Speculating on Higher Order Beliefs*

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July 28, 2023

Abstract

Higher order beliefs – beliefs about others’ beliefs – may be important for trading behavior and asset prices, but have received little systematic empirical examination due to challenges in measurement. We study more than twenty years of evidence from the Robert Shiller Investor Confidence surveys, which directly elicit details on individual and institutional investors’ higher order beliefs about the U.S. stock market. We find that investors’ higher order beliefs provide substantial motivations for non-fundamental speculation, e.g., to buy into a stock market perceived to be overvalued. Guided by the evidence, we construct a theoretical model that reveals that higher order beliefs may substantially amplify stock market fluctuations. When investors exhibit the same fundamental belief biases that they attribute to other investors, phenomena such as overreaction, momentum, and reversal can persist in equilibrium even though everybody knows about them.

*This is a substantially revised version of “Asset Pricing with Higher Order Beliefs.” We thank Federica Ambrosio, Natee Amornsiripanitch, Marios Angeletos, Nick Barberis, Paul Fontanier, Stefano Giglio, Will Goetzmann, Leandro Gomes, Paul Goldsmith-Pinkham, Gary Gorton, Peter Hansen, Zhen Huo, Jon Ingersoll, Toomas Laarits, Yukun Liu, Ben Matthies, Toby Moskowitz, Milad Nozari, Cameron Peng, Julia Selgrad, Kelly Shue, Alp Simsek, Sam Slocum, Aleh Tsyvinski, Joao Paulo Valente, and seminar participants at Yale SOM, Purdue Daniels, Chicago Booth, Caltech, UNC Kenan-Flagler, Harvard Business School, Emory Goizueta, the New York Fed, NYU Stern, Columbia Business School, the Treasury OFR, BC Carroll, and Stanford GSB for helpful comments and conversations. We are grateful to Bob Shiller for sharing data, and the Yale ICF for data support.

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Higher order beliefs – beliefs about others’ beliefs – are important in many equilibrium models in economics and finance. As emphasized by a strand of primarily theoretical literature starting with Keynes (1936), higher order beliefs may be particularly important for understanding investor behavior and financial market fluctuations. This is because investors have strong profit-based incentives to trade based on predictions of others’ beliefs rather than their own valuations, potentially leading asset prices to deviate substantially from fundamental values.\(^1\) Despite their ubiquity and potential importance, higher order beliefs have received little systematic empirical examination, primarily due to inherent challenges in their measurement.

In this paper, we examine survey data on investors’ higher order beliefs, and consider the corresponding asset pricing implications. Our data come from the Robert Shiller Investor Confidence survey, which directly elicits investors’ perceptions of other investors’ beliefs. The survey also reports investors’ return expectations, which embed additional information about higher order beliefs. This is because an investor’s return expectations reflect her forecasts of other investors’ future stock market demand, which, in turn, depend on her forecasts of their beliefs. Guided by the evidence, we construct a theoretical model that illustrates the potential importance of higher order beliefs in driving stock market fluctuations. Below, we summarize our empirical and theoretical findings.

**Empirical Results.** While previous work has documented episodes of non-fundamental speculation – e.g., buying into risky assets perceived to be overvalued – our headline finding is that for the U.S. stock market, *non-fundamental speculation is the rule, not the exception.*\(^2\) Higher order beliefs play an important role in driving non-fundamental speculation.

To illustrate this point, we examine the Shiller survey, which, for more than 20 years, has asked individual and institutional investors a number of questions regarding their stock market expectations. Particularly relevant for our exploration, the survey asks investors if they perceive other investors to be overly optimistic (or pessimistic) about the U.S. stock market’s prospects, as well as if they perceive the stock market to be over- or under-valued. The majority of survey respondents report that others have mistaken beliefs, and the direction of investors’ reported higher order beliefs coincide with their perception of the under- or over-valuation of the stock market.

We find that when investors report that others are overly optimistic, they also report expectations of higher than average returns for the short term (1-to-3 months ahead), before expecting reversion in subsequent periods. A natural, higher order belief-based interpretation is that investors forecast other investors will become even more optimistic in the near future.

\(^{1}\)For example, De Long et al. (1990) note that George Soros “has been generally successful... by betting not on fundamentals but, he claims, on future crowd behavior.” Brunnermeier and Nagel (2004) argue the evidence from the dot-com bubble is “consistent with the view that hedge fund managers were able to predict some of the investor sentiment that was arguably behind the wild fluctuations in valuations of technology stocks.”

\(^{2}\)This is speculation in the sense of Keynes (1936) and Harrison and Kreps (1978). We use the prefix ‘non-fundamental’ to distinguish from another common use of the term, where investors buy or sell an asset based on their relative optimism and pessimism about the asset’s fundamental value, e.g., Simsek (2021).
term, fueling short horizon returns, before optimism and prices revert. These expectations provide investors with strong incentives to buy into a stock market they see as overvalued. Consistent with such speculation, surveyed investors recommend holding positions aligned with their short term expectations, and misaligned with their valuations. Moreover, investors’ short term return expectations have strong explanatory power for the stock market positioning of asset managers. The speculative motives accompanying differently signed short and long term return expectations may help explain the insensitivity of investors’ stock allocations to their long term return expectations (Giglio et al. (2021)).

We also explore the source of investors’ higher order beliefs. We find that public macroeconomic news plays a key role in driving investors’ higher order beliefs. For example, following positive macroeconomic news, investors report beliefs that others have become increasingly optimistic and markets have become overvalued. At the same time, they report expectations of higher short term returns followed by lower long term returns, which are also consistent with a belief in momentum and reversal. We interpret the evidence as indicating that investors believe that other investors overreact to fundamental news, and will continue to overreact, which induces them to engage in non-fundamental speculation.

**Model.** Our empirical results pose a challenge for existing models, as we discuss in more detail in the main text. Most notably, models without an explicit focus on higher order beliefs cannot simultaneously explain investors’ return expectations, their stock market valuations, and their perceptions of other investors’ beliefs.

To rationalize the evidence and explore the implications of investors’ higher order beliefs, we construct a theoretical model. The model features a riskless asset and a risky asset (the stock market) that pays a publicly observed dividend each period, which is drawn from persistent but unobserved fundamentals. There is a mass of identical short-lived Bayesian investors, who trade against a downward sloping demand curve for the risky asset. Investors form their beliefs about the unobserved fundamentals each period by placing some weight (their Kalman gain) on the observed dividend, and some weight on their prior belief. Investors trade based on their forecasts of short term returns, which are determined by how they believe other investors (“the market”) will update their beliefs in the near term. Investors hold fixed beliefs about others’ belief updating, and attribute information from prices inconsistent with their higher order beliefs to persistent noise.

Our first result is that we can capture the empirical evidence under the assumptions that investors believe that other investors (1) persistently update their beliefs in the direction of past news (others’ Kalman gains are too low) and (2) have valuations that overreact to news, driven by overestimation of the persistence of fundamentals. The two belief updating frictions that investors attribute to others coincide with effectively identical frictions that can explain the forecast errors of macroeconomic forecasters (Angeletos, Huo and Sastry (2021)).

Our second and third results explore the asset pricing implications of investors’ higher order beliefs. Investors’ demand is strategically substitutable with others’ present demand and strategically complementary to others’ future demand, so the driver of investment choices is the forecasted change in others’ beliefs.
order beliefs. Our second result is that investors’ higher order beliefs can amplify asset price fluctuations even when all investors have correct fundamental beliefs. Investors’ beliefs that others will become more optimistic and continue to buy into the risky asset in the near future pushes asset prices to overreact to fundamental news in the present. Investors willingly buy into the market at a valuation that exceeds their own due to their forecasts of others’ increasing optimism. This speculative behavior results in overreaction to fundamental news, excess volatility, and predictable reversals.

Our third result relaxes the assumption that investors have correct fundamental beliefs, and considers the interaction between fundamental and higher order beliefs. When investors are subject to the same belief updating biases they attribute to other investors, the risky asset can exhibit momentum in addition to overreaction and reversal. This is because investors continue to update their fundamental beliefs in periods following news, which, in turn, leads them to update their higher order beliefs and return expectations, fueling further price movements. This result provides one perspective on the question “How can a strategy everyone knows about still work?” (Asness (2015)). Investors in the model all know about momentum, reversal, and overreaction, but they mistakenly attribute other investors as driving those phenomena, and the phenomena persist in equilibrium.

In our second and third results, investors neglect the information content of prices – they attribute information inconsistent with their beliefs to persistent noise. Our final result examines how higher order beliefs influence inference from prices. When investors extract information from prices, their perceptions of others’ belief mistakes may induce them to extract a biased signal from prices about fundamentals. That is, higher order beliefs can cause mistakes in fundamental beliefs. This result complements recent work indicating that (mis)inference from prices may contribute to fundamental belief biases found in survey data (e.g., Bordalo et al. (2021), Bastianello and Fontanier (2022a), Chaudhry (2023)).

**Related Literature.** Our paper relates to work on higher order beliefs in asset pricing, to which we bring empirical discipline using survey data. The literature on higher order beliefs in asset pricing can be partitioned into two traditions: noisy rational expectations models, where rational investors face frictions that prevent them from observing others’ beliefs and fundamentals (Singleton (1987), Allen, Morris and Shin (2006), Bacchetta and van Wincoop (2006, 2008), Makarov and Rytchkov (2012), Kasa, Walker and Whiteman (2014), Cespa and Vives (2015), and Nimark (2017)); and differences-of-opinion models, where investors know and disagree with other investors’ private valuations (Harrison and Kreps (1978), Harris and Raviv (1993), Kandel and Pearson (1995), Scheinkman and Xiong (2003), and Banerjee and Kremer (2010)). Relative to both, our paper introduces empirically motivated ingredients to characterize investors’ higher order beliefs. Our paper also highlights survey evidence that may be useful for future work – namely direct questions about in-

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4There are two notable exceptions that empirically examine higher order beliefs. Egan, Merkle and Weber (2014) survey private investors and find that beliefs about others’ return expectations affect investment decisions. Coibion et al. (2021) survey firm managers in New Zealand, with a focus on testing noisy information models.  
5A literature in macroeconomics also considers higher order beliefs, e.g., see Angeletos and Lian (2022).
vestors’ higher order beliefs and data on the term structure of expected returns.

While the literature on higher order beliefs emphasizes higher orders of reasoning (e.g., investors may form third order beliefs about others’ second order beliefs, and so on and so forth), our model assumes that investors only form second order beliefs and treat other investors as non-strategic. Multiple structures of belief hierarchies can rationalize the same distribution of equilibrium returns and return expectations. Return expectations capture the aggregated effect of all levels of higher order reasoning. Our model is but one of many that may explain the data, and illustrates ingredients that can capture the empirical evidence. Moreover, our model can be thought of as representing a form of level-k thinking, which places bounds on people’s strategic sophistication. Level-k thinking finds strong empirical support (Eyster (2019)), and has been used to construct tractable models of financial market behavior (e.g., Eyster, Rabin and Vayanos (2019), Bastianello and Fontanier (2022a,b)).

Our paper also relates to a literature on non-fundamental speculation in financial markets. The literature documents a number of instances of non-fundamental speculation in bubble-like episodes (e.g. McKay (1841), Kindleberger (1978), Temin and Voth (2004), Brunnermeier and Nagel (2004), Soros (2015)) and prominent theoretical work on the topic includes De Long et al. (1990), Harrison and Kreps (1978), Scheinkman and Xiong (2003), Abreu and Brunnermeier (2002, 2003), and Martin and Papadimitriou (2022). Our paper illustrates that non-fundamental speculation is a pervasive feature of the U.S. stock market, and illuminates role that higher order beliefs may play in giving rise to such speculation.

Finally, our paper is related to a literature in finance using survey data to understand market participants’ beliefs (Adam and Nagel (2022) provide a survey). A sizeable literature has studied the importance of fundamental and return expectations for investor behavior and stock market returns. Work on return expectations includes Vissing-Jorgensen (2003), Bacchetta, Mertens and van Wincoop (2009), Greenwood and Shleifer (2014), Amromin and Sharpe (2014), Barberis et al. (2015, 2018), Adam, Marcet and Beutel (2017), and Nagel and Xu (2022b), while work on fundamental expectations includes Chen, Da and Zhao (2013), Bordalo et al. (2020), De La O and Myers (2021), and Nagel and Xu (2022a). Our results bridge together fundamental and return expectations via higher order beliefs. See also Giglio et al. (2021), Jin and Sui (2022), and McCarthy and Hillenbrand (2021) for work relating fundamental and return expectations.

The rest of the paper proceeds as follows. In Section 1, we discuss survey data and present empirical evidence on investors’ return expectations. In Section 2, we present a theoretical asset pricing model, illustrate how higher order beliefs may explain the empirical evidence, and explore the corresponding asset pricing implications. Section 3 concludes.

Prior work on return expectations largely focuses on expectations of returns at a fixed future horizon (e.g., one-year ahead). We place particular focus on the future path of returns that investors expect. In contemporaneous work, Gandhi, Gormsen and Lazarus (2022) also study the term structure of return expectations.
1 Empirical Evidence from Survey Data

We study expectations of U.S. equity market returns reported by retail and institutional investors in the Robert Shiller Investor Confidence survey. Whereas other available investor surveys mostly ask a single or a few questions to investors about their return expectations at a fixed horizon, the Shiller survey is unique in providing a long time-series where investors are simultaneously asked about their higher order beliefs, their stock market valuations, and their return expectations over multiple horizons. This makes it particularly well-suited for studying the questions of interest.

We find that investors often believe that other investors hold incorrect stock market valuations, but find it profitable to speculate in the direction of these incorrect valuations. We also find that investors report a belief that stock markets overreact to news upon its arrival, and report return expectations that are consistent with the stock market exhibiting momentum and reversal. We discuss the ingredients required of higher order beliefs to match the empirical evidence, and the predictions made by existing models, which are able to match some, but not all of the evidence.

1.1 Data Description

The main data used in our empirical analysis come from the Shiller surveys, which are the microdata underlying the Robert Shiller Stock Market Confidence indices. The survey data have been collected continuously since 1989 – semi-annually for a decade, and then monthly by the International Center for Finance at the Yale School of Management since July 2001. Shiller (2000) discusses the survey questions in more detail. For our analysis, we focus on the continuous sample from July 2001 through April 2023.

1.1.1 Survey Respondents

The surveys are conducted by a market survey firm, which mails 500 surveys to high net-worth individual investors, and 500 surveys to institutional investors each month, with a sampling goal of 20 to 50 responses by each of the two types - individual and institutional. For both institutional and individual investors, the investor mailing lists are purchased from Data Axle (previously known as InfoUSA).

The micro data do not provide detailed demographic information on survey respondents (or non-respondents). There is likely to be selection into responding to the survey, as in other work surveying investors. For example, Giglio et al. (2021) find in a survey of Vanguard investors that their survey respondents tend to be older, wealthier, more likely to be male, and tend to trade more often than nonrespondents. The selection criteria for both individual and institutional investors in the Shiller survey, and the data that are available on investor characteristics, indicate that individual investor respondents are likely to have high income and be wealthy, and that institutional respondents manage large portfolios. While likely
not fully representative of the investor population, survey respondents are a substantial and important class of investors. In Appendix C.1, we test and find no evidence of systematic business cycle variation in the number of responses to the survey.

For individual investors, the mailing list for the surveys is constructed by sampling households with a household income of greater than $150,000 per year from the Infogroup Consumer Database. We have no additional demographic information on the respondents.

For institutional investors, the mailing list is constructed by sampling companies from the Infogroup Business Database with the SIC codes 628202 (Investment Management), 628203 (Financial Advisory Services), 628204 (Financing Consultants), and 628205 (Financial Planning Consultants). Survey respondents are asked to provide the ‘Size of the common stock portfolio(s) you make decisions about.” In the sample, the 25th, 50th, and 75th percentiles of responses are $1.8 million, $25 million, and $100 million. Summing across respondents by month, the 25th, 50th, and 75th percentiles of the sum of responses are $642 million, $1.57 billion, and $9.00 billion.

1.1.2 Questions of Interest

Especially relevant to us, the survey data contain investor responses to a series of questions regarding investors’ beliefs about other investors’ beliefs, their beliefs about stock market valuations, and their expectations of returns over different time horizons. These questions include (with responses in parentheses):

(i) Questions regarding higher order beliefs

(a) Many people are showing a great deal of excitement and optimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them. (True; False; No opinion)

(b) Many people are showing a great deal of pessimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them. (True; False; No opinion)

(ii) Questions regarding stock market returns, valuations, and behavior

(a) How much of a change in percentage terms do you expect [for the Dow Jones index] in the following 1 month? 3 months? 6 months? 1 year? 10 years?

(b) Stock prices in the United States, when compared with measures of true fundamental value or sensible investment value are (Too low; Too high; About right; Do not know)

(c) Although I expect a substantial drop in stock prices in the U.S. ultimately, I advise being relatively heavily invested in stocks for the time being because I think that prices are likely to rise for a while (True, False, No opinion; if True, indicate best guess for date of peak).

(d) Although I expect a substantial rise in stock prices in the U.S. ultimately, I advise being less invested in stocks for the time being because I think that prices are likely to drop for a while (True, False, No opinion; if True, indicate best guess for date of bottom).
(iii) **Questions regarding drivers of higher order beliefs**

(a) *What do you think is the cause of the trend of stock prices in the United States in the past six months? (It properly reflects the fundamentals of the U.S. economy and firms; It is based on speculative thinking among investors or overreaction to current news; Other (write-in); No opinion)*

Answers to question (i.a) and (i.b) directly provide information regarding investors’ higher order beliefs. The questions themselves may submit multiple interpretations, especially given the multi-part nature of the questions, and the fact that respondents may answer ‘True’ to both questions. We present cross-sectional and time-series evidence in this section that when investors respond that they believe many others are overly optimistic and they must be careful not to be influenced by them, they also report beliefs that stock markets are overvalued, and that they expect long term stock market returns to be low. Based on this evidence, investors appear to interpret these questions as asking:

(i.a) *I believe that many other investors hold overly optimistic stock market valuations.*

(i.b) *I believe that many other investors hold overly pessimistic stock market valuations.*

While there may be some noise associated with assigning this interpretation to the question – it is likely that some respondents may have differing interpretations – the interpretation is consistent with responses to other questions on average and is informative about investors’ views. This is especially the case in light of the long-time series of evidence relative to other surveys that may ask similar questions.

Answers to questions (ii.a-e) provide information about investors’ return expectations at different horizons, and the behavior associated with those expectations. These questions provide additional validation for our interpretation of the questions regarding investors’ higher order beliefs. Moreover, the questions on investors’ return expectations at different horizons encode additional information about investors’ higher order beliefs, in particular capturing how investors expect others’ beliefs to evolve over time. Questions (ii.c-e) also link investors’ higher order beliefs and return expectations with potential speculative trading behavior, as we discuss in more detail.

While question (ii.a) asks investors to explicitly report numerical values for their expected stock market returns, the other questions are provided in multiple choice format. For empirical analysis, we map answers for most of the questions into numerical values. For questions (i.a-b) and (ii.c-d), we map the responses (True; False; No opinion) to (1, -1, 0), so that higher numbers indicate increasing agreement with the questions. For question (ii.b), we map the responses (Too low; Too high; About right; Do not know) to (-1; 1; 0; 0), so that lower numbers correspond with perceived undervaluation and higher numbers.

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7 There is particular ambiguity about the meaning of the second part of the questions – ‘I must be careful not to be influenced by them.’ Based on the evidence that we present in this section comparing investors’ responses across questions, particularly questions (ii.c) and (ii.d), investors seem to indicate that they believe others’ valuations may be overly- optimistic or pessimistic, but nevertheless that others’ valuations still enter into their investment decisions given the influence that others have on short term market returns.
correspond with perceived overvaluation; we find our results are robust to dropping observations where investors report ‘Do not know.’

1.2 Summary Statistics

We begin by summarizing responses in the Shiller survey. Table 1 reports summary statistics over the full sample in response to the relevant questions. For all questions except for question (ii.a), the table reports the proportion of respondents in the sample that gave a specific answer in response to that question; for question (ii.a), the table reports expected returns for 1-, 3-, 6-, and 12-months ahead, averaged across survey respondents. The table reports statistics separately for individual and institutional investor respondents; the results are qualitatively and quantitatively similar for the two groups.

The first two rows of the table show the proportion of respondents reporting that they believe many other investors to be overly optimistic or pessimistic about the prospects for the stock market. Among individual investors, 59% report that they believe many others to be overly optimistic and 62% report that they believe many others to be overly pessimistic (for institutional investors, the proportions are 52% in response to both questions). These results indicate that several respondents simultaneously indicate a belief that many others are overly optimistic and overly pessimistic. They also suggest the presence of substantial disagreement about the stock market; the majority of respondents in the sample report that other investors have incorrect beliefs.

The third and fourth rows of the table report the proportion of surveyed investors indicating that they either eventually expect stock prices to drop, but advise being overweight stocks because they expect the market to rise in the near term; or alternatively, that they expect stock prices to rise in the long term, but expect them to fall in the short term, and accordingly recommend being underweight. Among individual investor responses, 35% report that they expect markets to drop but recommend being overweight, while 35% report that they expect markets to eventually rise, but recommend being underweight (the numbers are 33% and 32% for institutional investors). Given that the questions are logically mutually exclusive, the responses suggest that a substantial majority of investors expect short- and long-term returns to be differently signed. This result is notable, because it suggests the presence of non-fundamental speculative motivations on the part of investors, e.g., many investors that believe markets to be presently overvalued (and that market returns will be low in the long run) still recommend being overweight stocks due to potential short term profits from doing so.

The fifth row of the table reports average expected stock market returns. On average, both individual and institutional investors report small return expectations for the next

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8By-and-large, survey respondents recognize that the questions are mutually exclusive, though some do not. Only 7.2% of institutional investors and 7.8% of institutional responses are True to both questions (ii.c) and (ii.d) at the same time. Among individual investors, 71% that respond True to (ii.c) response False to (ii.d) and 73% that report True to (ii.d) report False to (ii.c); these numbers are 70% and 72% for institutional investors.
month, with an expectation of more positive return expectations from 3- to 6-months ahead. Return expectations for 12-months ahead are 3.7% on average for individual investors and 4.9% on average for institutional investors.

The sixth row of the table reports investors’ responses regarding stock market valuations vis-a-vis fundamental values. Among individual investor responses, 11% indicate that stock valuations are low relative to fundamentals, 37% say they are high, 44% say they are about right, and 8% express no opinion (these numbers are 19%, 30%, 49% and 2% for institutional investors).

The last row of the table reports investors’ responses regarding stock market trends. Among individual investor responses, 24% indicate that market movements properly reflect fundamental news, while 52% indicate that the movements reflect speculative thinking and overreaction by other investors; these numbers are 28% and 37% among institutional investors.

1.3 Higher Order Beliefs and Perceived Market Valuations

We next examine responses to questions (i.a) and (i.b), regarding other investors’ optimism and pessimism. We provide evidence consistent with respondents generally interpreting the questions to indicate that other investors are overly optimistic or overly pessimistic in their stock market valuations. We do so by analyzing the relationship of investors’ responses to questions (i.a) and (i.b) with investors’ responses to question (ii.b), regarding their perceptions of stock market valuations vis-a-vis fundamentals. If investors believe others are too optimistic in their valuations, then we expect them to indicate a belief that the stock market is overvalued; similarly, we expect those that believe others are too pessimistic to indicate the stock market is undervalued. We find exactly this result.

We construct two variables: Higher Order Optimism and HO Pessimism, which map the responses to questions (i.a) and (i.b), (True; False; No Opinion), to the values (1; -1; 0). The HO Optimism variable is increasing in agreement with the statement about other investors’ optimism, while the HO Pessimism variable is increasing in agreement with the statement about other investors’ pessimism. We regress the HO variables on an Overvaluation variable, which we compute by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). Higher values of Overvaluation correspond with higher stock market prices relative to fundamentals.

Table 2 reports the regression results. Columns 1 to 3, on which we focus in our discussion, report regression results pooling together individual and institutional investor responses. Columns 1 and 2 report results from regressions using survey response level observations. With month fixed effects (column 2), the regressions capture cross-sectional comparisons, for example, whether an investor that believes others are more optimistic also is more likely to believe the stock market is overvalued. We also run the regressions as time-
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<td>19%</td>
<td>30%</td>
<td>49%</td>
<td>2%</td>
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<td></td>
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<tr>
<td>(iii.a) Cause of 6-month market trend</td>
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<tr>
<td>24%</td>
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<td>22%</td>
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<td>37%</td>
<td>34%</td>
<td>1%</td>
<td></td>
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</tr>
</tbody>
</table>

**Total Number of Responses**

6688

5591

**Table 1: Shiller Survey Summary Statistics**

*Note:* The table reports summary statistics of survey responses to the Shiller surveys, reporting statistics separately for individual and institutional investor respondents. For questions (i,a,b), (ii,b,c,d) and (iii,a), the table reports the proportion of survey respondents in the sample that gave a particular answer in response to a given question. For question (ii,a), the table reports the average $k$-month ahead expected return reported by respondents. Questions and potential responses are presented in abbreviated form in the table; details on the questions and responses are provided in Section 1.1.
series regressions (column 3), by using cross-sectional monthly averages of the variables as the observations in our regressions. The time-series regressions capture whether, for example, in time periods where investors believe others to be more optimistic, they are also more likely to believe that markets are overvalued.

Panel A reports results where \textit{HO Optimism} is the dependent variable. Across each of the regression specifications, we find a strong relationship between \textit{HO Optimism} and \textit{Overvaluation}, providing strong support that respondents interpret question (i.a) as indicating that other investors are overly optimistic about stock market valuations.

With survey responses as the unit of observation, the coefficient on \textit{Overvaluation} is 0.34 without fixed effects, and 0.29 with fixed effects. These coefficients can loosely be interpreted as indicating that relative to an investor that reports they believe the stock market properly reflects fundamentals, an investor that reports they believe the stock market is overvalued is about 30\% more likely to indicate that many other investors are overly optimistic. In the time-series regression with monthly averages as the unit of observation, the coefficient on \textit{Overvaluation} is 0.63 ($R^2$ of 0.37), indicating an especially strong time-series relationship between the variables.

Panel B reports results where \textit{HO Pessimism} is the dependent variable. The coefficients are -0.14 in response-level regression with no fixed effects, -0.13 in the response-level with time fixed effects, and -0.23 in the regression using monthly averages as observations. These results once again indicate a strong relationship between \textit{Overvaluation} and \textit{HO Pessimism}, though weaker than the one with \textit{HO Optimism}.

Panel C reports results where the dependent variable is \textit{HO Belief}, which we define as \textit{HO Optimism} - \textit{HO Pessimism}. The measure captures a given respondent’s higher order belief, incorporating information from their responses about others’ optimism and pessimism. The coefficients are 0.48 in the response-level regression with no fixed effects, 0.41 in the response-level with time fixed effects, and 0.85 in the regression using monthly averages as observations, with $R^2$ values of 0.08, 0.14, and 0.34. For the rest of our empirical analysis in the main text, we focus on the \textit{HO Belief} measure, which incorporates the information captured by both the \textit{HO Optimism} and \textit{HO Pessimism} measures.

In each of the panels in Table 2, columns 4-6 report regression results using the sample of individual investors and columns 7-9 report regression results using the sample of institutional investors. The qualitative results are the same across the individual and institutional investor samples, with some slight quantitative variation. For brevity, we report results in the pooled sample for the rest of the paper and provide additional breakdowns of the results for the individual versus institutional investor samples in the appendix. We note that our main conclusions are largely similar for individual and institutional investors.

The results indicate a strong relationship between the \textit{HO Belief} and the \textit{Overvaluation} measures; when investors report that they think markets are overvalued, they are substantially more likely to report many other investors are optimistic, and less likely to report that other investors are pessimistic.
### Panel A: $y=HO$ Optimism

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<tr>
<td>Time FE</td>
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<td>Yes</td>
<td>NA</td>
</tr>
<tr>
<td>N</td>
<td>12135</td>
<td>12135</td>
<td>259</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.04</td>
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### Panel B: $y=HO$ Pessimism

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</thead>
<tbody>
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<td>Overvaluation</td>
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<td>-0.13</td>
<td>-0.16</td>
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<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
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<tr>
<td>Time FE</td>
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<td>NA</td>
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<tr>
<td>N</td>
<td>12135</td>
<td>12135</td>
<td>259</td>
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<tr>
<td>$R^2$</td>
<td>0.01</td>
<td>0.01</td>
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### Panel C: $y=HO$ Belief:=HO Optimism - HO Pessimism

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<td>(0.02)</td>
<td>(0.03)</td>
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<td>Time FE</td>
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<td>Yes</td>
<td>NA</td>
</tr>
<tr>
<td>N</td>
<td>12135</td>
<td>12135</td>
<td>259</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.08</td>
<td>0.05</td>
<td>0.12</td>
</tr>
</tbody>
</table>

### Table 2: Higher Order Optimism and Perceived Valuations

*Note:* The table reports results from regressions of Higher Order Belief variables constructed from the Shiller surveys on a perceived Overvaluation measure constructed from the surveys. The HO Optimism and HO Pessimism measures are constructed by mapping the responses to questions (i.a) and (i.b) regarding other investors’ optimism and pessimism, (True; False; No Opinion), to the values (1; -1; 0). The Overvaluation measure is constructed by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-a-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). Columns 1-3 pool together observations across the individual and institutional investor samples, and columns 4-6 and 7-9 separately report results for the two samples. The unit of observation for Columns 3, 6, and 9 is the monthly cross-sectional average of the variables; Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observations for columns 1, 2, 4, 5, 7, and 8 are survey responses; Driscoll-Kraay standard errors (12 lags) for coefficients are reported in parentheses.
1.4 Higher Order Beliefs and Return Expectations

We next turn to studying the relationship between higher order beliefs and return expectations of different horizons. Responses to questions (i.a) and (i.b) capture investors' beliefs about other investors' present beliefs. Return expectations capture how stock prices are expected to evolve in the future, embedding beliefs about other investors' future stock market demand and beliefs. That is, return expectations capture an additional dimension of higher order beliefs that is important for understanding investor behavior.

We regress investors' reported return expectations of different horizons (multiplied by 100) on the \( HO \) Belief variable. Panel A of Table 3 reports the regression results.

The first four columns report regression results for time-series regressions, where the unit of observation is the cross-sectional average of survey responses in a given month. The coefficient on \( HO \) Belief for 1-month return expectations is 1.57, indicating that a unit increase in the \( HO \) Belief variable corresponds with a 1.57% higher expected return for the following month. The coefficients decline with horizon; the coefficients on 3-, 6-, and 12-month return expectations are 0.59, -0.32, and -2.46. Appendix Table C.4 verifies that these results hold for both \( HO \) Optimism and \( HO \) Pessimism as independent variables.

The results reveal an interesting, hump-shaped pattern of cumulative return expectations corresponding with higher order beliefs. On average, in periods where investors report a belief that other investors are overly optimistic (and that stock markets are overvalued), they expect stock markets to rise over the next month – they expect stock market demand to rise. Though other factors may contribute, a natural, higher order belief-based interpretation is that investors expect others to become even more optimistic in the short term, fueling the increasing demand.

These expectations provide motivations for non-fundamental speculative trade, whereby investors may choose to take long (or overweight) positions in the stock market even when they perceive it to be overvalued, because they perceive the market may continue to rise before valuations correct and returns are lower.

The last four columns in Panel A of Table 3 report regression results for cross-sectional regressions, which include month fixed-effects, and where the unit of observation is a survey response. The coefficients for 1-, 3-, 6-, and 12-month ahead returns in the cross-sectional regressions are 0.05, -0.31, -1.03, and -1.79. The results indicate that in cross-sectional comparisons, an investor that holds a stronger belief that others are optimistic about stock market valuations does not necessarily believe that short term returns will be higher than an investor that holds a weaker belief that others are optimistic about stock market valuations, though they do expect worse long-term market performance. That is, the hump-shaped pattern of cumulative return expectations is largely a time-series phenomenon.

We provide additional validation for the time-series pattern of investors' return expectations by using investors' responses to questions (ii.c) and (ii.d), which ask investors whether they expect the stock market to reach a peak (trough) in the short run though they expect it to decline (rise) in the long run. We construct \( \text{Short-Term Peak} \) and \( \text{ST Trough} \) variables...
by mapping responses to questions (ii.c) and (ii.d), (True; False; No Opinion), to the values (1, -1, 0). We run time-series regressions of ST Peak and ST Trough on HO Belief and Overvaluation, with cross-sectional monthly averages as the unit of observation.

Panel B of Table 3 reports the regression results. The first two columns report results where the independent variable is HO Belief. The coefficient on HO Belief is 0.41 for ST Peak ($R^2$ of 0.22) and -0.28 for ST Trough ($R^2$ of 0.16). These results provide important validation of the relationship between investors’ beliefs regarding others’ optimism and their belief that markets will continue to rise before eventually declining, using a qualitative elicitation method. This evidence also helps assuage potential concerns with respondents struggling in providing quantitative responses.

Affirmative responses to (ii.c) and (ii.d), used to construct the ST Peak and ST Trough measures, also indicate investors’ recommendations to be overweight stocks even though they expect an eventual decline in stocks, or underweight despite expecting an eventual rise in stocks. The relationship between HO Belief, ST Peak, and ST Trough provide further evidence in support of non-fundamental speculation induced by higher order beliefs.

The last two columns in Panel B of Table 3 reports results where the independent variable in the regression is Overvaluation, capturing investors’ beliefs that the market is overvalued. We observe a similarly strong relationship between ST Peak and Overvaluation (coefficient of 0.65, $R^2$ of 0.26), though a weaker relationship between ST Trough and Overvaluation (coefficient of -0.06, $R^2$ of 0.00).

### 1.4.1 Higher Order Beliefs and Realized Returns

Given the evidence relating investors’ higher order beliefs and their return expectations, a natural question to ask is how investors’ higher order beliefs correspond with realized returns. We address this question by regressing realized returns of different horizons on the cross-sectional average of HO Belief, and find some evidence of similar patterns in realized returns as in return expectations.

Figure 1 plots coefficients from regressions of return expectations of different horizons (in solid blue) and return realizations of corresponding horizons (in dashed red) on the monthly cross-sectional average of HO Belief. For return expectations, the figure exactly reflects the results from Table 3. When investors believe others to be overly optimistic, they expect returns to be high in the short term (1 to 3 months), before expecting lower than average returns. For return realizations, the figure indicates that these expectations are qualitatively correct – returns are high in the short term (1 to 6 months), while they are lower than average in the long term. The standard errors on the return realization coefficients are wide (0.28, 0.92, 1.61, and 2.39 for 1-, 3-, 6-, and 12-month returns as the dependent variables), so there is little statistical certainty that equilibrium returns obey the hump-shaped returns that we find (or that they don’t). Nevertheless, it is notable that in this 20-year sample, investors seem to (qualitatively) ‘know’ about patterns in the data.
### Panel A: Term Structure of Expected Cumulative Returns

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<th>Cross-Sectional</th>
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<tr>
<td></td>
<td>$E_t(R_{t,t+1})$</td>
<td>$E_t(R_{t,t+3})$</td>
</tr>
<tr>
<td>$HO$ Belief</td>
<td>1.57 (0.31)</td>
<td>0.05 (0.05)</td>
</tr>
<tr>
<td>Time FE</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$N$</td>
<td>259</td>
<td>259</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.14</td>
<td>0.00</td>
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</table>

### Panel B: Short-term Peaks and Troughs

<table>
<thead>
<tr>
<th></th>
<th>ST Peak</th>
<th>ST Trough</th>
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<tr>
<td>$HO$ Belief</td>
<td>0.41 (0.07)</td>
<td>-0.28 (0.04)</td>
</tr>
<tr>
<td>Overvaluation</td>
<td>0.65 (0.12)</td>
<td>-0.06 (0.15)</td>
</tr>
<tr>
<td>Time FE</td>
<td>NA</td>
<td>NA</td>
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<tr>
<td>$N$</td>
<td>259</td>
<td>259</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.22</td>
<td>0.16</td>
</tr>
</tbody>
</table>

**Table 3: Higher Order Beliefs and Return Expectations**

*Note: Panel A of the table reports results from regressions of cumulative return expectations on the Higher Order Belief variable constructed from the Shiller surveys, pooling together observations across individual and institutional investors. Each column, labeled $E_t(R_{t,t+k})$, represents cumulative return expectations in month $t$ for returns from month $t$ to month $t + k$. The unit of observation in the first four columns is the monthly cross-sectional average of survey responses. Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observation for the last four columns are individual survey responses. Driscoll-Kraay standard errors (12 lags) of coefficients are reported in parentheses. Panel B of the table reports regressions of Short-Term Peak and ST Trough on the HO Belief and Overvaluation measures. The variable ST Peak is constructed from question (ii.c), which asks whether investors expect markets to eventually fall but reach a peak in the near term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The variable ST Trough is constructed from question (ii.d), which asks whether investors expect markets to eventually rise but reach a trough in the near term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The units of observation in the regressions are monthly cross-sectional averages of the variables. Newey-West standard errors (12 lags) of coefficients are reported in parentheses.*
Figure 1: Return Expectations and Higher Order Beliefs

*Note:* The figure plots coefficients from regressions of 1-, 3-, 6-, and 12-month return realizations and average return expectations on monthly cross-sectional averages of HO Belief. Dependent variables are multiplied by 100, so that coefficients can be interpreted as expected returns with a unit change in HO Belief. Standard errors are Newey-West standard errors (12 lags). The figure also plots plus and minus two standard errors for the estimated coefficients for return expectations.

1.5 Non-Fundamental Speculation

The evidence in this section indicates that investors have an incentive to engage in non-fundamental speculation, e.g., to ‘ride the bubble’ and buy into a stock market they perceived as overvalued due to the expectation of short term positive returns. Indeed, survey respondents’ affirmative responses to questions (ii.c) and (ii.d), advising taking positions in the stock market that are opposite their long-term stock market expectations, provide support of non-fundamental speculation motives. An important question is whether the expectations data actually capture investors’ trading behavior, and in particular, whether investors speculate based on their short term return expectations.

We provide additional evidence of non-fundamental speculation using data on futures positions. We find that the trading behavior of buy-side investors (asset managers, hedge funds, etc.) tracks short-horizon return expectations reported in the Shiller survey. Investors increase their market exposure corresponding with higher short term return expectations, and reduce their market exposure corresponding with lower short term expectations.

We obtain weekly data on the positions of investors in S&P500 equity index futures from the Traders in Financial Futures report from the Commodity Futures Trading Commission. The data have been published weekly since 2010, and have been backfilled to 2006. The report presents the number of long and short contracts held in aggregate by investors classified into one of four categories based on self-reported business purposes: futures dealers, levered funds (i.e., hedge funds), institutional asset managers, and other. Hazelkorn,
Moskowitz and Vasudevan (2023) find that positions in futures contracts from the report capture demand for equity market exposure that is reflected in equity market valuations.\(^9\) Changes in futures positions reflect a combination of funds’ rebalancing in response to flows (which may, for example, reflect individual investors’ expected returns and demand for equity market exposure), as well as changes in the return expectations of fund managers. That is, both individual and institutional investors’ expectations may be important for explaining changes in futures positions.

Building on the previous literature, we construct $\Delta \text{Net Positioning}$ as the change in the number of short contracts minus the number of long contracts held by dealers, normalized by lagged open interest. Futures contracts are in zero net supply, and dealers meet the futures demand of other investor types, so $\Delta \text{Net Positioning}$ is a measure of the change in net long demand for equity market exposure by buy-side investors (Hazelkorn, Moskowitz and Vasudevan (2023)).\(^10\) We standardize $\Delta \text{Net Positioning}$ to have zero mean and unit standard deviation. We run contemporaneous regressions of monthly and quarterly changes in $\text{Net Positioning}$ on changes in the cross-sectional average of investors’ return expectations in the same period as the independent variables.

Table 4 reports the regression results. Panel A reports results for monthly regressions and Panel B reports results for the quarterly regressions. In Panel A, the strongest relationship is between changes in 1-month return expectations and changes in net positioning. In a univariate regression, the coefficient on changes in 1-month return expectations is 0.090 (standard error of 0.046), indicating that a 1% change in expected returns correspond with a 0.09 standard deviation change in $\text{Net Positioning}$. In the multivariate regression, the coefficient on 1-month return expectations is 0.10 (standard error of 0.049). The coefficients on 3-, 6-, and 12-month return expectations are positive, but smaller in magnitude, and the coefficient on 6-month return expectations is negative in the multivariate regressions.

The results in Panel B, with quarterly observations, are considerably sharper. The coefficient on 1-month return expectations is 0.27 (standard error of 0.08, $R^2$ of 0.16) in the univariate regression, and 0.20 (standard error of 0.09) in the multivariate regression. The coefficient on 3-month return expectations is 0.25 (standard error of 0.07, $R^2$ of 0.12) and 0.14 (standard error of 0.10) in the multivariate regression. The coefficients for 6-month return expectations are 0.22 (standard error of 0.12) and 0.02 (standard error of 0.13), while the coefficients for 12-month return expectations are 0.12 (standard error of 0.10) and 0.04 (standard error of 0.11). The evidence indicates that return expectations over the next 1-3 months are strongly related to investors’ futures positions, while return expectations over

\(^9\)Other papers, for example Brunnermeier, Nagel and Pedersen (2008) and Moskowitz, Ooi and Pedersen (2012), use futures positioning data from the Commitment of Traders Report, a similar report also published by the Commodity Futures Trading Commission that groups traders into more coarse categories, and similarly find that the futures positions capture futures demand that is relevant for asset price behavior.

\(^10\)Dealers generally do not take on equity market exposure, but hedge their futures positions by trading in the spot market, as discussed in Hazelkorn, Moskowitz and Vasudevan (2023).
the next 6-12 months are weakly related to investors’ futures positions.\textsuperscript{11}

We also examine the joint dynamics of return expectations and futures positions. Figure 2 plots the impulse response functions of net positioning and return expectations to return expectation shocks. The top two panels of the figure plot impulse responses of quarterly net positioning and 1-month return expectations to an AR(1) innovation in 1-month return expectations. The bottom two panels plot impulse responses of quarterly net positioning and 12-month return expectations to an AR(1) innovation in 12-month return expectations. The dynamics of investors’ futures positions closely track their 1-month ahead return expectations. In comparison, there is little relationship between the dynamics of 12-month return expectations and investors’ futures positions.\textsuperscript{12}

One interpretation of the results is that the survey data reflect buy side investors’ expectations, which are accordingly reflected in their positions. Under this interpretation, the results indicate that investors’ short term return expectations lead them to engage in non-fundamental speculation, and they increase their positions because of perceived short term profits. While our evidence is consistent with such an interpretation, we are also cautious, in that we cannot link the identity of survey respondents with their trades.

Our results help explain a puzzle posed in the literature, that investors’ trading is surprisingly insensitive to changes in their 1-year expected returns (Giglio et al. (2021)).\textsuperscript{13} Our results similarly indicate a weak relationship between changes in investor positioning and 1-year ahead return expectations, which may be driven by the fact that investors may expect high returns at the one year horizon but low returns at shorter horizons, or vice-versa. In the presence of such misalignment of short- and long horizon return expectations, investors’ speculative trading motivations weaken the contemporaneous relationship between changes in long term return expectations and trading, consistent with the results we find.

1.6 What Drives Higher Order Beliefs and Return Expectations?

Given the observed time-series relationship between investors’ reported higher order beliefs and return expectations, we next seek to better understand drivers of these beliefs. We find positive macroeconomic news increases \textit{HO Belief} and short term return expectations, while decreasing long term return expectations.

We use two measures of macroeconomic news in our analysis. The first is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index,\textsuperscript{11}One explanation for the stronger relationship using quarterly observations is that investors may trade on changes in their beliefs with a lag.\textsuperscript{12}Breaking down the results by investor types in the appendix, our results indicate a particularly close relationship between 1-month return expectations and institutional asset managers’ (e.g., mutual funds’) futures positioning.\textsuperscript{13}In an experimental setting, Beutel and Weber (2022) find much stronger evidence of pass through of beliefs into risky asset shares. They attribute the wedge between their results and previous results as coming from endogeneity concerns in measuring the relationship between beliefs and portfolios in non-experimental settings.
Panel A: Monthly Futures Positions

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</tr>
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Panel B: Quarterly Futures Positions

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<th>(5)</th>
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<tbody>
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<td>$\mathbb{E}<em>t(R</em>{t,t+1})$</td>
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<td>(0.08)</td>
<td>(0.09)</td>
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</tr>
<tr>
<td>$\mathbb{E}<em>t(R</em>{t,t+12})$</td>
<td></td>
<td>0.04</td>
<td>0.12</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.10)</td>
<td>(0.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.16</td>
<td>0.12</td>
<td>0.10</td>
<td>0.04</td>
<td>0.21</td>
</tr>
<tr>
<td>$N$</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
</tr>
</tbody>
</table>

Table 4: Return Expectations and Investor Futures Positions

Note: The table reports results from regressions of changes in investors' futures positions on changes in return expectations. Our measure of futures positions is $\Delta$Net Positioning, defined as the change in the number of short minus long S&P500 futures contracts held by futures dealers in aggregate, normalized by lagged open interest. Data are from the Traders in Financial Futures report. Return expectations are the average return expectations in a given period from the Shiller survey. Panel A corresponds with monthly changes and Panel B corresponds with quarterly changes. Newey-West standard errors (12 lags for monthly, 4 lags for quarterly observations) of coefficients are reported in parentheses.
Note: The figure plots impulse responses of Net Positioning, 1-month, and 12-month return expectations in response to AR(1) return expectation innovations. Innovations are standardized to have zero mean and unit standard deviation. Net positioning is the change in net positioning from the quarter before the shock to the given period, normalized by open interest, and standardized to have zero mean and unit standard deviation. Return expectations are the change in the cross-sectional average of investors’ return expectations from the quarter before the shock to the given period, in percentage points. Standard errors are Newey-West standard errors (4 lags). The figure also plots plus and minus two standard errors for the estimated coefficients.
which is a composite index of 10 leading macroeconomic indicators.\textsuperscript{14} The second is quarterly AR(1) innovations in discussion of recessions in the Wall Street Journal, from Bybee et al. (2021).\textsuperscript{15} The first measure corresponds with positive macroeconomic news, while the second corresponds with negative macroeconomic news.

We regress quarterly changes in the cross-sectional averages of \textit{HO Belief}, \textit{Overvaluation}, and return expectations of different horizons on the measures of macroeconomic news. We standardize the independent variables, and changes in \textit{HO Belief} and \textit{Overvaluation}, to have zero mean and unit standard deviation. The coefficients for return expectations can be interpreted as the change in expected returns (in percentage points) corresponding with a one standard deviation innovation to the independent variable, and the coefficients for the other dependent variables can be interpreted as correlation coefficients.

Figure 3 plots the regression coefficients. With innovations to leading economic indicators as the independent variable, the coefficient on 1-month return expectations is 0.49, indicating that a one standard deviation innovation corresponds with a 49 basis point higher return expectation for the next month. The coefficients for 3-, 6-, and 12-month return expectations are 0.09, -0.21, and -0.44, indicating that investors lower their return expectations for the next year contemporaneous with the arrival of positive macroeconomic news. Innovations to the leading economic indicators index are 0.45 correlated with changes in \textit{HO Belief} and \textit{Overvaluation}, indicating that investors increasingly report that others are overly optimistic and that markets are overvalued in quarters with positive macroeconomic news. With innovations to recession attention as the independent variable, the coefficients on 1-, 3-, 6-, and 12-month return expectations are -0.73, -0.21, -0.08, and 0.18, indicating expectations of strong negative short term performance that will revert in the future. Innovations to recession attention are -0.40 and -0.24 correlated to \textit{HO Belief} and \textit{Overvaluation}.

We can interpret the evidence as follows: in quarters with positive macroeconomic news, the stock market tends to appreciate. The contemporaneous quarterly return associated with a one standard deviation shock to leading indicators is 2.09%. Investors perceive that in the month following macroeconomic news, returns will be 49 basis points higher, but that in the subsequent 11 months, this short term return will entirely revert, and further, that returns will be lower by nearly a quarter of the contemporaneous response to the news. That is, the evidence is consistent with the 2.09% return reflecting overreaction to news, with investors having a sense that the initial reaction is about 25% larger than justified by fundamentals. The interpretation of perceived overreaction is also consistent with investors’ direct responses when asked about the drivers of stock markets in question (iii.a). 52% of individual investors and 37% of institutional investors answer that the cause of the 6-month

\textsuperscript{14}The data are provided as an index – we construct innovations in the percent change in the index. We lag observations by one month to ensure that innovations are in investors’ information sets. We also report results in the appendix using coincident business cycle indicators, and find similar results.

\textsuperscript{15}Bybee et al. (2021) find that discussion of recessions has substantive predictive power for macroeconomic outcomes. Bybee, Kelly and Su (2023) find that the recession topic has substantial power to explain risk premia as an asset pricing factor. We use an updated series from the authors, which contains data through January 2021.
market trend is overreaction and speculative thinking by other investors.\footnote{The true proportions are a bit higher; several respondents select ‘other,’ and choose to fill in custom responses that indicate a view that stock prices are driven by others’ overreaction or speculation acting in conjunction with additional forces, such as monetary policy.}

### 1.7 Summarizing the Evidence and Implications for Theory

We can summarize the evidence presented in this section as follows:

(i) **(Non-fundamental speculation).** In time periods when investors perceive others to be overly optimistic and markets to be overvalued, they forecast short term returns to be high and long term returns to be low. Speculators seek to ‘ride-the-bubble’ and buy into an overvalued stock market.

(ii) **(Overreaction).** Investors perceive that other investors become overly optimistic, and stock markets become overvalued with the arrival of fundamental news. That is, the stock market displays initial overreaction to fundamental news.

(iii) **(Time-series momentum and reversal).** Investors forecast that the stock market exhibits momentum and reversal in response to fundamental news. In periods following positive fundamental news, investors forecast positive returns in the short term and negative returns multiple periods into the future.

The results provide systematic evidence from survey data that for the U.S. stock market, non-fundamental speculation is the rule not the exception, where previous work documents such behavior in particular episodes (e.g., Brunnermeier and Nagel (2004), Temin and Voth (2004)). Investors believe in patterns such as overreaction-driven momentum and reversal, but choose to buy into the stock market when they perceive it to be driven up by overly optimistic investors, because they see short term profits from doing so.

Below, we discuss mechanisms presented in the literature, and how they may help explain the results. A belief that other investors make systematic errors in forecasting fundamentals is sufficient to explain the results. Two other mechanisms highlighted in the literature – higher order uncertainty, and beliefs that other investors may form their return expectations on the basis of past price changes – may also help explain the results.

**Higher Order Uncertainty.** Higher order uncertainty – uncertainty about whether other investors agree with one’s beliefs – is a form of higher order beliefs that has received particular attention in the literature. In the presence of higher order uncertainty, investors with short investment horizons may not trade fully towards their beliefs, because of uncertainty regarding whether prices will reflect their beliefs in the near future. In one strand of literature, higher order uncertainty leads asset prices to underreact to fundamental news upon its arrival, and monotonically drift towards fundamental values in subsequent periods. This can happen even when the average belief is an unbiased signal of fundamentals (Allen,
Figure 3: Macroeconomic News, Return Expectations, and Higher Order Beliefs

Note: The figure plots coefficients from contemporaneous regressions of changes in quarterly average 1-, 3-, 6-, and 12-month return expectations, HO Belief, and Overvaluation on measures of macroeconomic news. The measure of macroeconomic news in the first panel is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, which is a composite index of 10 leading macroeconomic indicators. The measure of macroeconomic news in the second panel is AR(1) innovations in attention paid to recession news in the Wall Street Journal, from Bybee et al. (2021). The independent variables, HO Belief, and Overvaluation are scaled to have zero mean and unit standard deviation, and return expectations are multiplied by 100. Standard errors are Newey-West standard errors (4 lags). The figure also plots plus and minus two standard errors for the estimated coefficients.
Morris and Shin (2006), Banerjee, Kaniel and Kremer (2009)). However, such uncertainty does not capture the patterns of expectations we document, namely that asset prices initially overreact to news, and that they continue to overreact before reversing.

A form of higher order uncertainty that generates synchronization risk can help explain our results. In Abreu and Brunnermeier (2002, 2003), arbitrageurs become aware of mispricing sequentially, and have higher order uncertainty about other arbitrageurs’ awareness of the mispricing. With short investment horizons and the need for coordination to correct mispricings, arbitrageurs may engage in non-fundamental speculation and hold long positions in a stock market they see as overvalued.

In and of itself, however, this form of uncertainty is not sufficient to explain the patterns we observe, and must be combined with additional assumptions about the beliefs and behavior of non-arbitrageurs to explain investors’ beliefs in initial market overreaction to fundamental news, and their beliefs in time-series momentum and reversal.

**Return Extrapolation.** Recent work emphasizes that return extrapolation – investors forming their expected returns based on past returns – may explain investor behavior. Our empirical results provide some evidence consistent with return extrapolation: investors’ short term return expectations are correlated with past returns. However, without additional assumptions, return extrapolation cannot speak to investors’ multi-period return expectations, and perceptions of others’ beliefs.

Interestingly, beliefs that other investors return extrapolate can lead to non-fundamental speculation. In an economy populated by non-speculative fundamental traders, return extrapolators, and rational speculators, the rational speculators can push prices to overreact to fundamental news due to a recognition that such speculation may trigger future excitement by return extrapolators (De Long et al. (1990)). While the rational speculators in the model of De Long et al. (1990) would report return expectations and valuations consistent with the survey responses we observe, we would not expect them to report that they increasingly find others to be overly optimistic contemporaneous with positive news shocks (as in the data). The speculators in De Long et al. (1990) expect excess optimism to arise from return extrapolators in the periods following news. Beliefs in other investors’ return extrapolation may help explain the patterns, but existing models do not fully match the evidence.

**Errors in Forecasting Fundamentals.** A growing literature has suggested that investors may make systematic mistakes in forecasting asset price fundamentals (Chen, Da and Zhao (2013), Bordalo et al. (2020), De La O and Myers (2021), Nagel and Xu (2022a)). In the

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17 In Allen, Morris and Shin (2006), noisy signals about fundamentals lead to underreaction of prices to fundamental news, though Banerjee, Kaniel and Kremer (2009) note that if investors learn from prices, this effect may disappear. Banerjee, Kaniel and Kremer (2009) argue that price drift can reliably occur when investors agree-to-disagree about fundamental values, but have uncertainty about other investors’ higher order beliefs. Banerjee, Kaniel and Kremer (2009) also note that in noisy rational expectations models, speculation must be driven by information orthogonal to public information; we document speculation in response to public information.

18 If fundamental traders and return extrapolators are the only investors in the economy, and there is no third type, as in Barberis et al. (2018), the fundamental traders would believe that stock markets exhibit initial underreaction rather than forecasting initial overreaction (as the investors in the Shiller surveys do).
absence of assumptions about higher order beliefs, such theories can make vastly different predictions about return expectations. For example, if investors with mistaken fundamental beliefs believe all other investors share their beliefs, they always expect constant returns in the absence of time-varying risk premia, inconsistent with our evidence.

However, fundamental belief mistakes do feature in our explanation. In the model we present in the next section, investors believe that other investors (1) face informational frictions that lead them to persistently update their beliefs in the direction of past news, and (2) overestimate the persistence of fundamentals. These belief updating biases are effectively identical to the ones in Angeletos, Huo and Sastry (2021), who show that these frictions help match macroeconomic forecasters’ expectations.\footnote{Valente, Vasudevan and Wu (2021) use a model based on Angeletos, Huo and Sastry (2021), to match survey data on investors’ interest rate expectations in order to understand puzzles in foreign exchange markets.} We also find that potential interactions of fundamental and higher order beliefs can help explain financial market phenomena.

Martin and Papadimitriou (2022) propose a model where investors have differences-of-opinion about the (fixed) fundamental value of an asset. Investors that are correct in hindsight become wealthier, and so the belief and identity of the representative agent becomes more optimistic following good news and more pessimistic following bad news. Investors internalize the role of this shifting ‘sentiment,’ and may engage in non-fundamental speculation. Investors’ beliefs regarding the shifting relative wealth of optimists and pessimists in response to news may also help explain the facts we document.

Other Models. The previously discussed forms of higher order beliefs – higher order uncertainty and beliefs about other investors’ fundamental and return expectations – may help explain the results. We briefly discuss other common models in the literature.

Expectations of time-varying returns are commonly attributed to time-varying risk premia (Campbell and Cochrane (1999), Bansal and Yaron (2004), Gabaix (2008), Wachter (2013)). Such models typically assume a representative agent with correct beliefs, and accordingly struggle to confront the evidence that other investors hold incorrect expectations, and that the stock market is mispriced. They also generally predict that risk premia and return expectations decrease contemporaneous with positive news (which makes agents less risk-averse), whereas we find evidence that investors’ short term return expectations and equity market exposures increase contemporaneous with positive news.\footnote{Though, with specific parametrization of preferences and the cash flow process, models of time-varying risk premia may be able to generate hump-shaped pattern of return expectations in response to fundamental news.}

A common explanation for non-fundamental speculation is that investors may face short-sale costs and constraints (Harrison and Kreps (1978), Scheinkman and Xiong (2003), Duffie, Garleanu and Pedersen (2002)). The presence of short-sale constraints and costs leads pessimistic investors to avoid taking short positions, and asset prices reflect the valuations of optimistic investors, plus the added benefit they receive from the possibility of reselling to more optimistic investors that arrive in the future. In our setting – the aggregate U.S. stock market – short-sale constraints are unlikely to be binding for most investors.\footnote{It is essentially costless to short the stock market via futures (Hazelkorn, Moskowitz and Vasudevan (2023)).} While this
mechanism may contribute to our results, we think it is unlikely to be the primary driver for the aggregate U.S. market.

2 A Model of Non-Fundamental Speculation

We present a stylized asset pricing model that can rationalize the evidence from Section 1. The model illustrates that we can match the empirical evidence if investors believe that (1) other investors’ valuations’ overreact to news and (2) that other investors persistently update their beliefs in the direction of past news.

We then consider the implications of such higher order beliefs for asset prices. Our model reveals that financial markets can display pure higher order belief driven fluctuations and overreact to news, even when investors all have correct fundamental beliefs. We also find that when investors have the same fundamental belief biases that they attribute to other investors, phenomena such as momentum, reversal, and overreaction can all persist in equilibrium despite the fact that all investors know about them. Lastly, an alternative specification of our model reveals that when investors extract information from prices, their higher order beliefs can cause mistakes in their fundamental beliefs.

2.1 Model Setup

We begin by laying out the model environment, and describing investors’ beliefs.

2.1.1 Model Environment

There is a risky asset (the stock market) in zero net supply, and a riskless asset in perfectly elastic supply. The payoff of the riskless asset is normalized to zero. The asset pays a dividend $D_t$ each period, where $D_t$ evolves according to the process

$$D_t = d_t + v_t, \text{ where } v_t \sim N(0, \sigma_v^2), \text{ and}$$
$$d_t = \rho d_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma_{\epsilon}^2).$$

The term $d_t$ captures the persistent component of dividends, which we refer to as the asset’s fundamentals, while $v_t$ captures a transitory component of dividends. While dividends are observed each period, the underlying fundamentals are never revealed to investors.

The model follows an overlapping generations structure. Each period, a unit mass of identical investors are born. Investors born in period $t$ make an investment decision in that period. In period $t + 1$ they liquidate their investments, consume the proceeds, and pass their beliefs onto the new generation of investors. Each investor is individually infinitesimal, and does not consider the effect of their demand on prices.
Investors have exponential utility. Aggregate investor demand for the risky asset is

\[ Q_t^S = \theta \mathbb{E}^S_t \left( P_{t+1} + D_{t+1} - P_t \right), \]

\[ \equiv R_{t,t+1}(\text{Asset’s Return}), \]  \hspace{1cm} (2)

where \( P_t \) is the price of the risky asset; \( \theta \) captures the aggressiveness with which investors trade, increasing in their aggregate risk bearing capacity and decreasing in the risk they perceive; and \( \mathbb{E}^S_t \) are investors’ subjective expectations. We refer to \( R_{t,t+1} \equiv P_{t+1} + D_{t+1} - P_t \) as the return of the risky asset.

Investors trade against residual demand for the asset, denoted as \( \psi_t \), on which we later impose additional structure. The market clearing condition is given by

\[ 0 = Q_t^S + \psi_t. \]

\hspace{1cm} (3)

### 2.1.2 Assumptions about Investors’ Beliefs

We make three assumptions about investors’ expectations formation.

**Assumption 1 (Fundamental Beliefs).** Investors perceive the parameters \( (\rho, \sigma_v) \), the persistence of fundamentals and the transitory noise reflected in dividends each period, as \( (\rho_S, \sigma_{S,v}) \).

Assumption 1 indicates that investors have subjective beliefs about the parameters of the fundamentals process that may deviate from the true parameters.\(^{22}\)

Investors in the model are identical, but perceive belief disagreements with others. To make notationally clear perceived disagreements with other investors (“the market”), we denote the market’s first order beliefs about fundamental persistence and transitory noise as \( (\rho_M, \sigma_{M,v}) \). By construction, \( (\rho_M, \sigma_{M,v}) = (\rho_S, \sigma_{S,v}) \).

Investors each know their own belief parameters to be \( (\rho_S, \sigma_{S,v}) \). However, their second order beliefs about the market’s belief parameters are denoted by \( (\rho_{SM}, \sigma_{SM,v}) \).

**Assumption 2 (Second Order Beliefs About Fundamental Beliefs).** Investors believe that all other investors perceive the parameters \( (\rho, \sigma_v) \) as \( (\rho_{SM}, \sigma_{SM,v}) \).

**Assumption 3 (Additional Higher Order Beliefs).** Investors believe that other investors forecast that all investors born in period \( t + 1 \) will perceive the parameters \( (\rho, \sigma_v) \) as \( (\rho_{SM}, \sigma_{SM,v}) \), and will see this perception as common knowledge. That is, investors believe that other investors act as fundamental (non-strategic) traders.

Together, Assumptions 2 and 3 characterize investors’ higher order beliefs in the model. Assumption 2 indicates perceived belief disagreements with the market about the dividend

\[^{22}\text{We assume that investors hold fixed belief parameters that they do not update primarily for simplicity, but also because previous work studying survey data indicate that investors make systematic mistakes in forecasting cash flows. Realistic models of learning may still result in investors exhibiting substantial deviations from the full information rational expectations benchmark (e.g., see Johannes, Lochstoer and Mou (2016)).}\]
process. Assumption 3 captures that investors represent the market as a fundamental (non-strategic) trader.\footnote{Assumption 3 allows us to avoid the ‘infinite regress’ problem, where investors may want to forecast others’ forecasts of others’ forecasts of others’ forecasts, and so on and so forth (Townsend (1983)).}

We make an additional assumption regarding how investors reason about residual demand. It is not essential, but helps us present our discussion in a concise way.

**Assumption 4 (Beliefs about Residual Demand).** Investors believe that $\psi_t$ follows a random walk, i.e., they believe that $\psi_t \sim N(\psi_{t-1}, \sigma^2_{\psi})$.

We next turn to describing investors’ fundamental and return expectations.

### 2.1.3 Investors’ Fundamental Beliefs

Investors are Bayesian in forming their beliefs about fundamentals, $d_t$. Using their beliefs about the dividend process and their observations of past dividends, they form their expectations of $d_t$ by Kalman filtering. For ease of notation, investors’ beliefs about fundamentals in period $t$ after observing the period $t$ dividend, $D_t$, are denoted as

$$d^S_t \equiv E^S_t(d_t).$$

(Investors’ Fundamental Beliefs)

We follow the common assumption that a sufficient number of periods have passed such that investors are in a learning steady state. This means that investors’ Kalman gain – the weight they place on the observed dividend versus their prior in their fundamental beliefs – is constant each period. Lemma 1 outlines how investors’ fundamental beliefs evolve.

**Lemma 1 (Fundamental Beliefs).** Investors’ beliefs about fundamentals, $d_t$, evolve according to the updating process

$$d^S_t = (1 - \kappa_S)\rho_S d^S_{t-1} + \kappa_S D_t,$$

(4)

where

$$\kappa_S = \frac{\rho_S^2 \Sigma + \sigma^2_e}{\rho_S^2 \Sigma_S + \sigma^2_e + \sigma^2_{S,e}},$$

and

$$\Sigma_S = (1 - \kappa_S)(\rho_S^2 \Sigma_S + \sigma^2_e).$$

Proof. All proofs are provided in Appendix A. \qed

The subjective Kalman gain, $\kappa_S$, depends on investors’ perception of the signal-to-noise ratio of dividends, $\sigma^2_{S,e}/\sigma^2_e$. If investors believe that dividends are a noisy signal of fundamentals (e.g., larger $\sigma^2_{S,e}$), they put less weight on the most recently observed dividend and more weight on their prior in their updated belief each period.\footnote{Introducing a higher $\sigma_{S,e}$, as we do in our applications, induces sluggishness in the belief updating process by lowering the Kalman gain $\kappa_S$. This is approximately equivalent to assuming sticky expectations, e.g., Mankiw and Reis (2002), and subsequent literature that builds on their approach. Bouchaud et al. (2019) study the asset pricing implications of sticky expectations.}

We assume that all investors know and agree on the importance of persistent shocks to fundamentals, as captured by $\sigma^2_e$. Therefore, a higher perceived amount of transitory noise is equivalent to a
lower signal-to-noise ratio and a lower Kalman gain. For convenience, we focus our discussion on investors’ Kalman gains.

2.1.4 Investors’ Higher Order Beliefs and Return Expectations

Investors forecast asset returns by forecasting the beliefs of the market (i.e., all other investors). All investors have identical beliefs about fundamentals in period $t$, which we denote as $d_t^S$. However, because investors believe that others have different beliefs about the dividend process, they also mistakenly perceive a belief disagreement with the market.

To make clear the perceived belief disagreement, we denote the market’s belief about fundamentals in period $t$ as $d_t^M$. This is always equal to $d_t^S$. However, investors’ second order belief about the market’s fundamental belief in period $t$ is $d_t^{SM}$. With this notation, we can write investors’ perceived pricing rule.

**Lemma 2 (Perceived Pricing Rule).** Investors perceive that the form of the equilibrium pricing rule for the risky asset is

$$P_t = \frac{\rho_M d_t^M}{1 - \rho_M} + \frac{\psi_t}{\theta}.$$  \hspace{1cm} (5)

Investors perceive the equilibrium pricing function as sum of two components: the market’s fundamental valuation of the risky asset and a residual term. To rationalize the period $t$ price with the perceived pricing rule in Equation (5) and their second order beliefs, investors form a subjective belief about $\psi_t$ as

$$\psi_t^S \equiv \theta^{-1}(P_t - \frac{\rho_{SM} d_t^{SM}}{1 - \rho_{SM}}).$$ \hspace{1cm} (6)

Investors attribute the component of prices that is not consistent with their perceived model as stemming from unpredictable noise.\(^{25}\)

As noted in Equation (2), investors’ speculative demand depends on their return expectations, which include their forecasts of risky asset prices and future dividends. Investors forecast the dividend in period $t + k$ as $\mathbb{E}^S_t(D_{t+k}) = \rho_t^K d_t^S$. Their forecast of prices depends on their forecast of future demand, which, in turn, depends on their higher order beliefs.

**Lemma 3 (Forecasts of the Market’s Fundamental Beliefs).** In period $t$, investors forecast the market’s belief about fundamentals in period $t + h$ as

$$\mathbb{E}^S_t(d_t^{M+h}) = (1 - \kappa_{SM})^h \rho_{SM}^h d_t^{SM} + \kappa_{SM} \rho_{SM} \frac{((1 - \kappa_{SM})^h - \rho_{SM}^h)}{(1 - \kappa_{SM}) \rho_{SM} - \rho_S} d_t^S.$$ \hspace{1cm} (7)

---

\(^{25}\)They can equivalently perceive persistent shocks to the intercept of the demand curve, $\phi_t$. The failure of investors to invert prices is related to *cursedness* (Eyster and Rabin (2005), Eyster, Rabin and Vayanos (2019)). Because investors see prices as consistent with their beliefs about residual demand, their errors are *attentionally stable* in the sense of Gagnon-Bartsch, Rabin and Schwartzstein (2021).
Using Lemma 3, we can derive investors’ return expectations from period $t$ to $t + h$.

**Lemma 4 (Investors’ Return Expectations).** Investors’ cumulative expected return for the risky asset from period $t$ to period $t + h$ is given by

$$
\mathbb{E}^S_t(R_{t,t+h}) = \mathbb{E}^S_t\left(P_{t+h} - P_t + \sum_{k=1}^h D_{t+k}\right) = \frac{\rho_{SM}}{1 - \rho_{SM}} \left(\mathbb{E}^S_t(d^M_{t+h}) - d^S_t\right) + \frac{\rho_{S}(1 - \rho_{S}^h)}{1 - \rho_{S}} d^S_t,
$$

(8)

where $\mathbb{E}^S_t(d^M_{t+h})$ is defined as in Lemma 3. Investors’ return expectations for the risky asset in period $t + h$ satisfy

$$
\text{Sign}\left(\mathbb{E}^S_t(R_{t,t+1})\right) = \text{Sign}\left(\mathbb{E}^S_t(d^SM_t - \mathbb{E}^S_t(d^SM_{t+1}))\right).
$$

(9)

Together, Lemmas 3 and 4 connect investors’ cash flow and return expectations. Return expectations are determined by a combination of the future cash flows investors expect, and how they expect other investors to update their beliefs in response to those cash flows. Lemma 4 also illustrates that the sign of return expectations is determined by the direction in which investors forecast that other investors will revise their beliefs and valuations. Investors forecast that other investors will be surprised by future dividends and revise their beliefs, generating predictable returns in the direction of their surprise.

Before solving for the equilibrium pricing rule, we specify that residual demand is a downward sloping demand curve given by

$$
\psi_t = \lambda (\phi_t - P_t),
$$

(10)

where $\lambda$ captures the elasticity of the demand curve, and $\phi_t$ captures a residual valuation term.$^{26}$ $\phi_t$ need not be specified for understanding investors’ expectations in the model, but we assume it to be an informed valuation of the asset $\left(\frac{P}{1-p}d_t\right)$ in our applications.

We can now solve for the equilibrium pricing rule for the risky asset.

**Proposition 1 (Equilibrium Pricing Rule).** The market clearing price of the risky asset is

$$
P_t = \frac{\theta}{\lambda} \mathbb{E}^S_t(R_{t,t+1}) + \phi_t = \frac{\theta}{\lambda} Y d^S_t + \frac{\theta}{\lambda} Z d^{SM}_t + \phi_t,
$$

(11)

$^{26}$The demand curve can be micro-founded as a mass of long term investors with exponential utility. Speculatively, their expectations may coincide with those a different class of investors considered in the Shiller surveys, e.g., the managers studied in Dahlquist and Ibert (2022). The assumption that investors face a downward sloping demand curve finds long empirical support. Prior work includes Shleifer (1986), Chan and Lakonishok (1995), and Wurgler and Zhuravskaya (2002). Gabaix and Koijen (2021) provide recent evidence in the context of the aggregate stock market.
where

\[ Y = \frac{\theta (1 - (1 - \kappa_{SM}) \rho_{SM}) \rho_S}{1 - \rho_{SM}}, \]  
and

\[ Z = \frac{-\theta (1 - (1 - \kappa_{SM}) \rho_{SM}) \rho_{SM}}{1 - \rho_{SM}}. \]  

Proposition 1 indicates that the equilibrium asset price is a linear function of three variables: investors’ fundamental beliefs, \( d_i^S \), investors’ higher order beliefs about fundamentals, \( d_i^{SM} \), and a residual valuation term. Unlike the perceived pricing rule, higher order beliefs now enter, because each investor’s trading is driven by their higher order beliefs.

2.2 Matching the Evidence on Expectations

We first present a simple result that illustrate the role of fundamental and higher order beliefs in investors’ return expectations.

Lemma 5 (Long Horizon Return Expectation). At long horizons, investors’ return expectations converge to the difference in their valuation of the asset versus their perception of the market’s valuation: \( \lim_{h \to \infty} E_t^S(R_{t,t+h}) \to \frac{\rho_S}{1-\rho_S} d_t^S - \frac{\rho_{SM}}{1-\rho_{SM}} d_t^{SM} \).

Proposition 5 illustrates that investors’ long horizon return expectations are equal to the difference in the sum of dividends they expect versus the sum of dividends they perceive others expect. When the quantity \( \frac{\rho_S}{1-\rho_S} d_t^S - \frac{\rho_{SM}}{1-\rho_{SM}} d_t^{SM} \) is negative, investors perceive the risky asset as overvalued, as they expect a negative return from holding it in perpetuity.

Proposition 2 (Non-Fundamental Speculation in Response to News). Denote \( \kappa \equiv \frac{\rho_{SM}}{1-\rho_S} \kappa_S \), \( \bar{\kappa} \equiv \frac{\rho_{SM}}{1-\rho_S} \kappa_S \). \( \kappa < \kappa_{SM} < \bar{\kappa} \) implies that

(i) Investors perceive that the risky asset price overreacts to news on average. That is, for a positive shock to dividends, \( \eta_t > 0 \), on average, investors perceive that \( \lim_{h \to \infty} E_t^S(R_{t,t+h}) < 0 \), i.e., that long term returns to holding the asset are negative.

(ii) Investors engage in non-fundamental speculation following news. That is, on average, for a shock \( \eta_t > 0 \) to dividends, investors take on long positions in the risky asset they perceive to be overvalued, because they expect a positive short term return from doing so.

(iii) Investors perceive that the risky asset exhibits time-series momentum and reversal. They perceive positive returns in period \( t \) contemporaneous with a positive shock to dividends, \( \eta_t > 0 \). They perceive that returns to holding the risky asset will be positive from \( t+1 \) to \( t+\bar{h} \) for some \( \bar{h} \geq 1 \), and that returns holding the risky asset will be negative for \( t+h, h > \bar{h} \).

Proposition 2 indicates that by placing joint restrictions on \( \kappa_{SM} \) and \( \rho_{SM} \) (relative to \( \kappa_S \) and \( \rho_S \)), we can capture the empirical evidence on investors’ expectations.

\[ \text{In Section 2.3 with residual demand reflecting an informed valuation, we find perceived overreaction in a stronger sense, } P_t > \frac{\rho_S}{1-\rho_S} d_t^S. \]
Figure 4: Investors’ Expectations in the Model

Note: Panel A plots investors’ forecasts of other investors’ beliefs over time, given a unit change in their belief about fundamentals. Panel B plots investors’ forecasted expected cumulative returns from period $t$ onwards. The parameter values used to generate the figure are $(\rho_S, \kappa_S, \rho_{SM}, \kappa_{SM}) = (0.8, 1, 0.89, 0.6)$.

The inequality $\kappa_{SM} < \bar{\kappa}$ delivers that in response to a dividend shock, investors believe that others do not sufficiently incorporate information about the shock. They believe that others’ Kalman gains are too low; or equivalently, they believe that others perceive dividends as a noisier signal of fundamentals than they are. Accordingly, investors forecast that others will predictably revise their beliefs upwards in period $t + 1$ and expect a positive return in period $t + 1$ after a positive dividend shock in period $t$.

The inequality $\kappa_{SM} > \bar{\kappa}$ delivers perceived overreaction of other investors’ valuations to news upon its arrival. It is best understood as a joint restriction on $\kappa_{SM}$ and $\rho_{SM}$, which can be equivalently expressed as $\rho_{SM}\kappa_{SM} > \frac{1-(1-\kappa_S)\rho_S}{1-\rho_S}$. The logic is that investors believe that others react slowly to fundamental news, but they perceive fundamentals as more persistent than they are. If others sufficiently overestimate the persistence of fundamentals, their belief about the future sum of dividends is sufficiently high that their valuations overreact.

Crucial for matching the data, investors forecast that the market’s valuation can overreact in period $t$, and then continue to overreact in period $t + 1$ as it further incorporates the dividend news of the previous period. This forecast of continued overreaction leads investors to engage in non-fundamental speculation in period $t$. Even though they think the risky asset is overvalued in period $t$ upon the arrival of fundamental news, investors choose to buy into the risky asset on the forecast that returns will be positive in period $t + 1$. Notably, the model generates non-fundamental speculation in the absence of short-sale constraints. The mechanism within our model that generates this is that investors believe that other investors will predictably revise their beliefs upwards in the future (despite already being overly optimistic), and they dismiss information from prices that may be inconsistent.

Figure 4 illustrates the intuition behind Proposition 2. Panel A plots how investors expect others investors’ beliefs about fundamentals, $d^M_t$, to evolve over time, and Panel B
plots investors’ forecasted cumulative expected returns for the asset. Following a shock, investors expect other investors to revise their beliefs about \( d_t \) upwards, resulting in positive returns. This occurs despite the fact that investors already believe the asset price in period \( t \) has overreacted, exceeding what investors perceive as the long term cumulative returns associated with the shock. Investors expect other investors’ beliefs about \( d_t \) to eventually become overly optimistic, and for other investors to constantly be negatively surprised by dividends, leading to forecasts of subsequent reversals.

We highlight that to satisfy the conditions of Proposition 2, we must have that \( \kappa_{SM} < \kappa_S \) and \( \rho_{SM} > \rho_S \), i.e., investors believe that others’ Kalman gains are too low and that others’ perceived persistence of fundamentals is too high. This is effectively identical to the frictions that Angeletos, Huo and Sastry (2021) find can match empirical evidence on professional forecasters’ macroeconomic beliefs.\(^{28}\)

### 2.3 Higher Order Beliefs and Equilibrium Asset Prices

The previous section provides restrictions to match investors’ higher order beliefs in survey data. We next study how such higher order beliefs affect equilibrium asset prices, and in particular, how asset pricing facts such as overreaction, time-series momentum, and reversal may arise as equilibrium outcomes, despite investors’ knowledge and beliefs in these phenomena. In our applications, we specify that residual demand is informed.

**Assumption 5 (Informed Residual Demand).** Residual demand is informed; that is \( \phi_t = \frac{\rho - \rho_d}{1 - \rho} d_t \).

We first consider the case where investors’ fundamental beliefs are correct, and all asset price fluctuations are purely induced by higher order beliefs. Under the assumptions of Proposition 2, overreaction to fundamental news and reversals arise as equilibrium outcomes. We then allow investors to have mistaken fundamental beliefs. In addition to overreaction and reversals, momentum also arises as an equilibrium outcome when investors exhibit the same fundamental belief biases that they believe other investors exhibit. In both cases, the key friction that leads equilibrium asset prices to reflect observed patterns is that investors mistakenly attribute other investors as the cause of asset price fluctuations.

Before proceeding to the cases, we present a lemma that is helpful for understanding the behavior of equilibrium asset prices in the model.

**Lemma 6 (Objective Expected Returns).** Given Assumption 5, objective expected returns are given by

\[
\mathbb{E}_t(R_{t,t+1}) = \frac{\theta}{\lambda} \left( \mathbb{E}^S_{t+1}(R_{t+1,t+2}) - \mathbb{E}^S_t(R_{t,t+1}) \right). \tag{14}
\]

\(^{28}\) Angeletos, Huo and Sastry (2021) assume that perceived persistence exceeds true persistence. They use noisy private information to generate persistence in belief updating to past shocks. Reis (2020) notes that sticky expectations, following Mankiw and Reis (2002), generates a similar effect.
Lemma 6 indicates that the driver of predictable price and return variation in the model is changes in investors’ return expectations. When investors revise their one-period ahead return expectations upwards, equilibrium returns are positive, and when they revise their their one-period ahead return expectations downwards, equilibrium returns are negative.

2.3.1 Pure Higher Order Belief Induced Fluctuations

Assuming investors’ fundamental beliefs are correct, Proposition 2’s assumption that \( x < x_{SM} < \bar{x} \) bears immediate implications for equilibrium prices and returns.

**Proposition 3 (Higher Order Belief Induced Fluctuations).** If investors have correct fundamental beliefs, i.e., their beliefs about the persistence and signal-to-noise ratio of the dividend process match the objective values, \((\rho_s, \kappa_s) = (\rho, \kappa)\), and if \( x < x_{SM} < \bar{x} \), as defined in Proposition 2, then asset prices overreact relative to the rational expectations benchmark. Following a positive shock to dividends, \( \eta_t > 0 \), prices are higher than the rational expectations benchmark \( P_t > \phi_t \), returns are initially negative \( R_{t, t+1} < 0 \), and then are subsequently positive \( (\exists h > 1 \text{ such that } h > \bar{h} \implies R_{t+h-1, t+h} > 0) \).

We can understand Proposition 3 using Equation (11). The first line of Equation (11) indicates that equilibrium prices can be written as \( P_t = \phi_t + \mathbb{E}_t^S (R_{t, t+1}) \). If residual demand is informed, \( \phi_t \) reflects the price of the risky asset under the rational expectations benchmark.

Figure 5 illustrates asset prices in Proposition 3. The higher order belief that other investors will persistently react to past news and overreact to news \((x < x_{SM} < \bar{x}\) from Proposition 2) delivers that asset prices overreact contemporaneous with news. This is because the return expectations induced by investors’ higher order beliefs are positive in period \( t \), leading the asset price to overreact. Investors’ return expectations decrease immediately following the shock, leading to a reversal. Because residual demand is informed, investors’ return expectations induce mispricing in their direction. Investors in period \( t \) recognize that the stock market has overreacted, but attribute the overreaction to the mistaken fundamental beliefs of other investors, rather than to the speculative trading of investors like themselves.

2.3.2 Fundamental and Higher Order Beliefs

Higher order beliefs matching survey data generate overreaction and excess volatility, but alone, they do not generate time-series momentum in the AR(1) environment with Bayesian investors we study. With correct fundamental beliefs, investors revise their expected returns downwards in the period immediately following a positive shock, generating a sharp reversal. This is a function of perceiving others as Bayesian learners.\(^{29}\)

However, when investors have mistaken fundamental beliefs, then we may also observe time-series momentum as an equilibrium outcome, alongside overreaction and reversal. We

\(^{29}\)For Bayesian investors, (perceived) learning slows in the periods subsequent to a shock.
Proposition 4 (Momentum and Reversal in Equilibrium). Denote $\bar{\kappa}_S \equiv \left(1 - \alpha\right)\rho \left(\rho - \alpha^2 \rho_{SM}\right)\kappa$, where $\alpha = \frac{\kappa_{SM}}{\kappa_S}$, and $\kappa$ is the Kalman gain of a rational investor. If $\kappa_S < \kappa$, and $\kappa < \kappa_{SM} < \kappa$, then, in addition to overreaction and reversals, the risky asset also exhibits momentum. That is, given a shock to dividends in period $t$, $\eta_t > 0$, returns are positive in the next period, $R_{t,t+1} > 0$.

Proposition 4 indicates that the interaction between fundamental and higher order belief biases can lead asset prices to exhibit momentum, reversal, and overreaction in equilibrium, despite the fact that all investors believe in these phenomena. Again, the key mechanism here is that investors do not realize that their own trading drives the patterns of predictable return variation. The momentum effect is produced by investors themselves having Kalman gains that are too low. They update their fundamental beliefs in the direction of past dividend news in the same way they believe others do. When investors update their fundamental beliefs, their higher order beliefs induce them to increase their return expectations, as they expect others to react to the news later, further pushing the risky asset price in the future.

Figure 6 illustrates Proposition 4 for a parameterization where investors exhibit precisely the same belief-updating biases they attribute to other investors. Prices overreact and con-

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30E.g., see Svenson (1981), Camerer and Lovallo (1999), and Liu et al. (2022).
Figure 6: Fundamental and Higher Order Belief Driven Fluctuations

Note: The blue line in the figure plots (objective) expected cumulative returns to the risky asset assuming that residual demand is informed and if investors exhibit the same behavioral biases they attribute to other investors, given a positive shock to dividends in period $t$. The dashed red line plots the rational expectations valuation of the risky asset in period $t$. The parameter values used are $(\kappa, \rho, \kappa_S, \rho_M) = (1, 0.8, \alpha \kappa, \beta \rho, \alpha \kappa_S, \beta \rho_S)$, where $\alpha = 0.4$ and $\beta = 1.11$.

Investors' slow learning about fundamentals (low $\kappa_S$) also slows the speed with which reversals occur, as investors' perception of the arrival of additional news counteracts the fading response they forecast of other investors to the original shock.

The asset price dynamics in Proposition 4 and Figure 6 are qualitatively similar to the dynamics we would expect when investors have correct higher order beliefs (they do not perceive other investors as different from themselves), but they have incorrect fundamental beliefs, i.e., they overestimate the persistence of the fundamentals process and persistently react in the direction of past news. They key difference is that investors recognize that patterns such as overreaction, momentum, and reversal exist, consistent with the survey data, which would not happen in the alternate case with correct higher order beliefs.

2.4 Higher Order Beliefs and Inference from Prices

In our main specification, investors attribute any component of current prices that is inconsistent with their higher order beliefs to persistent noise, and they do not recognize the structure that residual demand takes. For our last result, we analyze a modified version of the model where investors form their fundamental beliefs by extracting information from prices based on their higher order beliefs rather than by observing dividends.\footnote{As with our main specification, the starkness of this assumption helps to illustrate the mechanism we have in mind. It is possible to combine elements of our main specification and this alternate specification.} We find that the form of higher order beliefs we study can co-exist with, interact with, and cause
fundamental belief mistakes.

We spell out the modified model in more detail in Appendix B. The structure is identical to our main specification except that investors recognize that residual demand takes the form of an informed downward sloping demand curve. Investors know the persistence of fundamentals, but their second order beliefs satisfy Proposition 2 and match the evidence on return expectations. Without loss of generality, we normalize $\theta + \lambda = 1$. Equilibrium coincides with the fixed point where investors’ perceived price, given their extracted fundamental belief, equals the market clearing price.

**Proposition 5 (Inference from Prices).** Assume that investors extract their fundamental beliefs from the equilibrium risky asset price rather than by direct inference from dividends. Starting from steady state, given a fundamental shock in period $t$, extracted fundamental beliefs satisfy

$$d_t^f = \left( (1 - \theta) \rho + \frac{\theta (\theta \rho + (1 - \theta \rho) \kappa_{SM}) \rho_{SM} + \rho}{1 - \rho_{SM}} \right)^{-1} \frac{\rho}{1 - \rho} d_t.$$  

(15)

In the absence of speculative investors ($\theta = 0$), the extracted signal from Proposition 5 coincides exactly with the true fundamentals. Outside of this case, however, the extracted signal differs from the true fundamentals. Investors perceive prices as being driven by other investors’ fundamental beliefs, when in fact, they are the outcome of speculative trade. Investors realize the market has overreacted to fundamental news, but extract incorrect fundamental information due to their inability to understand the source of this overreaction.

Investors’ extracted beliefs can reflect underreaction and overreaction to true fundamentals, depending on the exact parametrization. Figure 7 plots the magnitude of under- and overreaction of fundamental for feasible values of $\kappa_{SM}$ and $\rho_{SM}$ that correspond to patterns in investors’ return expectations, with overreaction in red and underreaction in blue.

Equation (15) reflects an equilibrium relationship that holds for investors’ fundamental beliefs and true fundamental values in equilibrium. It illustrates that inference from prices, which we do not include in our prior results, does not ‘fix’ asset price fluctuations that may be caused by higher order beliefs. Investors may rationalize equilibrium asset prices in a manner consistent with their higher order beliefs, and in the process, misinfer fundamental information. The equation also indicates that for investors to reconcile their higher order beliefs with observed equilibrium prices, they may necessarily hold mistaken fundamental beliefs. This finding complements recent work that suggests that (mis)inference from prices may contribute to fundamental belief mistakes (e.g., Bordalo et al. (2021), Bastianello and Fontanier (2022a), and Chaudhry (2023)).

### 3 Conclusion

We study investors’ higher order beliefs, using survey data from the Robert Shiller Investor Confidence surveys. While previous work has documented instances of non-fundamental
speculation – investors taking positions in a risky asset in a direction that conflicts with their fundamental views – we find that such speculation is the norm for the U.S. stock market. The majority of investors in the Shiller surveys, who represent an important class of investors, report that other investors have mistaken beliefs, but nevertheless report positive return expectations from speculating in the direction of these mistaken beliefs. In addition, investors report that they believe that stock markets overreact and exhibit momentum and reversal in response to news.

Guided by the empirical evidence, we construct a theoretical model that can match the survey evidence, with investors believing that other investors make systematic mistakes in interpreting fundamental news. We find that higher order beliefs may substantially amplify stock market fluctuations. When investors exhibit the same fundamental belief biases that they attribute to other investors, patterns such as overreaction, momentum, and reversal can persist in equilibrium, even though everybody knows about them.

Our paper also provides direction for future work. The types of higher order beliefs we study may be at play in other asset markets. For example, in a previous version of this paper, Schmidt-Engelbertz and Vasudevan (2023), we examine foreign exchange markets, and find similar patterns of investors’ return expectations that we document here. Additionally, quantitative work may be helpful for better understanding the extent to which stock market fluctuations can be attributed to speculation. We also do not provide microfoundations for investors’ higher order beliefs (and our approach to modeling them is admittedly ‘backwards-engineered’). But exploring the source of investors’ higher order beliefs, and how they interact with fundamental beliefs, may prove fruitful.
References


Bastianello, Francesca, and Paul Fontanier. 2022a. “Expectations and Learning from Prices.” 3, 4, 37

Bastianello, Francesca, and Paul Fontanier. 2022b. “Partial Equilibrium Thinking, Extrapolation, and Bubbles.” 4


McKay, Charles. 1841. Delusions of The Madness of Crowds. Harriman House Classics. 4


Nagel, Stefan, and Zhengyang Xu. 2022b. “Dynamics of subjective risk premia.” National Bureau of Economic Research. 4


Soros, George. 2015. *The alchemy of finance*. John Wiley & Sons. 4


A Proofs

Proof of Lemma 1

Proof. Given the linear-Gaussian environment, belief updating follows the standard Kalman filter recursion method derived in Hamilton (2020):

\[ d^S_t = \rho_S d^S_{t-1} + \kappa_{S,t-1} D_t, \]
\[ \kappa_{S,t-1} = \frac{\rho^2_S \Sigma_{S,t-1} + \sigma^2_{\epsilon}}{\rho^2_S \Sigma_{S,t-1} + \sigma^2_{\epsilon} + \sigma^2_{S,v}}, \text{ and } \Sigma_{S,t} = (1 - \kappa_{S,t-1})(\rho^2_S \Sigma_{S,t-1} + \sigma^2_{\epsilon}). \]

The Kalman gain, \( \kappa_{S,t} \) and conditional variance \( \Sigma_{S,t} \) converge to the steady state values reported in the main text.

Proof of Lemma 2

Proof. Investors perceive that the pricing rule is determined by

\[ 0 = Q^S_t + \psi_t, \text{ and } \]
\[ 0 = \theta \left( \frac{\rho_M}{1 - \rho_M} d^M_t - P_t \right) + \psi_t. \]  
(A.1)

Dividing both sides by \( \theta \) and solving for \( P_t \) provides the perceived pricing rule in the main text.

Proof of Lemma 3

Proof. Investors’ period \( t \) expectation of \( d^M_{t+h} \) is given by

\[ E^S_t(d^M_{t+h}) = \rho_{SM}(1 - \kappa_{SM}) E^S_t d^M_{t+h-1} + \rho_{SM} \kappa_{SM} E^S_t(d^S_{t+h-1}). \]  
(A.2)

We can compute \( E^S_t(d^M_{t+j}) = \rho_{S}^j d^S_t \), for \( j = 1, 2, \ldots, t + h - 1 \). Iterating and plugging into Equation (A.2), we get that

\[ E^S_t(d^M_{t+h}) = \rho_{SM}^h(1 - \kappa_{SM})^h d^S_t + \sum_{j=0}^{h-1} (1 - \kappa_{SM})^j \rho_{SM}^j \rho_{S}^{h-j} d^S_t, \]

(A.3)

which simplifies to Equation (7).
Proof of Lemma 4

Proof. Using investors’ perceived pricing formula, we can see that

$$E_t^S(P_{t+h}) = \frac{\rho_{SM}}{1-\rho_{SM}} E_t^S(d_{t+h}^M) + \theta^{-1} E_t^S(\psi_{t+1}). \quad (A.4)$$

Lemma 3 provides an expression for $E_{S,t}(d_{t+h}^M)$. Given investors’ perception of $\psi_t$ as perfectly persistent, $E_t^S(\psi_{t+1}) = \psi_t^S$. Plugging in investors’ perceived pricing formula, we can write the expected price change component for returns as

$$E_t^S(P_{t+h}) - P_t = \frac{\rho_{SM}}{1-\rho_{SM}} \left( E_{S,t}(d_{t+h}^M) - d_{t}^M \right). \quad (A.5)$$

Lastly, we can write investors’ expected dividends as

$$E_t^S \left( \sum_{k=1}^{h} D_{t+k} \right) = \sum_{k=1}^{h} \rho_k d_t^S$$

$$= \frac{\rho_{S}(1-\rho_{S}^h)}{1-\rho_{S}} d_t^S. \quad (A.6)$$

To prove the second part of the lemma, we note that

$$E_t^S(R_{t,t+1}) = \frac{\rho_{SM}}{1-\rho_{SM}} \left( d_{t+1}^M - d_t^M \right) + \rho_S d_t^S$$

$$= \frac{(1-(1-\kappa_{SM})\rho_{SM})(\rho_S d_t^S - \rho_M d_t^M)}{1-\rho_{SM}}. \quad (A.7)$$

The quantities $1-\rho_{SM}$ and $(1-(1-\kappa_{SM})\rho_{SM})$ are both positive, so subjective expected returns are positive if and only if $\rho_{S} d_t^S - \rho_M d_t^S > 0$. Note that

$$E_t^{SM}(d_{t+1}^SM) = \rho_{SM} d_t^SM, \quad (A.8)$$

and

$$E_t^S(d_t^SM) = \kappa_{SM} \rho_S d_t^S + (1-\kappa_{SM})\rho_{SM}(d_t^SM), \quad (A.9)$$

so,

$$E_t^S(d_t^SM) - E_t^{SM}(d_{t+1}^SM) = \kappa_{M}(\rho_S d_t^S - \rho_{SM} d_t^SM), \quad (A.10)$$

which is also positive if and only if $\rho_{S} d_t^S - \rho_M d_t^SM > 0$. \qed
Proof of Proposition 1

Proof. Investor demand is given by

\[ Q_t^S = \theta \mathbb{E}_i^S (R_{t,t+1}) = \theta \frac{(1 - (1 - \kappa_{SM}) \rho_{SM})}{1 - \rho_{SM}} (\rho_S d_t^S - \rho_M d_t^M). \]  

(A.11)

Residual demand is given by

\[ \lambda (\phi_t - P_t). \]  

(A.12)

Imposing market clearing \((Q_t^S + \psi_t = 0)\) and matching coefficients yields the expression in the main text. \(\square\)

Proof of Lemma 5

Proof. We can re-write investors’ perceived price in period \(t\) as

\[ \frac{\rho_{SM}}{1 - \rho_{SM}} d_t^{SM} + \theta^{-1} \psi_t^S. \]  

(A.13)

Given investors’ belief that noise follows a random walk, we can re-write investors’ expected price change as

\[ \lim_{h \to \infty} \mathbb{E}_{i}^{S} (P_{t+h} - P_t) = \frac{-\rho_{SM}}{1 - \rho_{SM}} d_t^{SM}. \]  

(A.14)

Investors’ expected dividends from holding the asset from period \(t\) in perpetuity are

\[ \mathbb{E}_i^S \left( \sum_{k=1}^{\infty} D_{t+k} \right) = \sum_{k=1}^{\infty} \rho_S d_t^S = \frac{\rho_S}{1 - \rho_S} d_t^{S}. \]

Hence, we can write investors’ expected returns as

\[ \frac{\rho_S}{1 - \rho_S} d_t^S = \frac{\rho_{SM}}{1 - \rho_{SM}} d_t^{SM}. \]  

(A.15)

\(\square\)

Proof of Proposition 2

Proof. We prove each of the parts of the proposition in sequence.

(i) Perceived overvaluation occurs if \(\frac{\rho_{SM}}{1 - \rho_{SM}} d_t^{SM} > \frac{\rho_S}{1 - \rho_S} d_t^S\). From steady state, given a shock to dividends, \(D_t - D_{t-1} = \eta_t > 0, d_t^{SM} = \kappa_{SM} \eta_t\) and \(d_t^S = \kappa_S \eta_t\). Then it is straightforward that perceived overvaluation happens when \(\kappa_{SM} > \frac{1}{1 - \rho_S} \frac{\rho_{SM}}{\rho_S} \kappa_S\), as assumed.
(ii) This follows immediately from Lemma 4, and the fact that \( \kappa_{SM} < \frac{\rho_S}{\rho_{SM}} \kappa_S \), as assumed.

(iii) Perceived time-series momentum immediately follows from (ii). Next, we show that for some period in the future \( \bar{h} \), for \( h > \bar{h} \), investors forecast returns to be negative.

Using Proposition 4, we can write investors’ return expectations \( h \) periods ahead as

\[
\mathbb{E}_{S,t}(R_{t+h-1,t+h}) = \frac{(1 - (1 - \kappa_{SM})\rho_{SM})(1 - \kappa_{SM})^{h} \rho_{SM}^h \rho_{SM}^h (\kappa_{SM} \rho_{S} d_{t}^S + (1 - \kappa_{SM}) \rho_{SM} d_{t}^{SM} - \rho_{S} d_{t}^{SM})}{(1 - \kappa_{SM})(1 - \rho_{SM})(\rho_{S} - (1 - \kappa_{SM}) \rho_{SM})} - \frac{(\rho_{SM} - \rho_{S})(1 - \kappa_{SM}) \rho_{SM}^h d_{t}^S}{(1 - \kappa_{SM})(1 - \rho_{SM})(\rho_{S} - (1 - \kappa_{SM}) \rho_{SM})}. \tag{A.16}
\]

We next proceed by cases, based on whether \( \rho_{S} > (1 - \kappa_{SM}) \rho_{SM} \) or \( \rho_{S} < (1 - \kappa_{SM}) \rho_{SM} \).

**Case 1:** \( \rho_{S} > (1 - \kappa_{SM}) \rho_{SM} \)

The denominator of Equation (A.16) is positive, since \( 0 < \kappa_{SM} < 1 \), \( 0 < \rho_{SM} < 1 \), and \( \rho_{S} > (1 - \kappa_{SM}) \rho_{SM} \) (by assumption). Hence, the claim requires that the numerator is negative. In the numerator, \( 1 - (1 - \kappa_{SM}) \rho_{SM} \) is positive. So our claim requires that

\[
(\rho_{SM} - \rho_{S})(1 - \kappa_{SM}) \rho_{SM}^h d_{t}^S > (1 - \kappa_{SM})^h \rho_{SM}^h (\kappa_{SM} \rho_{S} d_{t}^S + (1 - \kappa_{SM}) \rho_{SM} d_{t}^{SM} - \rho_{S} d_{t}^{SM}). \tag{A.17}
\]

Re-arranging terms, this is equivalent to

\[
\left( \frac{(1 - \kappa_{SM}) \rho_{SM}}{\rho_{S}} \right)^h < \frac{\kappa_{SM} (\rho_{S} d_{t}^S - \rho_{SM} d_{t}^{SM}) + (\rho_{SM} - \rho_{S}) d_{t}^{SM}}{(\rho_{SM} - \rho_{S})(1 - \kappa_{SM}) d_{t}^S}. \tag{A.18}
\]

Each of the terms on the right hand side is positive. Hence, the right hand side reduces to a positive constant. The term \( \left( \frac{1 - \kappa_{SM}}{\rho_{S}} \right) \rho_{SM} \) is less than 1 (since \( \rho_{S} > (1 - \kappa_{SM}) \rho_{SM} \) by assumption), and hence, the left hand side is decreasing in \( h \). As \( h \to \infty \), the left hand side converges to zero. Therefore, we can find an \( \bar{h} \) such that for \( h > \bar{h} \), the left hand side is less than right hand side, and the forecasted expected return is negative.

**Case 2:** \( \rho_{S} < (1 - \kappa_{SM}) \rho_{SM} \)

The denominator of Equation (A.16) is negative, since \( \rho_{S} < (1 - \kappa_{SM}) \rho_{SM} \). Hence, our claim requires that the numerator is positive, which requires that

\[
(\rho_{SM} - \rho_{S})(1 - \kappa_{SM}) \rho_{SM}^h d_{t}^S < (1 - \kappa_{SM})^h \rho_{SM}^h (\kappa_{SM} \rho_{S} d_{t}^S + (1 - \kappa_{SM}) \rho_{SM} d_{t}^{SM} - \rho_{S} d_{t}^{SM}). \tag{A.19}
\]

Re-arranging terms, this is equivalent to

\[
\left( \frac{(1 - \kappa_{SM}) \rho_{SM}}{\rho_{S}} \right)^h > \frac{\kappa_{SM} (\rho_{S} d_{t}^S - \rho_{SM} d_{t}^{SM}) + (\rho_{SM} - \rho_{S}) d_{t}^{SM}}{(\rho_{SM} - \rho_{S})(1 - \kappa_{SM}) d_{t}^S}. \tag{A.20}
\]

As in the previous case, the right-hand side is a positive constant. By assumption, the \( \left( \frac{1 - \kappa_{SM}}{\rho_{S}} \right) \rho_{SM} > 1 \). Hence, we can find a \( \bar{h} \) such that \( h > \bar{h} \) implies that the inequality holds. □
Proof of Lemma 6

Proof. We can write objective expected returns as

\[ E_t(R_{t,t+1}) = E_t(P_{t+1} + D_{t+1}) - P_t \]
\[ = \frac{\theta}{\lambda} \left( E_{t+1}^S(R_{t+1,t+2}) - E_t^S(R_{t,t+1}) \right) + \left( \phi_{t+1} - \phi_t \right) + \rho d_t. \]  

(A.21)

Proof of Proposition 3

Proof. For the first part of the proposition, the claim follows immediately from the fact that investors expect positive returns following a positive dividend shock, which leads \( P_t \) to exceed \( \phi_t \).

For the second part of the proposition

\[ E_t^S(R_{t,t+1}) = \frac{(1 - (1 - \kappa_{SM})\rho_{SM})E_t^S(\rho_Sd_t^S - \rho_{SM}d_t^{SM})}{1 - \rho_{SM}}. \]

Everything except \( \rho_Sd_t^S - \rho_{SM}d_t^{SM} \) is a constant (and expected returns are positive if and only if \( \rho_Sd_t^S - \rho_{SM}d_t^{SM} > 0 \)), so we focus on this term.

Claim 1: Whenever expected returns are positive, the next period’s returns are lower. That is \( E_t^S(R_{t,t+1}) > 0 \) \( \implies \) \( E_t^S(R_{t+1,t+2}) < E_t^S(R_{t,t+1}) \). To show this, we show that

\[ \rho_Sd_t^S - \rho_{SM}d_t^{SM} > \rho_Sd_{t+1}^S - \rho_{SM}E_t^S(d_{t+1}^{SM}). \]  

(A.22)

This comes from

\[ \rho_Sd_{t+1}^S - \rho_{SM}E_t^S(d_{t+1}^{SM}) = \rho_S^2d_t^S - \kappa_{SM}\rho_{SM}\rho_Sd_t^S - (1 - \kappa_{SM})\rho_{SM}^2d_t^{SM} \]
\[ = \rho_S((\rho_S - \kappa_{SM}\rho_{SM})d_t^S) - \rho_{SM}(1 - \kappa_{SM})\rho_{SM}d_t^{SM} \]
\[ \leq \rho_S((\rho_S - \kappa_{SM}\rho_{SM})d_t^S - (1 - \kappa_{SM})\rho_{SM}d_t^{SM}) \]
\[ = \rho_S((\rho_Sd_t^S - \rho_{SM}d_t^{SM}) - \kappa_{SM}(\rho_{SM}d_t^S - \rho_{SM}d_t^{SM})) \]  

(A.23)

\[ > \rho_Sd_t^S - \rho_{SM}d_t^{SM}. \]

Claim 2: Whenever expected returns are negative, the next period’s returns are higher. That is \( E_t^S(R_{t,t+1}) < 0 \) \( \implies \) \( E_{t+1}^S(R_{t+1,t+2}) > E_t^S(R_{t,t+1}) \). To show this, we show that

\[ \rho_Sd_t^S - \rho_{SM}d_t^{SM} < \rho_Sd_{t+1}^S - \rho_{SM}E_t^S(d_{t+1}^{SM}). \]  

(A.24)
This comes from:

\[
\rho_S d^s_{t+1} - \rho_{SM} E^S_t (d^{SM}_{t+1}) = \rho^2 S d^S_t - \kappa_{SM} \rho_{SM} \rho_S d^S_t - (1 - \kappa_{SM}) \rho^2_{SM} d^{SM}_t \\
= \rho^2 S d^S_t - \rho^2_{SM} d^{SM}_t + \kappa_{SM} \rho_{SM} (\rho_{SM} d^{SM}_t - \rho_S d^S_t) \\
\geq \rho_S (\rho_S d^S_t - \rho_{SM} d^{SM}_t) + \kappa_{SM} \rho_{SM} (\rho_{SM} d^{SM}_t - \rho_S d^S_t) \\
> \rho_S d^S_t - \rho_{SM} d^{SM}_t 
\]

(A.25)

Then we note that realized returns are proportional to the change in subjective expected returns, by Lemma 6. Expected returns are initially positive (by \(\kappa_{SM} < \bar{\kappa}\)), so the period \(t+1\) returns are negative by claim 1. Eventually, return expectations switch to being negative (from Proposition 2). Realized returns switch to being positive (by claim 2).

Proof of Proposition 4

Proof. Per Lemma 6, for momentum to exist in equilibrium following a shock in period \(t\), we need that

\[
\mathbb{E}^S_{t+1}(R_{t+1,t+2}) > \mathbb{E}^S_t (R_{t+1,t+1}). 
\]

(A.26)

Following previous proofs, this is true if and only if

\[
\rho_S d^S_{t+1} - \rho_{SM} d^{SM}_{t+1} > \rho_S d^S_t - \rho_{SM} d^{SM}_t. 
\]

(A.27)

Without loss of generality, we assume a unit shock to the persistent component of dividends, \(d_t = 1\). We denote \(\kappa_{SM} = \alpha \kappa_S\).

We know that \(d^S_t = \kappa_S\) and \(d^{SM}_t = \alpha \kappa_S\). In period \(t+1\), agents observe the true dividend, \(\rho\), and update their fundamental and higher order beliefs.

\[
d_{t+1} = \kappa_S \rho + (1 - \kappa_S) \rho_S \kappa_S, \text{ and} \\
d^{SM}_{t+1} = \kappa_{SM} \rho + (1 - \kappa_{SM}) \rho_{SM} \alpha \kappa_S. 
\]

(A.28)

Then, we get

\[
\rho_S d^S_{t+1} - \rho_{SM} d^{SM}_{t+1} - (\rho_S d^S_t - \rho_{SM} d^{SM}_t) \\
= \kappa_S (\rho - \alpha \rho + \kappa_S (\alpha^2 \rho_{SM} - \rho_S)). 
\]

(A.29)

We want this to be greater than zero, which, imposing \(\kappa_S > 0\), is satisfied if and only if

\[
\kappa_S < \frac{(1 - \alpha) \rho}{\rho_S - \alpha^2 \rho_{SM}}. 
\]

(A.30)

In the case that there is a non-trivial transitory component to dividends, then \(\kappa < 1\), and this requirement becomes \(\kappa_S < \frac{(1 - \alpha) \rho - \kappa}{\rho_S - \alpha^2 \rho_{SM}}\).

\(\square\)

49
B Model with Inference from Prices

The model environment follows the main model environment, with a few modifications. First, investors do not directly observe dividends, but believe that other investors do. Second, investors know that residual demand reflects an informed valuation about fundamentals, and they know the slope of the residual demand curve. We also normalize $\theta + \lambda = 1$, without loss of generality, and assume that investors’ have correct beliefs about the persistence of the dividend process ($\rho_S = \rho$).

On average, following a shock, investors perceive that equilibrium prices satisfy

$$P_t = \theta \rho_{SM} \kappa_{SM} d_t^S + \lambda \frac{\rho}{1 - \rho} d_t^S,$$

while prices are actually determined by the market clearing condition

$$0 = \theta \mathbb{E}_t^S(R_{t,t+1}) + \lambda \left( \frac{\rho}{1 - \rho} d_t - P_t \right),$$

$$\mathbb{E}_t^S(R_{t,t+1}) = \theta \frac{(1 - (1 - \kappa_{SM})\rho_{SM})(\rho - \kappa_{SM}\rho_{SM})}{1 - \rho_{SM}} d_t^S. \quad (B.2)$$

In equilibrium, $d_t^S$ solves a fixed point from setting $P_t$ from Equations (B.1) and (B.2) equal to each other. Then, the extracted fundamental belief, $d_t^S$, satisfies

$$d_t^S = \left( 1 - \theta \right) \frac{\rho}{1 - \rho} + \theta \frac{(\theta \rho + (1 - \theta)\kappa_{SM})\rho_{SM} + \rho}{1 - \rho_{SM}} \left( 1 - \theta \right) \frac{\rho}{1 - \rho} d_t.$$

Note that the condition $\kappa_{SM} < \frac{\rho}{\rho_{SM}}$ means that investors have positive expected returns (from Equation (B.2)) and the condition $\kappa_{SM} > \frac{1 - \rho_{SM}}{1 - \rho} \frac{\rho}{\rho_{SM}}$ guarantees that investors perceive the market to be overvalued (from Equation (B.1)).
C Additional Empirical Analyses

In this section, we present additional empirical analyses. We first analyze if the number of survey responses in the Shiller survey displays any business cycle variation, and find no evidence that it does. The rest of the section presents tables and figures that replicate the main results for different subsets of the data (e.g., individual versus institutional investors).

C.1 Survey Responses in the Shiller Survey

We analyze if there is any business cycle frequency variation in responses to the Shiller survey. We regress the quarterly change in the log number of survey responses to the survey each quarter on S&P 500 returns, and quarterly innovations in the Conference Board Coincident indicators index (labeled ‘Macro’). The independent variables are standardized to have zero mean and unit standard deviation. Table C.1 reports the results, and Newey-West standard errors (4 lags) are reported in parentheses. There is little evidence to indicate systematic business cycle variation in survey response counts.

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<th>Retail</th>
<th>Inst</th>
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<td>1.07</td>
</tr>
<tr>
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<td>(2.57)</td>
<td>(2.21)</td>
</tr>
<tr>
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<td>-2.58</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>(2.99)</td>
<td>(1.59)</td>
<td>(1.54)</td>
</tr>
</tbody>
</table>

Table C.1: Response Counts and Business Cycle Variation

---

32Unfortunately, we do not observe the number of questionnaires that were sent out each quarter, so we use changes in total responses to proxy for response rates.
Panel A: Term Structure of Expected Cumulative Returns

<table>
<thead>
<tr>
<th></th>
<th>E_t(R_{t,t+1})</th>
<th>E_t(R_{t,t+3})</th>
<th>E_t(R_{t,t+6})</th>
<th>E_t(R_{t,t+12})</th>
<th>E_t(R_{t,t+1})</th>
<th>E_t(R_{t,t+3})</th>
<th>E_t(R_{t,t+6})</th>
<th>E_t(R_{t,t+12})</th>
</tr>
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<tbody>
<tr>
<td>HO Belief</td>
<td>1.44 (0.53)</td>
<td>0.76 (0.57)</td>
<td>0.00 (0.60)</td>
<td>-1.38 (0.68)</td>
<td>0.21 (0.06)</td>
<td>-0.05 (0.08)</td>
<td>0.61 (0.10)</td>
<td>-1.19 (0.12)</td>
</tr>
<tr>
<td>Time FE</td>
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<td>NA</td>
<td>NA</td>
<td>NA</td>
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<td>Yes</td>
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<td>5651</td>
</tr>
<tr>
<td>R^2</td>
<td>0.09</td>
<td>0.02</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.02</td>
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Panel B: Short-term Peaks and Troughs

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<th>ST Peak</th>
<th>ST Trough</th>
<th>ST Peak</th>
<th>ST Trough</th>
</tr>
</thead>
<tbody>
<tr>
<td>HO Belief</td>
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<td>-0.25 (0.05)</td>
<td>0.54 (0.07)</td>
<td>-0.02 (0.17)</td>
</tr>
<tr>
<td>Overvaluation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time FE</td>
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<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>N</td>
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<td>259</td>
<td>259</td>
<td>259</td>
</tr>
<tr>
<td>R^2</td>
<td>0.13</td>
<td>0.10</td>
<td>0.21</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Table C.2: Higher Order Beliefs and Return Expectations (Individual Investors)**

*Note:* This table replicates Table 3 for the individual investor subset of our sample.
### Panel A: Term Structure of Expected Cumulative Returns

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</thead>
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<td>$\mathbb{E}<em>t(R</em>{t,t+1})$</td>
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<td>$HO$ Belief</td>
<td>0.13</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$ST$ Peak</td>
<td>-0.39</td>
<td>-0.61</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$ST$ Trough</td>
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<td>-1.52</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>$ST$ Peak</td>
<td>-2.57</td>
<td>-2.49</td>
</tr>
<tr>
<td></td>
<td>(0.70)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>$ST$ Trough</td>
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<td>-0.61</td>
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<tr>
<td></td>
<td>(0.05)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$ST$ Peak</td>
<td>-0.12</td>
<td>-0.61</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$ST$ Trough</td>
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<td>-0.61</td>
</tr>
<tr>
<td></td>
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<td>(0.31)</td>
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<tr>
<td>$ST$ Peak</td>
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<td>-1.52</td>
</tr>
<tr>
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<td>(0.39)</td>
<td>(0.13)</td>
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<tr>
<td>$ST$ Trough</td>
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<td>-2.49</td>
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<tr>
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<td>(0.08)</td>
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<tr>
<td>$ST$ Trough</td>
<td>-0.61</td>
<td>-0.61</td>
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<tr>
<td></td>
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<td>(0.31)</td>
</tr>
<tr>
<td>$ST$ Peak</td>
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<td>-1.52</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>$ST$ Trough</td>
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<td>-2.49</td>
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<tr>
<td></td>
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<td>(0.19)</td>
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<tr>
<td>$ST$ Peak</td>
<td>-0.12</td>
<td>-0.61</td>
</tr>
<tr>
<td></td>
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<td>(0.08)</td>
</tr>
<tr>
<td>$ST$ Trough</td>
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<td>-0.61</td>
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<tr>
<td></td>
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<td>(0.31)</td>
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<tr>
<td>$ST$ Peak</td>
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<td>(0.19)</td>
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<td>Yes</td>
</tr>
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<tr>
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<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
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### Panel B: Short-term Peaks and Troughs

<table>
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<tr>
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<th>ST Peak</th>
<th>ST Trough</th>
<th>ST Peak</th>
<th>ST Trough</th>
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</thead>
<tbody>
<tr>
<td>$HO$ Belief</td>
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</tr>
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<td>(0.07)</td>
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<tr>
<td>Overvaluation</td>
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<td>$N$</td>
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<tr>
<td>$R^2$</td>
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**Table C.3: Higher Order Beliefs and Return Expectations (Institutional Investors)**

*Note:* This table replicates Table 3 for the institutional investor subset of our sample.
Panel A: Term Structure of Expected Cumulative Returns and Higher Order Optimism

<table>
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<td></td>
<td>$E_t(R_{t,t+1})$</td>
<td>$E_t(R_{t,t+12})$</td>
</tr>
<tr>
<td><strong>HO Optimism</strong></td>
<td>1.83 (0.45)</td>
<td>0.06 (0.05)</td>
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<td><strong>Time FE</strong></td>
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<td>Yes</td>
</tr>
<tr>
<td><strong>N</strong></td>
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<td>10137</td>
</tr>
<tr>
<td><strong>$R^2$</strong></td>
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</tbody>
</table>

Panel B: Term Structure of Expected Cumulative Returns and Higher Order Pessimism

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</tr>
</thead>
<tbody>
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<td></td>
<td>$E_t(R_{t,t+1})$</td>
<td>$E_t(R_{t,t+12})$</td>
</tr>
<tr>
<td><strong>HO Optimism</strong></td>
<td>-2.03 (0.61)</td>
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</tr>
<tr>
<td><strong>Time FE</strong></td>
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<tr>
<td><strong>N</strong></td>
<td>259</td>
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<tr>
<td><strong>$R^2$</strong></td>
<td>0.08</td>
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</table>

**Table C.4: Higher Order Optimism, Pessimism, and Return Expectations**

*Note: This table replicates Table 3, separately breaking down the results for HO Optimism and HO Pessimism.*
Figure C.1: Macroeconomic News and Expectations (Individual Investors)

Note: The figure replicates Figure 3 for the individual investor subset of our sample.
Figure C.2: Macroeconomic News and Expectations (Institutional Investors)

Note: The figure replicates Figure 3 for the institutional investor subset of our sample.
FIGURE C.3: COINCIDENT INDICATORS AND EXPECTATIONS

Note: The figure replicates the top panel of Figure 3, using innovations to the Coincident Macroeconomic Indicators index from the Conference Board.
Figure C.4: Futures Positions and Return Expectations (Individual Investors)

Note: The figure replicates Figure 2 for the individual investor subsample.
**FIGURE C.5: FUTURES POSITIONS AND RETURN EXPECTATIONS (INSTITUTIONAL INVESTORS)**

*Note: The figure replicates Figure 2 for the institutional investor subsample.*
Figure C.6: Futures Positions and Return Expectations (Asset Managers)

Note: The figure replicates Figure 2 using the futures positions of asset managers.
Figure C.7: Futures Positions and Return Expectations (Levered Money)

Note: The figure replicates Figure 2 using the futures positions of leveraged money investors.