{d}_t^s \\ &= (1 - \rho(1 - \kappa_1 - \kappa_2)) \times \left(\frac{\rho}{1-\rho} \mathbf{d}_t^i - \underbrace{\frac{\rho}{1-\rho} \mathbf{d}_t^s}_{=P_t} \right). \end{aligned}$$

The claim follows since $Q_t^i \propto \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t)$. □

B Model with Learning about Fundamentals from Prices

In this section, we lay out the details of a variant of our baseline model where investors use prices to learn about fundamentals in a manner that is consistent with their higher order beliefs.

B.1 Investors' Beliefs

We remove assumption 1, that the idiosyncratic noise in investors' signals are idiosyncratic interpretations, and that investors believe other investors' private signals to be uninformative about fundamentals. Then, investor i 's belief updating is given by

$$\mathbf{d}_t^i = (1 - \kappa_1 - \kappa_2)\rho\mathbf{d}_{t-1}^i + \kappa_1 D_t + \kappa_2 \mathbb{E}_t^i(s_t).$$

Given the information structure, prices, in principle, fully reveal all private information in the economy. We add an assumption about fundamental investors' belief updating:

Assumption 4 (Fundamental Investors' Belief Updating) *Fundamental investors correctly understand the information structure and equilibrium structure of the economy.*

Assumption 4 means that all fundamental investors correctly extract s_t from prices ($\mathbb{E}_t^i(s_t) = s_t$), and that each fundamental investors' beliefs corresponds with the rational expectations belief. Given the information structure, speculators also all share the same beliefs with one another, but their higher order beliefs lead them to extract a distorted signal, which, in turn distorts their fundamental belief.

To accommodate the survey evidence with learning about fundamentals from prices, we also replace assumption 2 with assumption 5.

Assumption 5 (Second Order Beliefs) *Speculators believe that all other investors overestimate the amount of noise in new information and the persistence of fundamentals. That is, speculators perceive others' fundamental belief-updating parameters as $(\sigma_v^2, \sigma_\eta^2, \rho)$ as $(\mu\sigma_v^2, \mu\sigma_\eta^2, \rho_{sf})$, for some $\mu > 1$.*

Assumption 5 means that speculators' beliefs about how other investors' fundamental beliefs evolve can be expressed as

$$\mathbf{d}_t^{sf} = (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_{t-1}^{sf} + \alpha\kappa_1 D_t + \alpha\kappa_2 \mathbb{E}_t^i(s_t),$$

where $\alpha < 1$. This is similar to the expression for speculators' second order beliefs provided in the main specification, though α now also relies on the amount of noise that speculators believe that other investors perceive. Note that (1) Assumption 3 still holds, so speculators perceive all other investors as fundamental investors, and (2) speculators perceive that other

investors correctly extract s_t from prices but do not appropriately incorporate the signal into their beliefs.

The definition of equilibrium remains the same, and speculators' perceived pricing function and equilibrium pricing function are still determined by Lemmas 2 and 3. However, with learning about fundamentals from prices, \mathbf{d}_t^s no longer is equal to the average belief of fundamental investors, \mathbf{d}_t^f , and reflects the potentially incorrect signal that speculators extract from prices.

Lemma B.1 (Equilibrium Beliefs with Learning from Prices) *In equilibrium, speculators' first and second order beliefs are given by*

$$\mathbf{d}_t^s = (1 - \kappa_1 - \kappa_2)\rho\mathbf{d}_{t-1}^s + \kappa_1 D_t + \kappa_2 s_t^s, \text{ and} \quad (\text{B.1})$$

$$\mathbf{d}_t^{sf} = (1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_{t-1}^{sf} + \alpha\kappa_1 D_t + \alpha\kappa_2 s_t^s, \text{ where} \quad (\text{B.2})$$

$$s_t^s \equiv \mathbb{E}_t^s(s_t) = e_1 D_t + e_2 \mathbf{d}_{t-1}^f + e_3 \mathbf{d}_{t-1}^s + e_4 \mathbf{d}_t^f, \quad (\text{B.3})$$

where the coefficients e_1, e_2, e_3 , and e_4 are defined in the appendix.

Proof. See the end of the section. □

As Lemma B.1 illustrates, with learning from prices, speculators' higher order beliefs lead them to extract a biased signal from prices, for the price they observe to be consistent with their perceived pricing function. After understanding the parameter values that match the survey data, we then more closely explore the implications of this biased signal.

B.2 Matching the Survey Evidence and Implications for Asset Prices

To match the survey evidence, we place joint restriction on speculators' second order beliefs about persistence (ρ_{sf}) and the signal-to-noise ratio of new information (governed by α):

Proposition B.1 (Matching the Survey Evidence with Fundamental Learning from Prices) *If $\frac{\rho}{\rho(1-\alpha)+\alpha} < \rho_{sf} < \frac{\rho}{\alpha}$ and $\theta < \frac{\alpha\rho_{sf}}{\rho - \rho_{sf}(1-\alpha(\kappa_1+\kappa_2))(\rho - \alpha\rho_{sf})}$, then speculators*

- (i) *Have positive return expectations on average following a positive shock to fundamentals;*
- (ii) *Believe that markets are overvalued on average following the arrival of a positive shock to fundamentals;*
- (iii) *Engage in non-fundamental speculation.*

Proof. See the end of the section. □

Proposition B.1 indicates that to match the survey data with learning about fundamentals from prices, we must place joint restrictions on α , ρ_{sf} , and θ . First, these joint restrictions

imply that $\alpha < 1$, i.e., speculators believe that other investors do not fully incorporate new information into their beliefs. This condition is required for $\mathbf{d}_t^{sf} < \mathbf{d}_t^s$ following a positive shock, which generates positive return expectations.³² Additionally, non-fundamental speculation still requires $\rho_{sf} > \rho$, i.e., speculators believe that other investors overestimate the persistence of fundamentals. However, there is an upper bound on ρ_{sf} now, which is required to guarantee speculators believe that there will be positive returns in the period following positive news despite a belief that others overreact to news. Lastly, the restriction on θ ensures against a corner case: that there are so many speculators in the economy that the average response to a positive fundamental shock is for prices to decline.³³

With learning from prices, the behavior of equilibrium asset prices does not submit to the same sharp analytical characterization as before. Nevertheless, we find that the patterns discussed in the main text – that asset prices exhibit reversals, and fall below their fundamental values – persist for all parameter values evaluated.

In general, the larger the magnitude of the overvaluation in period t , the larger the crash in period $t + 1$. Figure B.1 illustrates the relative magnitudes of period t overvaluation (in the right panel) and period $t + 1$ returns (in the left panel), for the parameter values of α and ρ_{sf} where we observe non-fundamental speculation. Darker colors in the plots coincide with higher period t overvaluation and lower period $t + 1$ returns following a positive shock. The more distorted that speculators believe other investors' beliefs to be (lower α and higher ρ_{sf}), the higher overvaluation we tend to observe in period t , and the larger the crash that we observe in period $t + 1$.

B.3 Proofs

Proof of Lemma B.1

Proof. We start from steady state in $t - 1$ (taking unconditional expectations), where

$$\mathbb{E}(\mathbf{d}_{t-1}^{sf}) = \mathbb{E}(\mathbf{d}_{t-1}^s) = \mathbb{E}(\mathbf{d}_{t-1}^f) = 0.$$

Then, given a shock to D_t and s_t , $\mathbf{d}_t^{sf} = \alpha \mathbf{d}_t^s$. Following a positive shock, for speculators to have positive return expectations in equilibrium, we must have $P_t > 0$, by the proof of Proposition 1. Since the equilibrium price must satisfy $P_t = \frac{\rho_{sf}}{1-\rho_{sf}} \mathbf{d}_t^{sf}$, we must also have that $\mathbf{d}_t^s > 0$ in equilibrium following a positive shock. We first derive the restriction on θ that guarantees this.

³²Without learning about fundamentals from prices, the restriction on α is not required given the lack of feedback of second order beliefs into first order beliefs, as $\mathbf{d}_t^{sf} < \mathbf{d}_t^s$ is guaranteed by $\rho_{sf} > \rho$.

³³Because speculators believe that prices overreact to news, they adjust the signal they extract from prices downwards following a positive shock. If there are a sufficient number of speculators doing this, then the market clearing price might decline following good news on average.

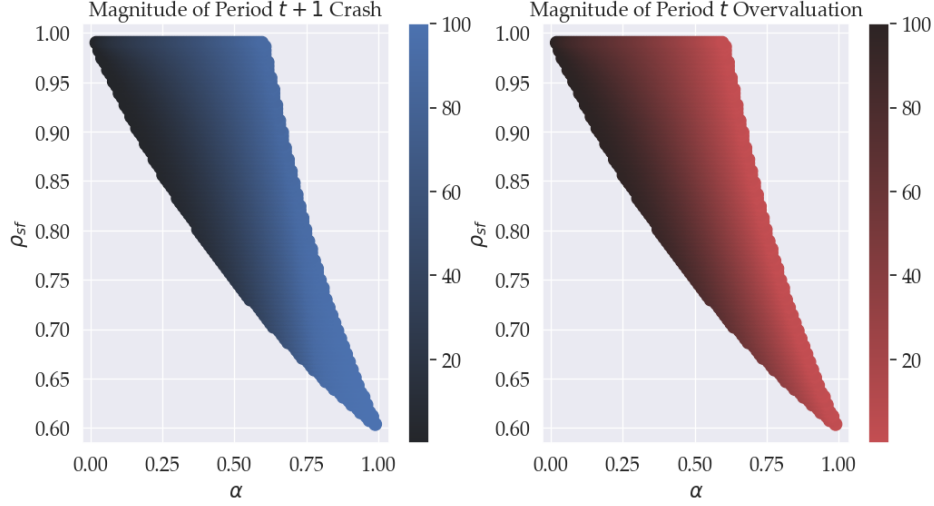


FIGURE B.1: MAGNITUDES OF BUBBLES AND CRASHES WITH LEARNING FROM PRICES

Note: The figure displays the relative magnitudes of overvaluation (in the right panel) in period t and the magnitude of the crash in period $t + 1$ (in the left panel). The x-axis of the plot is α , where lower values corresponding to speculators' perceiving that other investors' Kalman gains are too low, and the y-axis of the plot is ρ_{sf} , the speculators' belief about other investors' beliefs about the persistence of fundamentals. Values are plotted for parameter values where speculators engage in non-fundamental speculation, and are colored based on the percentile of period t valuations and period $t + 1$ returns. Darker colors correspond with higher period t valuations and larger period $t + 1$ crashes. The values of $(\theta, \rho, \kappa_1, \kappa_2)$ used in the plots are $(0.5, 0.6, 0.2, 0.2)$.

Given the period $t - 1$ beliefs, we can write the period t subjective beliefs of speculators in equilibrium as

$$\mathbf{d}_t^s = \frac{(1 - \theta)\rho(1 - \rho_{sf})}{(1 - \rho)(\alpha\rho_{sf} - \theta\rho + \theta\rho_{sf}(1 - \alpha(\kappa_1 + \kappa_2))(\rho - \alpha\rho_{sf}))} \mathbf{d}_t^f. \quad (\text{B.4})$$

For a positive shock to fundamentals, \mathbf{d}_t^f is positive since it is the rational expectations belief, and the numerator of Equation (B.4) is always positive. Hence, \mathbf{d}_t^s is positive if and only if

$$\theta < \frac{\alpha\rho_{sf}}{\rho - \rho_{sf}(1 - \alpha(\kappa_1 + \kappa_2))(\rho - \alpha\rho_{sf})}, \quad (\text{B.5})$$

which provides the restriction on θ .

Next, we note that speculators' return expectations are given by

$$\begin{aligned} \mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) &= \underbrace{\frac{\rho_{sf}}{1 - \rho_{sf}}((1 - \alpha(\kappa_1 + \kappa_2))\rho_{sf}\mathbf{d}_t^{sf} + \alpha(\kappa_1 + \kappa_2)\rho\mathbf{d}_t^s)}_{=\mathbb{E}_t^s(P_{t+1})} \\ &+ \underbrace{\rho\mathbf{d}_t^s}_{=\mathbb{E}_t^s(D_{t+1})} - \underbrace{\frac{\rho_{sf}}{1 - \rho_{sf}}\mathbf{d}_t^{sf}}_{=P_t}. \end{aligned}$$

Plugging this into return expectations, given $\mathbf{d}_t^s > 0$, we get that return expectations are positive if and only if $\rho_{sf} < \frac{\rho}{\alpha}$.

For (ii), subjective overvaluation follows from $\frac{\rho}{1-\rho}\mathbf{d}_t^s < \frac{\rho_{sf}}{1-\rho_{sf}}\mathbf{d}_t^{sf}$. Given that $\mathbf{d}_t^{sf} = \alpha\mathbf{d}_t$, we can solve and note that speculators perceive markets to be overvalued when $\rho_{sf} > \frac{\rho}{\rho(1-\alpha)+\alpha}$.

Claim (iii) follows directly from claims (i) and (ii). \square

Proof of Lemma B.1

Proof. The expressions for e_1, e_2, e_3 , and e_4 are

$$\begin{aligned} e_1 &= -\frac{\kappa_1}{\kappa_2}, \\ e_2 &= -\frac{(1-\theta)\rho(1-\alpha(\kappa_1+\kappa_2))(1-\rho_{sf})\rho_{sf}}{(1-\rho)\kappa_2(\alpha\rho_{sf}-\theta\rho+\theta\rho(1-\alpha(\kappa_1+\kappa_2))(\rho-\alpha\rho_{sf}))}, \\ e_3 &= -\frac{\theta\rho(1-(1-\alpha(\kappa_1+\kappa_2))\rho_{sf})(\rho_{sf}-\rho+(\kappa_1+\kappa_2)(\rho-\alpha\rho_{sf}))}{\kappa_2(\alpha\rho_{sf}-\theta\rho+\theta\rho(1-\alpha(\kappa_1+\kappa_2))(\rho-\alpha\rho_{sf}))}, \text{ and} \\ e_4 &= -\frac{e_2}{(1-\alpha(\kappa_1+\kappa_2))\rho_{sf}}. \end{aligned}$$

Equations (B.1) and (B.2) are given by (perceived) Bayesian updating. Additionally, in equilibrium, we must have that the perceived price coincides with the true price. This means that

$$\mathbf{d}_t^{sf} = \left(\frac{\rho_{sf}}{1-\rho_{sf}} - b_1 \right)^{-1} \left(a_1 \mathbf{d}_t^s + c_1 \mathbf{d}_t^f \right). \quad (\text{B.6})$$

Equation (B.3) comes from setting Equations (B.6) and Equations (B.2) equal, providing an expression for \mathbf{d}_t^s , and plugging the resulting expression into Equation (B.1), and solving for s_t^s . \square

C Additional Empirical Analyses

In this section, we present additional empirical analyses. We first analyze if the number of survey responses in the Shiller survey displays any business cycle variation, and find no evidence that it does. The rest of the section presents tables and figures that replicate the main results for different subsets of the data (e.g., individual versus institutional investors).

C.1 Survey Responses in the Shiller Survey

We analyze if there is any business cycle frequency variation in responses to the Shiller survey. We regress the quarterly change in the log number of survey responses to the survey each quarter on S&P 500 returns, and quarterly innovations in the Conference Board Coincident indicators index (labeled ‘Macro’).³⁴ The independent variables are standardized to have zero mean and unit standard deviation. Table C.1 reports the results, and Newey-West standard errors (4 lags) are reported in parentheses. There is little evidence to indicate systematic business cycle variation in survey response counts.

	All	Retail	Inst
Returns	1.52 (4.12)	0.44 (2.57)	1.07 (2.21)
Macro	-0.61 (2.99)	-2.58 (1.59)	1.97 (1.54)

TABLE C.1: RESPONSE COUNTS AND BUSINESS CYCLE VARIATION

³⁴Unfortunately, we do not observe the number of questionnaires that were sent out each quarter, so we use changes in total responses to proxy for response rates.

Panel A: Term Structure of Expected Cumulative Returns								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Belief</i>	1.44 (0.53)	0.76 (0.57)	0.00 (0.60)	-1.38 (0.68)	0.21 (0.06)	-0.05 (0.08)	0.61 (0.10)	-1.19 (0.12)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	258	258	258	258	5651	5651	5651	5651
<i>R</i> ²	0.09	0.02	0.00	0.03	0.00	0.00	0.01	0.02

Panel B: Short term Peaks and Troughs				
	<i>ST Peak</i>	<i>ST Trough</i>	<i>ST Peak</i>	<i>ST Trough</i>
<i>HO Belief</i>	0.28 (0.04)	-0.25 (0.05)		
<i>Overvaluation</i>			0.54 (0.07)	-0.02 (0.17)
Time FE	NA	NA	NA	NA
<i>N</i>	259	259	259	259
<i>R</i> ²	0.13	0.10	0.21	0.00

TABLE C.2: HIGHER ORDER BELIEFS AND RETURN EXPECTATIONS (INDIVIDUAL INVESTORS)

Note: This table replicates Table 3 for the individual investor subset of our sample.

Panel A: Term Structure of Expected Cumulative Returns								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Belief</i>	0.13 (0.17)	-0.39 (0.31)	-1.28 (0.39)	-2.57 (0.70)	-0.12 (0.05)	-0.61 (0.08)	-1.52 (0.13)	-2.49 (0.19)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	258	258	258	258	5651	5651	5651	5651
<i>R</i> ²	0.00	0.01	0.05	0.09	0.00	0.02	0.05	0.07

Panel B: Short term Peaks and Troughs				
	<i>ST Peak</i>	<i>ST Trough</i>	<i>ST Peak</i>	<i>ST Trough</i>
<i>HO Belief</i>	0.28 (0.07)	-0.10 (0.07)		
<i>Overvaluation</i>			0.61 (0.14)	0.04 (0.13)
Time FE	NA	NA	NA	NA
<i>N</i>	259	259	259	259
<i>R</i> ²	0.08	0.02	0.16	0.00

TABLE C.3: HIGHER ORDER BELIEFS AND RETURN EXPECTATIONS (INSTITUTIONAL INVESTORS)

Note: This table replicates Table 3 for the institutional investor subset of our sample.

Panel A: Term Structure of Expected Cumulative Returns and Higher Order Optimism								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Optimism</i>	1.83 (0.45)	0.73 (0.49)	-0.70 (0.51)	-3.50 (0.79)	0.06 (0.05)	-0.40 (0.08)	-1.06 (0.11)	-1.88 (0.15)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	259	259	259	259	10137	10137	10137	10137
<i>R</i> ²	0.09	0.01	0.01	0.09	0.00	0.00	0.02	0.03
Panel B: Term Structure of Expected Cumulative Returns and Higher Order Pessimism								
	Time-Series				Cross-Sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
<i>HO Pessimism</i>	-2.03 (0.61)	-0.71 (0.65)	-0.10 (0.92)	2.19 (1.34)	-0.04 (0.05)	0.13 (0.07)	0.75 (0.10)	1.28 (0.15)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
<i>N</i>	259	259	259	259	10137	10137	10137	10137
<i>R</i> ²	0.08	0.01	0.00	0.02	0.00	0.00	0.01	0.01

TABLE C.4: HIGHER ORDER OPTIMISM, PESSIMISM, AND RETURN EXPECTATIONS

Note: This table replicates Table 3, separately breaking down the results for *HO Optimism* and *HO Pessimism*.

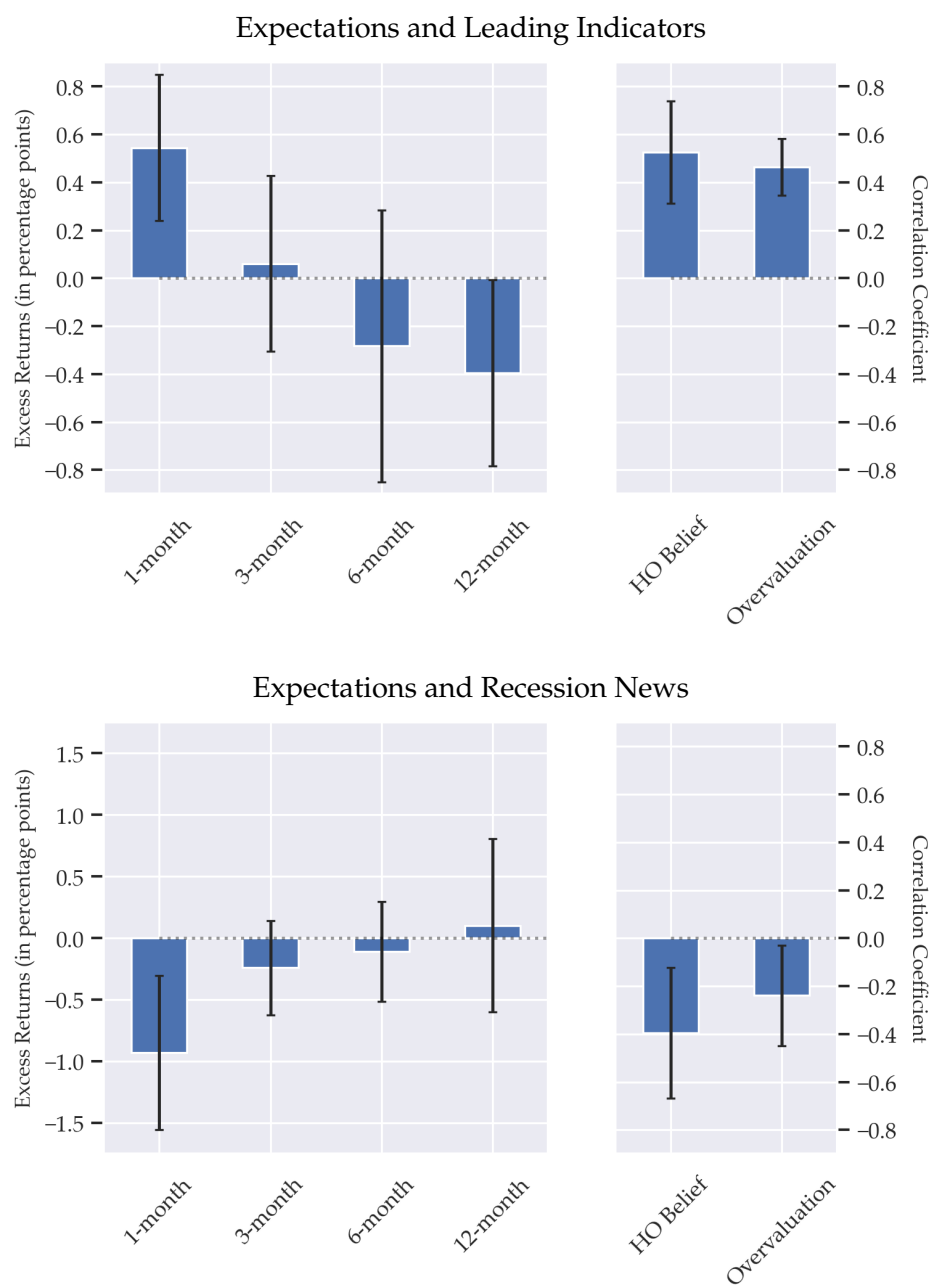


FIGURE C.1: MACROECONOMIC NEWS AND EXPECTATIONS (INDIVIDUAL INVESTORS)

Note: The figure replicates Figure 3 for the individual investor subset of our sample.

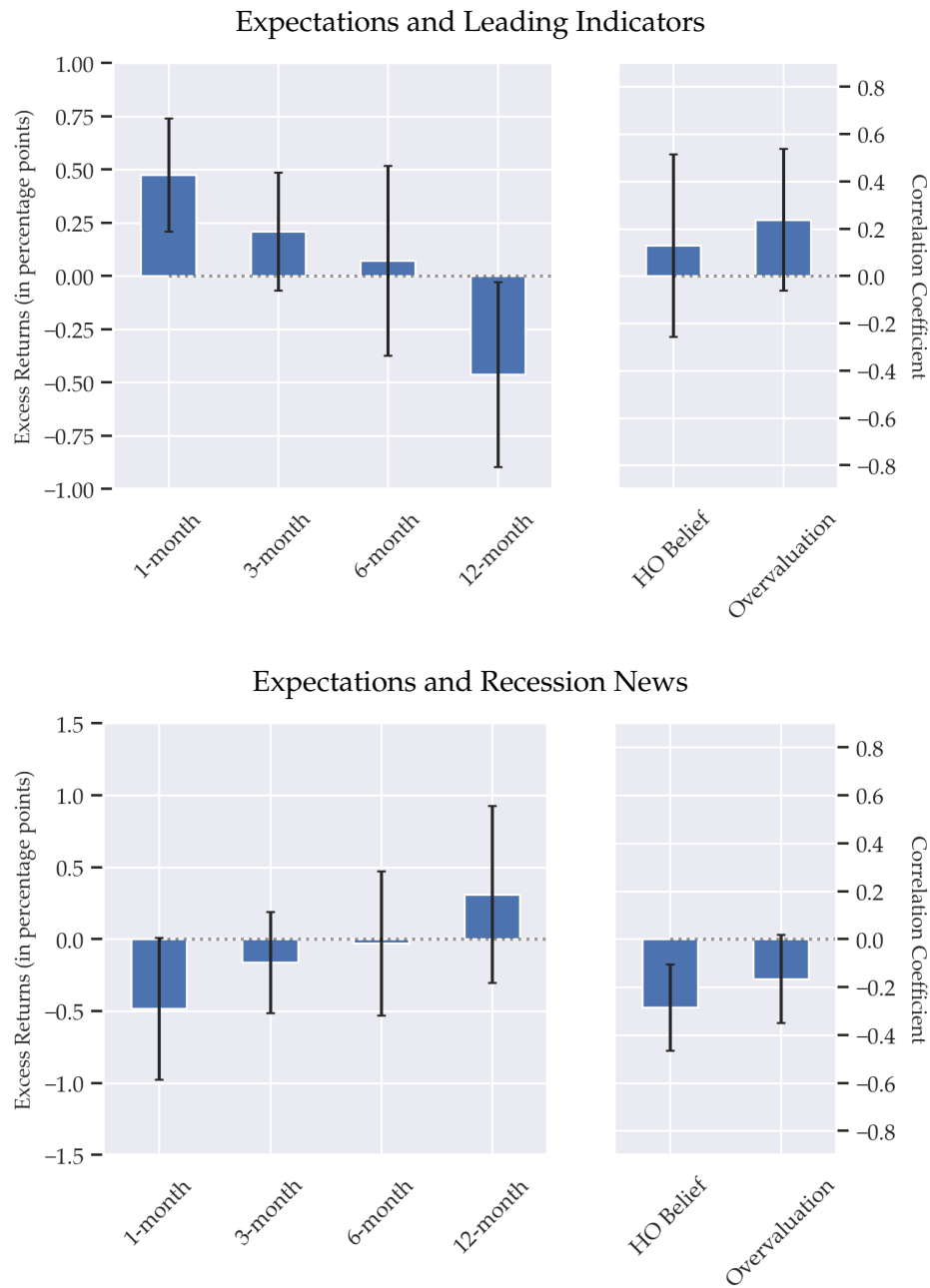


FIGURE C.2: MACROECONOMIC NEWS AND EXPECTATIONS (INSTITUTIONAL INVESTORS)

Note: The figure replicates Figure 3 for the institutional investor subset of our sample.

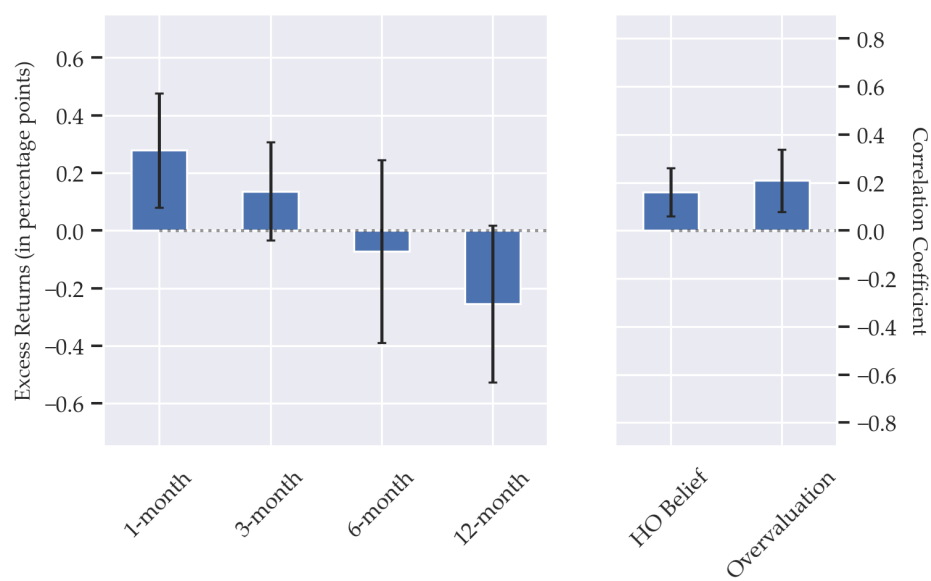


FIGURE C.3: COINCIDENT INDICATORS AND EXPECTATIONS

Note: The figure replicates the top panel of Figure 3, using innovations to the Coincident Macroeconomic Indicators index from the Conference Board.

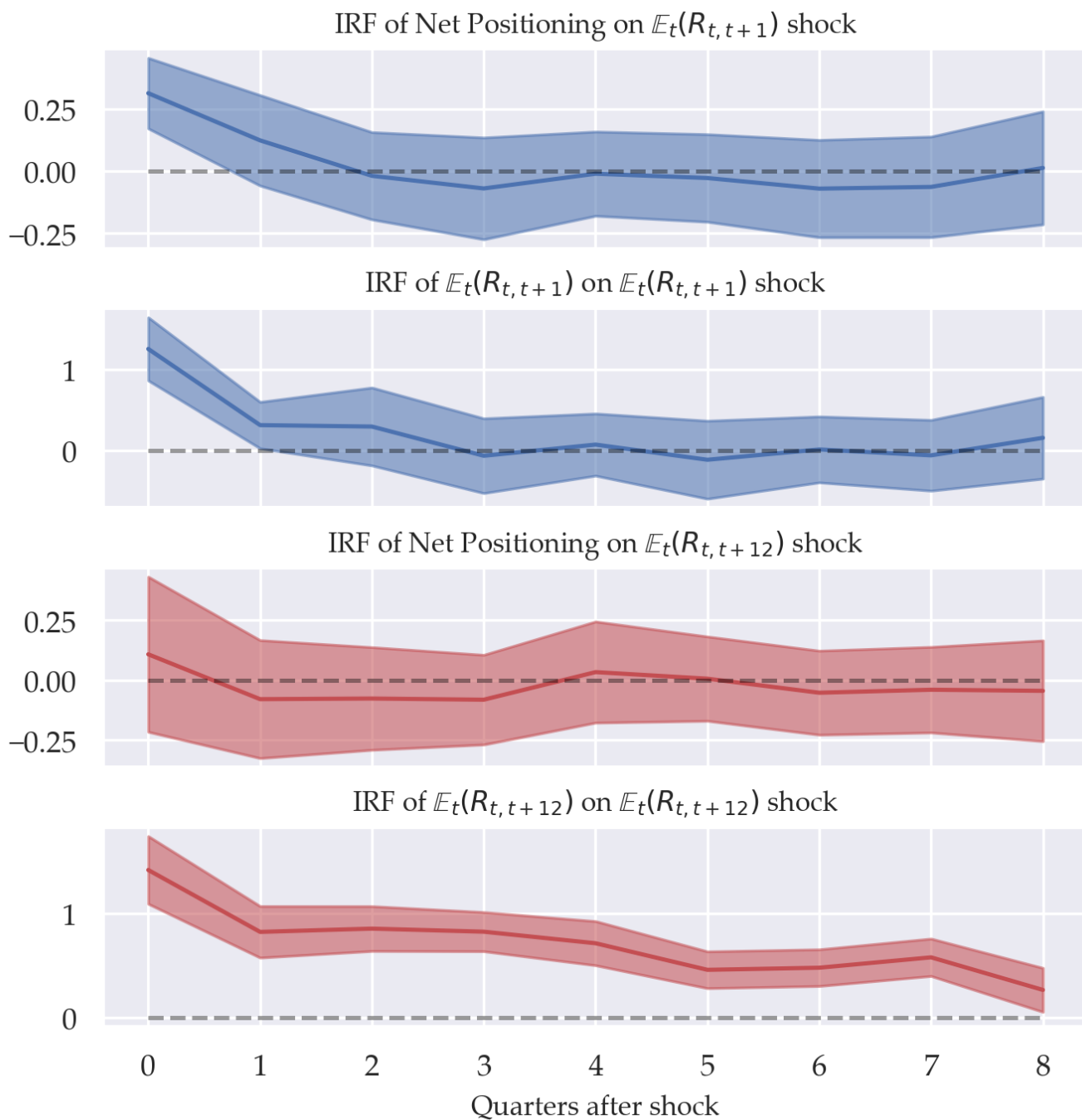


FIGURE C.4: FUTURES POSITIONS AND RETURN EXPECTATIONS (INDIVIDUAL INVESTORS)

Note: The figure replicates Figure 2 for the individual investor subsample.

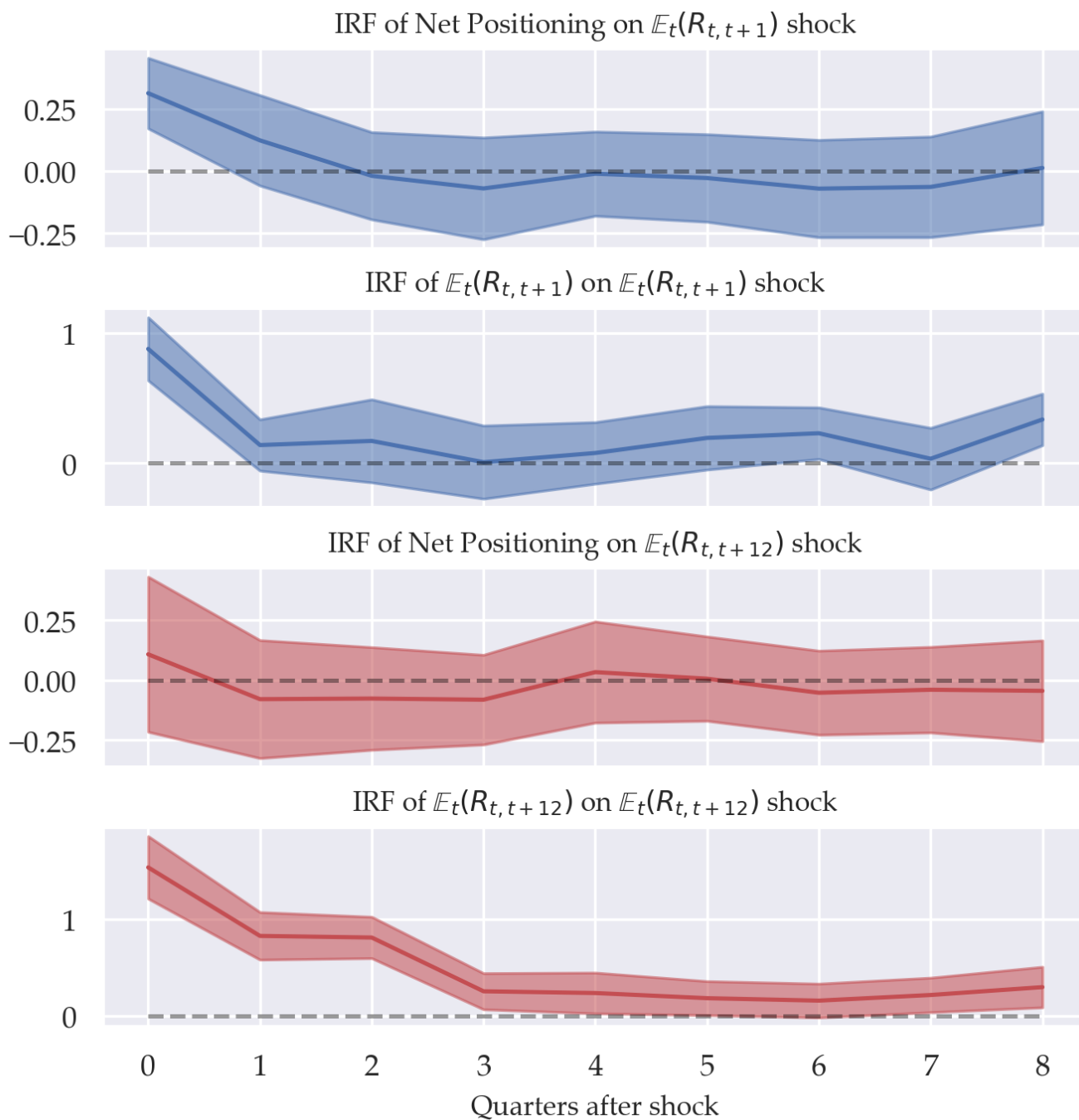


FIGURE C.5: FUTURES POSITIONS AND RETURN EXPECTATIONS (INSTITUTIONAL INVESTORS)

Note: The figure replicates Figure 2 for the institutional investor subsample.

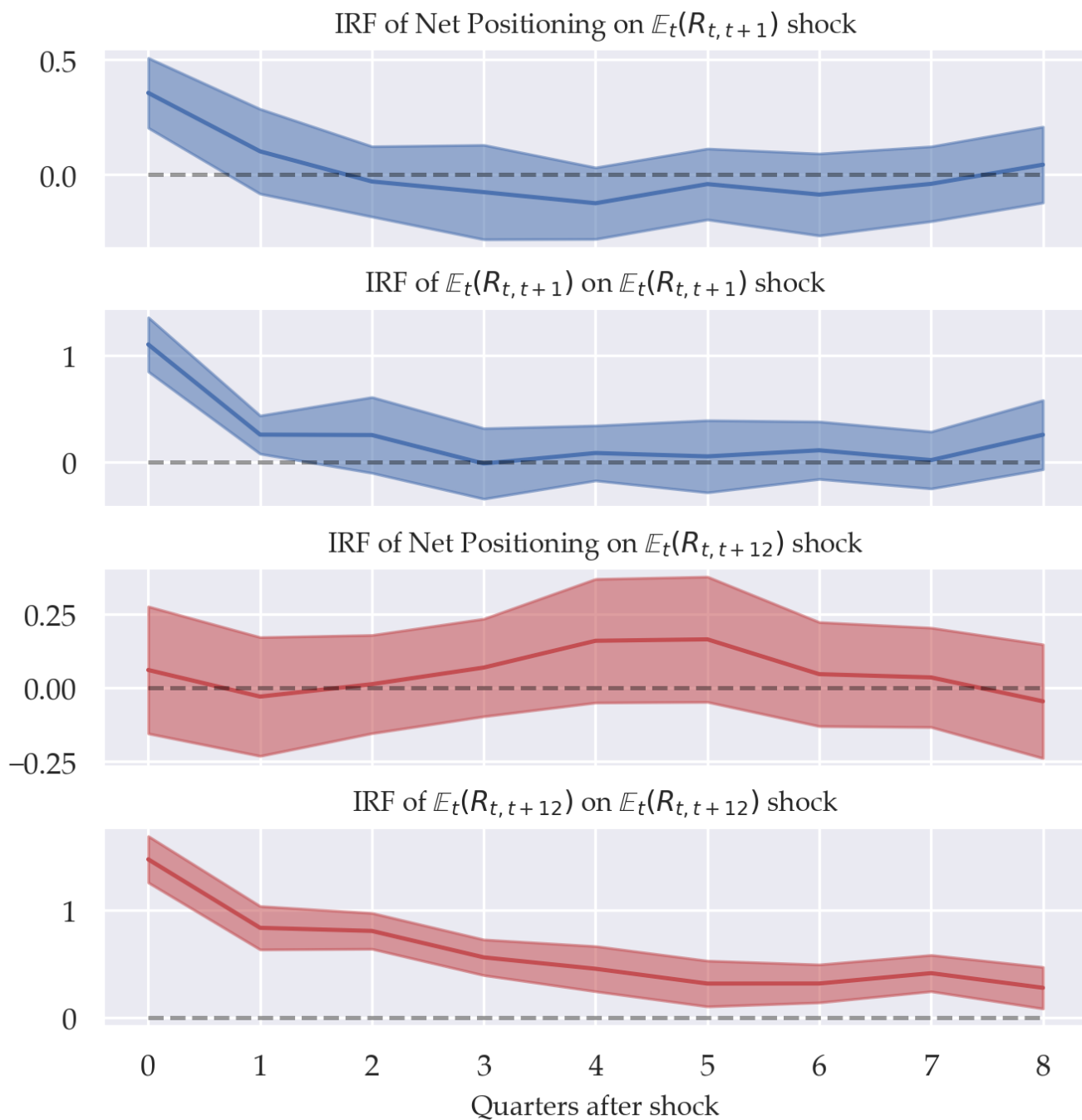


FIGURE C.6: FUTURES POSITIONS AND RETURN EXPECTATIONS (ASSET MANAGERS)

Note: The figure replicates Figure 2 using the futures positions of asset managers.

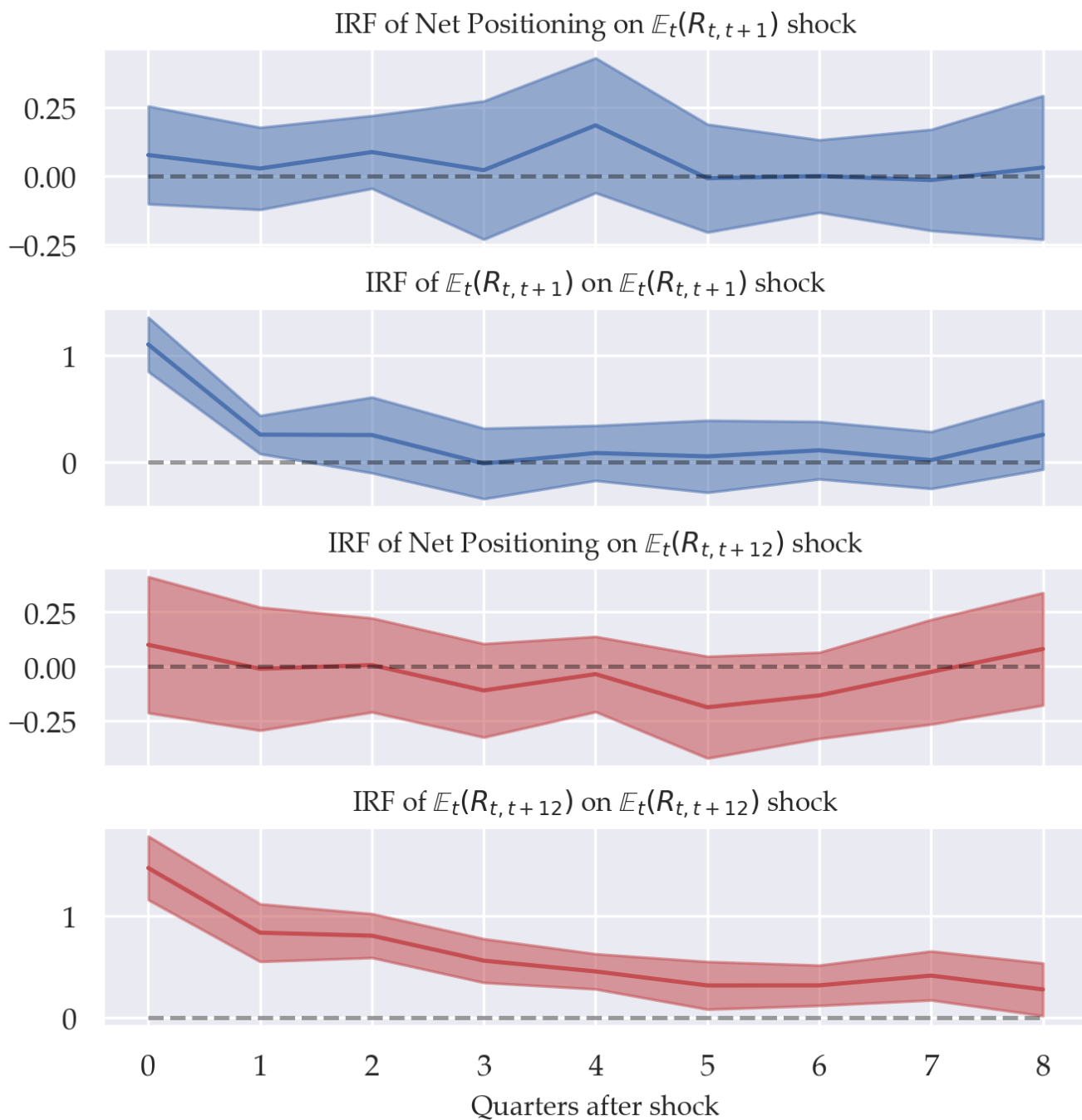


FIGURE C.7: FUTURES POSITIONS AND RETURN EXPECTATIONS (LEVERED MONEY)

Note: The figure replicates Figure 2 using the futures positions of leveraged money investors.