Asset Pricing with Higher Order Beliefs*

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Abstract

Higher order beliefs – beliefs about others’ beliefs – may be important for trading behavior and asset prices, but have received little systematic empirical examination due to challenges in measurement. We circumvent these challenges using the insight that return expectations for an asset encode higher order beliefs, as they depend on forecasts of other investors’ future demand. Analyzing survey data on the beliefs of U.S. equity market and global currency market investors, we find that the term structure of investors’ cumulative return expectations follows a hump shape: when investors report high return expectations for the following month or quarter, they report low return expectations for subsequent quarters. We use novel survey data to directly relate this pattern to investors’ higher order beliefs. Guided by the evidence, we construct a tractable asset pricing model that we use to theoretically and quantitatively explore investors’ higher order beliefs and their impact on asset prices. In a quantitative application, we find that the higher order beliefs of financial institutions exhibit systematic biases, but nevertheless play a corrective role for exchange rates.

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Higher order beliefs – beliefs about others’ beliefs – are important in many equilibrium models in economics and finance. As emphasized by a strand of primarily theoretical literature starting with Keynes (1936), higher order beliefs may be particularly important for understanding investor behavior and financial market fluctuations. This is because investors seeking to earn high returns have strong incentives to trade based on predictions of other investors’ beliefs rather than their own fundamental asset valuations. Indeed, successful investors claim to follow strategies based on predicting others’ beliefs. Such trading behavior can lead asset prices to substantially deviate from fundamental values. Despite their ubiquity and potential importance, higher order beliefs have received little systematic empirical examination, primarily due to inherent challenges in their measurement.

In this paper, we examine survey data on investors’ beliefs in U.S. equity and global currency markets, explore what they imply about investors’ higher order beliefs, and consider the corresponding asset pricing implications. Our exploration is guided by the insight that an investor’s return expectations depend on her beliefs about the asset demand of other investors in the future, which, in turn, depend on her forecasts of other investors’ beliefs – her higher order beliefs. For example, she might expect high returns to investing in an asset if she believes that other investors will revise their beliefs about the asset’s valuation upwards in the future. We draw out this intuition in a stylized model, which reveals a rich interaction of investors’ higher order beliefs with their beliefs about fundamentals, their trading motivations, and their impact on asset prices. In a quantitative application, we estimate our model using data on survey-based expectations in currency markets, and use it to examine the role of higher order beliefs for asset price behavior.

We begin our analysis by documenting new empirical evidence on investors’ return expectations and higher order beliefs. First, examining the term structure of investors’ return expectations at a given point in time, we find that investors have hump-shaped cumulative return expectations across surveys and markets. In particular, when investors expect high returns 1-month or 1-quarter ahead, they often expect low returns in subsequent quarters, a pattern that is particularly pronounced following fundamental news. While the hump-shaped pattern of return expectations poses a challenge for standard models, we directly empirically link the pattern with investors’ higher order beliefs.

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\[1\] De Long et al. (1990) note that George Soros “has been generally successful... by betting not on fundamentals but, he claims, on future crowd behavior.” They also point to other successful investors that trade by predicting other investors’ beliefs. Brunnermeier and Nagel (2004) argue that hedge funds’ trading behavior during the dot-com bubble is consistent with higher order belief induced trading and “consistent with the view that hedge fund managers were able to predict some of the investor sentiment that was arguably behind the wild fluctuations in valuations of technology stocks at the time.”

\[2\] For beliefs about time-varying risk premia to explain the evidence, equity market investors would have to believe that risk premia are high upon the arrival of good news, and low immediately thereafter, counter to traditional assumptions. Behavioral models where investors make systematic errors in forecasting cash flows can make vastly different predictions about return expectations depending upon assumptions about risk premia and higher order beliefs.
optimistic about the stock market’s prospects. We find that the Higher Order Optimism index is strongly positively correlated with past market returns, macroeconomic news, and 1-month ahead return expectations, but strongly negatively correlated with expectations of returns for 3- to 12-months ahead.

We rationalize the link between the hump-shaped pattern of investors’ return expectations and their higher order beliefs with a simple theoretical model. The model is rich enough to capture the empirical evidence on investors’ return expectations and higher order beliefs, but importantly, also provides a tractable way to estimate features of investors’ higher order beliefs using survey data on fundamental and return expectations. This tractability, in turn, allow us to quantitatively investigate the role that higher order beliefs may play in investors’ trading behavior and asset prices.

The model features a riskless asset, and a single risky asset that pays a dividend each period, which is observed by all investors. Each period, the dividend is drawn from a distribution centered around an unobserved fundamental that follows an AR(1) process. The model features short-lived Bayesian investors who trade against residual demand for the risky asset. They form their beliefs about the unobserved fundamental and future dividends by Kalman filtering, which involves placing some weight (their Kalman gain) on the observed dividend and some weight on their prior belief about dividends.

Investors in the model have hump-shaped cumulative return expectations following the arrival of fundamental news. Their return forecasts are based on their higher order beliefs about how other investors (“the market”) will update their beliefs about fundamentals in the future. We capture the upwards portion of hump-shaped return expectations with the assumption that investors perceive others to persistently update their beliefs in the direction of past news. This perception drives a belief that asset returns will be in the same direction of news for multiple periods, as the market continues to incorporate past information. We capture the downwards portion of hump-shaped return expectations with the assumption that investors perceive the market as overestimating the persistence of fundamentals. This perception drives a belief that once the market incorporates past fundamental news, it expects dividends to remain high for longer than they do in reality. Investors believe that dividends will eventually be lower than prevailing expectations, which will lead to low asset returns as the market revises its valuation downwards.

The model illustrates a set of rich implications regarding investors’ higher order beliefs, their predicted trading behavior, and the resulting effects on equilibrium asset prices. A belief that returns will be high following positive fundamental news leads investors to trade aggressively upon the arrival of news. However, investors may have differing motivations for trading aggressively. For example, due to a belief in other investors’ overestimation of fundamental persistence, investors may perceive that market valuations overreact, e.g., they believe the risky asset becomes overvalued upon the arrival of good news. In this case, investors’ motivations in trading aggressively are destabilizing (as opposed to corrective): investors believe the asset will become even more overvalued in the future, and seek to profit
by purchasing the asset in the present and reselling it in the future at an increasingly inflated value. The empirical evidence on investors’ higher order beliefs in the U.S. equity market points in the direction of destabilizing trading motives, with investors expecting short-term positive returns from investing in the stock market in times when they believe other investors to be overly optimistic.

Separate from investors’ motivations for trading, the true impact of their higher order beliefs on equilibrium asset prices depends upon whether their beliefs are correct or incorrect. If investors have correct beliefs, then their motivations for trading correspond with the true impact that their trading has on asset prices. However, if investors have incorrect fundamental beliefs (e.g., their own fundamental valuations overreact to news), or if they have incorrect higher order beliefs (e.g., they underestimate the speed with which other investors incorporate information), then their motivations for trading may differ from the true impact of their trading. For example, investors’ trading behavior might over-correct and destabilize asset prices if their own fundamental beliefs overreact to news, even if their motivations for trading are corrective.

To understand how these interactions play out, we take our model to foreign exchange markets, and estimate it using survey data of large financial institutions that are active market participants. The parameters that require estimating are those governing investors’ fundamental and higher order beliefs, and those governing residual demand – demand not captured by beliefs in survey data. Following Angeletos, Huo and Sastry (2021), we estimate investors’ fundamental belief parameters by matching the impulse responses of model-implied forecast errors in response to fundamental shocks with the corresponding empirical impulse response functions estimated using survey data. We then use these estimated fundamental belief parameters to estimate investors’ higher order belief parameters by matching their model-implied, multi-period return expectations in response to fundamental shocks with corresponding survey-based expectations. Finally, we estimate parameters governing residual demand to match the empirically observed returns process when combined with investors’ trading behavior.

With the estimated model in hand, we turn to understanding how higher order beliefs affect asset prices. The cash flow associated with borrowing in a currency and lending in another currency is the interest rate differential between the currencies. Currency market returns are characterized by strong inertia and initial underreaction, as well as delayed overreaction to news about interest rate differentials. In survey data, investors forecast exchange rates to rise following an interest rate news shock. They expect the market to continue to revise its beliefs upwards and incorporate the news of the shock for multiple periods. Based on this belief, upon the arrival of news, investors trade aggressively, providing a corrective effect against the initial underreaction that is prevalent in currency markets. The model implies that exchange rates under react by 2.9% relative to their fundamental value following a one standard deviation shock to fundamentals, but would under react by 4.7% without investors’ trading. Investors also believe that the market overestimates the persistence of
fundamentals. This perception leads them to believe that returns will eventually reverse following a positive fundamental shock, once the market becomes disappointed upon observing lower than expected interest differentials. Accordingly, investors in the model then switch to trading against perceived overreaction by taking short positions several periods after a shock. This behavior once again stabilizes the exchange rate, and reduces its eventual overreaction to news by 1.6%.

Interestingly, investors in the model exert a corrective influence on exchange rates in spite of the fact that their fundamental and higher order beliefs exhibit systematic biases. In a counterfactual exercise, we find that correcting investors’ higher order belief biases does not improve informational efficiency. This is because investors’ fundamental valuations overreact to news, as they themselves overestimate the persistence of interest rate differentials. With correct knowledge of other investors’ fundamental beliefs, investors in our model over-correct the underreaction in currency markets, destabilizing the exchange rate. The results highlight that the impact of investors’ higher order beliefs on equilibrium asset prices crucially depends on the interaction of these higher order beliefs with investors’ fundamental beliefs.

For equity markets, we provide suggestive evidence that surveyed investors’ higher order beliefs may push equity valuations away from fundamental values and destabilize prices. We do not conduct a similar quantitative exercise as in currency markets, as we do not have data on the fundamental expectations of the same types of investors for whom we have return expectations. However, our empirical analysis, interpreted through the lens of our model, indicates that investors believe that the equity market overreacts to news. Despite this belief, investors are motivated to take positions in the direction of recent past news due to a belief that others will continue to update their beliefs in the same direction. This destabilizing trading behavior may amplify the overreaction of fundamental beliefs found in survey data of equity analysts, which other work has suggested contributes to excess stock market volatility and return predictability (Nagel and Xu (2022a), Bordalo et al. (2020)). An interesting avenue for further exploration, which our paper provides a framework to study, is the relative contribution of fundamental beliefs versus higher order beliefs in driving stock market fluctuations.

Taken together, our evidence highlights the role of higher order beliefs in asset prices. Higher order beliefs implied by survey data suggest that investors believe that other investors persistently update their beliefs in the same direction of past news for several periods, and that others overestimate the persistence of fundamentals. However, such beliefs have an ex-ante ambiguous effect on informational efficiency, depending upon whether they are correct, and depending upon their interaction with the prevailing fundamental beliefs in the market. Our quantitative analysis indicates that higher order beliefs implied by survey data of financial institutions may play a corrective role in currency markets, but we find suggestive evidence that the higher order beliefs of equity market investors may play a destabilizing role.
Related Literature

Our paper is related to other work on higher order beliefs in asset pricing, which, as noted previously, has primarily been theoretical in nature, and to which we bring empirical content using survey data.\(^3\) The literature on higher order beliefs in asset pricing can largely be partitioned into two traditions: ‘beauty contest’ models in the noisy rational expectations tradition, where investors form beliefs in a rational manner but face informational frictions that prevent them from observing other investors’ beliefs and the true fundamentals (Singleton (1987), Allen, Morris and Shin (2006), Bacchetta and van Wincoop (2006, 2008), Makarov and Rytchkov (2012), Kasa, Walker and Whiteman (2014), Cespa and Vives (2015), and Ni- mark (2017)); and differences-of-opinion models, where investors know and disagree with other investors’ private valuations, and speculate based on the resulting belief disagreements (Harrison and Kreps (1978), Harris and Raviv (1993), Kandel and Pearson (1995), Scheinkman and Xiong (2003), and Banerjee and Kremer (2010)).\(^4\) Relative to both bodies of work, our paper’s focus is primarily empirical, and our paper also highlights that the impact of investors’ higher order beliefs for asset markets depends crucially on the interactions of those higher order beliefs with investors’ fundamental beliefs.

While the literature on higher order beliefs emphasizes higher orders of reasoning (e.g., if all investors have second order beliefs about other investors’ fundamental beliefs, then investors may optimally form third order beliefs about those second order beliefs, and so on and so forth), our approach in this paper abstracts away from such considerations, and focuses on the aspects of higher order beliefs that are measurable using readily available survey data. Multiple structures of belief hierarchies can rationalize the same distribution of equilibrium returns and return expectations, so data on returns and return expectations alone cannot distinguish features of higher order reasoning. Return expectations capture the aggregated effect of all levels of higher order reasoning that investors may engage in. Accordingly, we interpret our results as providing a measurement of the impact of higher order beliefs on asset prices, and as providing micro-founded theoretical models of higher order reasoning and beliefs with empirical guidance. We provide a more detailed discussion of how our results relate to other models of higher order beliefs in Section 4.

Our paper is also related to a literature in finance using survey data as a means of understanding market participants’ beliefs (Adam and Nagel (2022) provide a survey). A sizeable prior literature on equity markets has studied the importance of fundamental and return

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\(^3\) There are two notable exceptions in the literature that seek to provide some empirical characterization of higher order beliefs. One is Egan, Merkle and Weber (2014), who conduct a survey of private investors and study how expectations of other investors’ return expectations influence private investors’ investment decisions. They find evidence that second-order beliefs do matter for investment decisions. The second exception, from macroeconomics, is Coibion et al. (2021), who survey firm managers in New Zealand regarding their higher order beliefs, with a specific focus on testing the predictions of noisy information models.

\(^4\) Outside of finance, there is an important strand of experimental work that suggests that people often make systematic errors in predicting others’ behavior, with some additional supporting field evidence; see Eyster (2019) for a recent survey. There is also growing literature in macroeconomics that deals with higher order beliefs e.g., as discussed in a recent survey by Angeletos and Lian (2022).

Whereas the extant literature has largely studied fundamental and return expectations in isolation from one another, we seek to tie the two together by higher order beliefs (see also Giglio et al. (2021), Jin and Sui (2022), and McCarthy and Hillenbrand (2021) for other work relating fundamental and return expectations). In doing so, we bring to light a set of rich interactions between investors’ fundamental and return expectations that may be missed when studying the two in isolation. For example, investors with extrapolative return expectations – beliefs that high past market returns predict future market returns – trade more aggressively following past returns. In doing so, depending upon their own fundamental beliefs, they may believe they are correcting or destabilizing asset prices, and depending upon the prevailing beliefs of market participants, their behavior may actually correct or destabilize prices. In currency markets, our quantitative evidence suggests that extrapolative return expectations play a corrective role, whereas we find suggestive evidence that such return expectations may destabilize prices and contribute to overreaction in the U.S. equity market.

The rest of the paper proceeds as follows. In Section 1 we discuss survey data and present empirical evidence on investors’ return expectations. In Section 2, we present a theoretical asset pricing model, and illustrate how higher order beliefs may explain the empirical evidence. Section 3 calibrates the model using survey data in currency markets, and presents quantitative analysis on the importance of higher order beliefs. Section 4 discusses alternative explanations for our empirical evidence and compares our perspective on higher order beliefs with other work in the literature. Section 5 concludes.

5Relative to prior work on return expectations, which largely focuses on expectations of returns at a fixed future horizon (e.g., one-year ahead), our work places particular focus on the future path of returns that investors expect. In contemporaneous work to our own, Gandhi, Gormsen and Lazarus (2022) study the term structure of return expectations implied by options prices.

6Bacchetta, Mertens and van Wincoop (2009) and Nagel and Xu (2022b) focus on survey evidence on return expectations across asset classes.
1 Empirical Evidence from Survey Data

To guide our investigation of higher order beliefs, we study expectations of U.S. equity market returns reported in the Shiller/Yale ICF survey by retail and institutional investors, as well as return expectations of financial institutions for developed market currencies from FX4casts. Across each of the surveys we consider, focusing on the term structure of investors’ return expectations, we find evidence that investors have hump-shaped return expectations – when they expect high returns 1-month to 1-quarter ahead, they expect low or negative returns in subsequent quarters. While these expectations are not consistent with the predictions of standard models, we show that they are strongly related to investors’ reported higher order beliefs, and later argue that they can be rationalized by focusing on investors’ higher order beliefs about other investors’ belief updating process.

1.1 Data Description

Below we briefly describe the survey data used for our empirical analysis in this section and in our quantitative analysis in Section 3.

Shiller / Yale ICF Investor Survey

The Robert Shiller Stock Market Confidence indices are based on survey data collected continuously since 1989 - semi-annually for a decade, and then monthly by the International Center for Finance at the Yale School of Management since July 2001. About 300 questionnaires each month are mailed to individuals identified by a market survey firm as high-net-worth investors and institutional investors, with a sampling goal of 20 to 50 responses by each of the two types - individual and institutional. Shiller (2000) discusses the survey questions and the stock market confidence indices constructed from the survey in more detail. We use individual-level responses to the survey questions in our analysis.

Especially relevant to us, the survey data contain investor responses to a series of questions regarding investors’ expectations of returns over different time horizons, and their beliefs about other investors’ beliefs. In particular, the survey asks

(i) How much of a change in percentage terms do you expect [for the Dow Jones index] in the following 1 month? 3 months? 6 months? 1 year? 10 years?

(ii) Although I expect a substantial drop in stock prices in the U.S. ultimately, I advise being relatively heavily invested in stocks for the time being because I think that prices are likely to rise for a while (True, False, No opinion; if True, indicate best guess for date of peak).

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7Other papers that have used the Shiller / Yale ICF survey data to study investor return expectations include Bacchetta, Mertens and van Wincoop (2009), who find that U.S. institutional investors’ subjective expected excess returns are acylical and Greenwood and Shleifer (2014), who find that the Shiller monthly investor confidence index is correlated with other investor surveys and to mutual fund flows.
Although I expect a substantial rise in stock prices in the U.S. ultimately, I advise being less invested in stocks for the time being because I think that prices are likely to drop for a while (True, False, No opinion; if True, indicate best guess for date of bottom).

Many people are showing a great deal of excitement and optimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them.

Many people are showing a great deal of pessimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them.

Answers to question (i) provide direct evidence of investors’ return expectations over different horizons. Answers to questions (ii) and (iii) provide explicit evidence of a distinctive elements of hump-shaped return expectations, differently signed expectations of returns over different future horizons. Answers to questions (iv) and (v) provide direct evidence on investors’ higher order beliefs.

FX4casts

FX4casts provides the average forecast of exchange rates and interest rates from a number of large financial institutions that actively participate in foreign exchange markets across the world. Our sample begins in August 1986 and ends in December 2019, and contains monthly observations. The data on exchange rate forecasts include 3-, 6-, and 12-month ahead forecasts of spot exchange rates for 32 currencies over the full sample. The dataset also contains forecasts of 1- and 24-month ahead spot exchange rates from September 2008 onwards. As is common in the literature, we focus our study on developed market G11 currencies.

We use the survey data to construct expected returns for a given foreign currency from period \( t \) to \( t + h \) as \( E_{S,t}(r_{t,t+h}) = E_{S,t}(s_{t+h}) - s_t + i_t^d \), where \( s_t \) is the log exchange rate in USD per unit of foreign currency, \( i_t^d \) is the forward implied interest rate differential (foreign minus U.S. interest rate), and \( E_{S,t}(\cdot) \) captures subjective expectations as reported in survey data.

Consensus Economics

In our quantitative application in currency markets (presented in Section 3), we also use data on interest rate forecasts for all countries in our sample from Consensus Economics. For each month, the dataset provides forecasts of interest rates (and other macroeconomic quantities) from a number of large financial institutions across the world, with a large overlap in respondents with the FX4casts survey. The sample begins in October 1989.\(^8\)

\(^8\)Consensus Economics also separately provides exchange rate forecasts, which are not part of our data sample. Other papers have found a strong relationship between exchange rate expectations reported across the surveys (e.g., see Candia and De Leo (2021)). This is to be expected, as there is a substantial overlap between the institutions reporting their forecasted beliefs about exchange rates in the Consensus Economics and FX4casts samples.
The data on interest rate forecasts include forecaster level data for forecasts of the short-term interest rate three months from the forecast date and twelve months from the forecast date. We form consensus forecasts of interest rate differentials by taking the average forecast for each country at each point in time and subtracting the average forecast of the US interest rate in the same period.

1.2 Empirical Evidence

In this section, we analyze how investors’ return expectations behave in response to past news. In both equity and currency markets, and across surveys, we find that investors report pronounced hump-shaped cumulative return expectations in response to fundamental news, expecting positive returns in the quarter immediately following positive news, but expecting negative returns in subsequent quarters. In Appendix D.1, we also report details on investors’ unconditional return expectations, and find that the hump-shaped pattern of returns also appears unconditionally.

1.2.1 Return Expectations in Response to News

To analyze the response of expectations to news, we run regressions of return expectations measured using survey data on past news. Regressions are of the form

\[ E_{S,t}(r_{t+h,t+h+k}) = \alpha_h + \beta_h \text{News}_t + \epsilon_{t,h}, \]

where \( E_{S,t}(r_{t+h,t+h+k}) \) are expectations reported in survey data in period \( t \) for excess returns from period \( t+h \) to period \( t+h+k \), \( \text{News}_t \) is a positively-signed measure of news that arrives in period \( t \), and \( \beta_h \) are the coefficients of interest. We measure news using measures of innovations to fundamentals (quarterly innovations to GDP growth for the U.S. equity market and quarterly innovations to interest rate differentials for currencies), and using 90-day trailing returns. We multiply return expectations by 100 and scale the independent variables in the regressions to have zero mean and unit standard deviation. This means that the coefficients in the regressions can be interpreted as expected returns in percentage points associated with a one standard deviation fundamental innovation.

Figure 1 illustrates the hump-shaped pattern of cumulative return expectations that we find in our regression analysis. The figure plots coefficients from regressions of expectations of \( k \)-month ahead returns on news shocks, where news shocks are measured as quarterly interest rate differential innovations in currency markets and as 90-day trailing returns for the U.S. equity market. We use consensus return expectations in currency markets, and we run a pooled regression including observations across the Shiller/Yale ICF individual and institutional investor surveys for the U.S. equity market.

Following a shock, in both equity and currency markets, survey respondents report initially positive returns, which they expect to revert in subsequent months. In equity markets,
FIGURE 1: HUMP-SHAPED CUMULATIVE RETURN EXPECTATIONS IN RESPONSE TO NEWS

Note: The figure plots coefficients from regressions of the form $E_{S,t}(r_{t,t+h}) = \alpha_h + \beta_h\text{News}_t + \epsilon_{t,h}$, where $E_{S,t}(r_{t,t+h})$ is the expected return from period $t$ to $t + h$, as reported in survey data, News$_t$ is a measure of news in period $t$, and $\beta_h$ is the coefficient of interest. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation news shock. News shocks are measured as quarterly interest rate differential innovations for currency markets, and 90-day trailing equity returns for the U.S. equity market. Standard errors are HAC-Panel standard errors in currency market regressions and are clustered by quarter for the U.S. equity market regressions. The figure reports plus and minus one standard deviation for the estimated coefficients.

The coefficient for 1-month return expectations is 0.31, indicating an expected return of 31 basis points in the month following a one standard deviation 90-day trailing return. However, the coefficient on 12-month return expectations is -0.75, indicating that investors expect the initial positive returns to completely reverse, and for the market to eventually drop by 75 basis points. In currency markets, the regression coefficient for 3-month return expectations is 0.63, indicating an expected 63 basis point higher return corresponding with a one standard deviation innovation to interest rate differentials. However, the coefficient for 12-month return expectations is 0.10, indicating that survey respondents expect nearly the entirety of the initial positive returns to reverse after the first quarter.

We next dive more deeply into the details of the regressions. Table 1 reports regression results for the U.S. equity market where news is measured using innovations to GDP growth. For individual investors, regression coefficients for 1- and 3-month returns are 0.62 and 0.55, indicating that a one standard deviation innovation to GDP growth corresponds with 62 and 55 basis point higher return expectations for the following month and quarter. The regression coefficients for 3- to 6- and 6- to 12-month ahead returns are -26 and -53 basis points, indicating hump-shaped return expectations: individual investors expect reversals

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9We do not report regression coefficients for 1-month return expectations for currency markets due to the mismatched, shorter sample of data available for 1-month expectations.
Individual Investors | 0.62 | 0.55 | -0.26 | -0.53
(0.17) | (0.13) | (0.14) | (0.40)
Institutional Investors | 0.06 | 0.00 | -0.34 | -0.83
(0.08) | (0.13) | (0.17) | (0.35)
Pooled | 0.17 | 0.09 | -0.28 | -0.75
(0.06) | (0.10) | (0.17) | (0.34)

| $E_{S,t}(r_{t,t+1})$ | $E_{S,t}(r_{t,t+3})$ | $E_{S,t}(r_{t+3,t+6})$ | $E_{S,t}(r_{t+6,t+12})$

**Table 1: Shiller Expected Returns in Response GDP News**

*Note:* The table reports results from regressions of survey respondents’ return expectations over different horizons on quarterly GDP innovations. $E_{S,t}(r_{t+h,t+h+k})$ represents a survey respondent’s $k$-month return expectation for $h$ months in the future. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation innovation to quarterly GDP Growth. Standard errors are clustered by quarter and reported in parentheses.

of the positive short horizon returns that they forecast. For institutional investors, the corresponding coefficients are 6, 0, -34, and -83 basis points, indicating a weaker hump-shape, with very strong negative returns forecasted after a quarter. The coefficients are 17, 9, -28, and -75 basis points when pooling together observations across individual and institutional investors. The results are consistent with a hump-shape in investors’ return expectations, with some heterogeneity in the exact form of return expectations for individual and institutional investors.

Table 2 reports regression results for the U.S. equity market where news is measured using 90-day trailing returns. We observe an even more pronounced hump-shape in investors’ return expectations. For individual investors, regression coefficients for 1- and 3-month returns are 96 and 60 basis points, while regression coefficients for 3- to 6- and 6- to 12-month ahead returns are -26 and -39 basis points. For institutional investors, the corresponding coefficients are 21, 4, -39, and -54 basis points, while the coefficients are 31, 14, -36, and -51 basis points when pooling together observations across individual and institutional investors. Once again, the results are consistent with a pronounced hump shape in investors’ cumulative return expectations, with individual investors expecting more positive short horizon returns following past news and institutional investors reporting more pessimistic return expectations.\(^{10}\)

\(^{10}\)In Appendix Table D.1, we find that institutional investors report even more pronounced hump-shaped returns in response to trailing 30-day returns, with a coefficient of 36 basis points ($t$-statistic of 2.9) for their 1-month return expectations and a coefficient of -38 basis points ($t$-statistic of -2.3) for their 6- to 12-month return expectations. The evidence broadly suggests that institutional investors exhibit hump-shaped return expectations, but, in comparison with individual investors, they believe that the persistence of returns following positive past news is
Table 2: Shiller/Yale ICF Expected Returns in Response to 90-Day Trailing Returns

Note: The table reports results from regressions of survey respondents’ return expectations over different horizons on trailing three month returns. \( \mathbb{E}_{S,t} (r_{t,t+1}) \) represents a survey respondent’s \( k \)-month return expectation for \( h \) months in the future. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation trailing 90-day return. Standard errors are clustered by quarter and reported in parentheses.

Table 3 reports regression results for currencies. The first row in the table reports results from regressions where the independent variable is AR(1) innovations to interest rate differentials. The coefficients for 3-month expected returns is 0.63, indicating that a one standard deviation shock to interest rate differentials leads survey respondents to forecast a 63 basis point return in the following quarter. The coefficients for 3- to 6- and 6- to 12-month expected returns are -2 and -51 basis points, indicating that survey respondents forecast the returns they expect in the next quarter to nearly fully revert over the subsequent three quarters. The regression results provide strong evidence that, on average, forecasters expect positive past fundamental news, in the form of higher interest rate differentials, to correspond with hump-shaped expected returns for a currency. The second row of the table reports regression results where the independent variable is a currency’s trailing 3-month return. The coefficients for 3-month, 3- to 6-month, and 6- to 12-month ahead expected returns are 39, 14, and -13 basis points. The results using trailing returns as a measure of news also suggest the presence of hump-shaped returns in response to past currency excess returns, though weaker than when using interest rate differentials to measure the arrival of news.

1.2.2 Additional Evidence on Return Expectations

As noted previously, we analyze investors’ unconditional return expectations reported in survey data in more detail in Appendix Section D.1. When not conditioning on past news, shorter and that the market will reverse course more strongly.
we also find evidence of hump-shaped return expectations, with investors reporting differently signed 1-month and 1-quarter ahead return expectations versus return expectations for subsequent quarters.

Related to but distinct from the hump-shaped pattern of cumulative return expectations, the analysis also reveals that investors often report cumulative return expectations that are differently signed than their 1-quarter ahead return expectations. That is, if investors expect a positive short horizon return, they often expect that positive return to completely reverse, and for the 12-month return for holding the stock market or a particular currency to become negative. This pattern is particularly pronounced in equity markets, where about 50% of survey respondents report differently signed 1-month return expectations and 12-month cumulative return expectations. Consistent with this evidence, more than 60% of respondents in the Shiller / Yale ICF survey advise holding over- or under-weight positions in the stock market that are opposite their beliefs about whether the stock market will eventually drop or rise from its current level.

### 1.3 Explaining the Evidence

We next turn to discussing potential explanations for the patterns in investors’ return expectations. We first present evidence suggesting that higher order beliefs play an important role in explaining the facts. We then briefly discuss alternative theories, but defer a more detailed discussion of them to Section 4.
1.3.1 Higher Order Beliefs

To understand how higher order beliefs may explain the hump-shaped pattern of return expectations, we analyze responses to questions (iv) and (v) from the Shiller surveys, regarding whether investors believe others to be overly optimistic or pessimistic about the stock market’s prospects. In doing so, we find a strong relationship between investors’ reported higher order beliefs and patterns in their return expectations.

We construct monthly Higher Order Optimism indices, which capture higher order beliefs about other investors’ optimism about the stock market. We construct one index for individual investors, one for institutional investors, and one pooling together responses across the two investor types. The indices are constructed as the proportion of respondents in a given month responding yes to question (iv), that they believe others to be overly optimistic, minus the proportion of respondents in a given month answering yes to question (v), that they believe others to be overly pessimistic. We construct the indices from 1999 onwards, the period over which we have monthly observations.

Figure 2 plots the monthly Higher Order Optimism index over time, pooling together individual and institutional survey responses. The index displays interesting and intuitive variation over time. The index exhibits peaks in 1999 to 2000 at the height of the ‘dot-com bubble,’ in 2014 following prolonged low interest rates, in 2017 following the election of Donald Trump, and towards the end of 2020 following government stimulus and news regarding vaccine authorization. The index exhibits troughs at the start of the Iraq war in 2003, during the great financial crisis, and during the U.S. debt ceiling and European debt crises in 2011.11 The individual and institutional investor Higher Order Optimism indices are highly correlated and exhibit largely similar dynamics, and are presented in Appendix Figure D.1. As we discuss in more detail in Appendix Section D.2, the Higher Order Optimism indices tend to be high following periods of strong stock market performance and positive macroeconomic news.

We next analyze the relationship between investors’ return expectations and their higher order beliefs. We regress the average 1-month, 3-month, 3- to 6-month, and 6- to 12-month ahead return expectations in a given month (multiplied by 100) on the Higher Order Optimism index for that month, which we standardize to have zero mean and unit standard deviation. We run regressions separately for the individual and institutional investor samples, as well as pooling the observations together.

Table 4 reports results from the regressions. For each of the samples, coefficients are positive for 1-month ahead return expectations, and negative for 3- to 6-month and 6- to 12-month ahead return expectations. The evidence indicates that when investors’ higher order beliefs are that other investors are overly optimistic about the stock market, they believe (as is to be expected) that long-term market returns will be lower than average. However, they also believe that shorter-term market returns will be higher than average. For indi-

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11These events are all mentioned in responses in the Shiller surveys where investors are given the opportunity to expound on the drivers of recent market performance and market valuations.
Note: The figure plots the Higher Order Optimism index, which, in a given month, is measured as the difference between the proportion of investors in the Shiller surveys reporting a belief that other investors are overly optimistic about the stock market’s prospects and the proportion of investors reporting a belief that other investors are overly pessimistic about the stock market’s prospects. The index is constructed using survey responses from both individual and institutional investors.

Individual investors, the regression coefficients for 1-month, 3-month, 3- to 6-month, and 6- to 12-month ahead return expectations are 58 bps, 13 bps, -37 bps, and -58 bps corresponding with a one standard deviation change in the Higher Order Optimism index. For institutional investors, the corresponding coefficients are 14 bps, -15 bps, -47 bps, and -56 bps, and in the pooled sample, the coefficients are 57 bps, 19 bps, -40 bps, and -73 bps. The regression evidence is broadly consistent across individual and institutional investors, with individual investors believing that other investors’ optimism corresponds with stronger short-term future returns than institutional investors.

Finally, we also compute correlations of the Higher Order Optimism index with (a) the proportion of investors that answer yes to question (iii) each month, that they expect stock markets to eventually drop substantially but advise being heavily invested because of expectations of positive short term return expectations; and with (b) the proportion of investors that answer yes to question (iv) each month, that they expect stock markets to eventually rise, but advise being less invested due to an expectation that prices will drop in the short run. The correlation of the Higher Order Optimism index with (a) is 0.39 (standard error of 0.10), and its correlation with (b) is -0.41 (standard error of 0.10). That is, when investors believe that other investors are overly optimistic, they advise taking overweight positions in the market despite believing the market will eventually decline substantially, and they similarly advise taking underweight positions in the market when they believe others are overly pessimistic, despite a belief that markets will eventually rise substantially. The evidence is consistent with investors’ higher order beliefs playing a role in their
Individual Investors 0.58 0.13 -0.37 -0.58  
(0.23) (0.26) (0.16) (0.18)

Institutional Investors 0.14 -0.15 -0.47 -0.56  
(0.07) (0.15) (0.16) (0.20)

Pooled 0.57 0.19 -0.40 -0.73  
(0.11) (0.13) (0.15) (0.20)

TABLE 4: HIGHER ORDER OPTIMISM AND RETURN EXPECTATIONS

Note: The table reports regression results from regressions of consensus return expectations over different horizons on the Higher Order Optimism indices. $E_{S,t}(r_{t,t+1})$ represents the consensus $k$-month return expectation for $h$ months in the future. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation change in the independent variable. The table reports the average coefficient across countries. Newey-West standard errors are reported in parentheses.

hump-shaped return expectations, and also with investors exhibiting destabilizing speculative trading motives, seeking to trade in the direction of other investors’ beliefs that they believe to be incorrect. We return to this point in our theoretical analysis.

Taken together, the evidence from the Higher Order Optimism indices suggests that investors’ higher order beliefs may play an important role in the hump-shaped pattern in their return expectations. In particular, the evidence is consistent with the idea that when investors believe that other investors’ are overly optimistic, they believe that market returns will eventually decline (around one year in the future), but they also believe that others’ optimism corresponds with positive short-horizon returns.

1.3.2 Alternative Explanations

We defer a more detailed discussion of alternative theories to explain the evidence on return expectations to Section 4. Here, we quickly note that the empirical evidence is not consistent with the predictions of standard models of time-varying risk premia. To rationalize the facts with time-varying risk premia, subjective risk premia would have to be high following good news, would have to vary much faster than business cycle frequency, behavior that is not consistent with standard models. The evidence is also not explained by prevailing behavioral theories that focus on investors making systematic errors in forecasting cash flows. These theories do not make predictions on investors’ subjective return expectations without introducing additional assumptions.
1.4 Return Expectations and Investor Behavior

An important question is whether the measures of survey-based return expectations that we use actually correspond with investor behavior. Previous work does suggest a connection, more broadly, between survey data on return expectations and investor behavior in equity and global currency markets. For example, Greenwood and Shleifer (2014) provide evidence on the relationship between fund flows and survey-based measures of equity return expectations, and Stavrakeva and Tang (2020) provide evidence on the relationship between survey-based measures of currency return expectations and investors’ futures positioning. In Appendix Section D.3, we study the relationship between return expectations and investor behavior, measuring investor behavior using mutual fund flows for the U.S. equity market, and futures positions for both equity and currency markets. Consistent with the prior literature, we find a strong relationship between investor behavior and return expectations.

Moreover, we also find that short horizon (1-month to 1-quarter) return expectations are especially important drivers of investors’ trading behavior, versus longer horizon return expectations. For example, even when survey data indicate expectations that returns will be positive for the next quarter but will revert in immediately subsequent quarters, investors have strong positive asset demand, as captured by flows and futures positioning data. Such behavior provides support for an assumption of myopic trading behavior that we make in our model, and is also useful for understanding investors’ trading motivations, as we discuss in more detail.

2 Model

We present a theoretical asset pricing model, and show that the hump-shaped pattern of cumulative return expectations found in survey data can be matched by introducing two assumptions regarding investors’ higher order beliefs: (1) a belief that other investors will persistently revise their valuations in the same direction of past news and (2) a belief that others’ beliefs about fundamentals eventually overreact to news. The model matches the survey evidence on investors’ higher order beliefs, and also helps to illustrate how investors’ higher order beliefs may influence investor behavior and equilibrium asset prices. Importantly, the model serves as a tractable base for our quantitative study of higher order beliefs in currency markets in Section 3.

2.1 Model Setup

We begin by laying out the model environment, and describing how investors in the model form their fundamental beliefs about cash flows and their higher order beliefs about other investors’ beliefs. Investors’ fundamental and higher order beliefs give rise to a perceived pricing rule that investors use in forming their return expectations. These return
expectations, in turn, determine investors’ risky asset demand. We then proceed to detailing equilibrium in the model, which is the market clearing outcome of investors’ demand and “residual demand” for the risky asset, where the latter is demand not related to surveyed investors’ beliefs. We let the data speak to residual demand, which we know little about ex-ante, in our quantitative exercise in Section 3.

2.1.1 Model Environment

Time is discrete and indexed by $t = \{1, 2, 3, \ldots \}$. There is a riskless asset in perfectly elastic supply and a risky asset in zero net supply. The riskless asset pays an exogenous per period interest rate normalized to zero, and the risky asset pays a dividend of $D_t$ in period $t$. $D_t$ evolves according to the process

\begin{equation}
D_t = d_t + v_t, \text{ where } v_t \sim N(0, \sigma_v^2),
\end{equation}

\begin{equation}
d_t = \rho d_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma^2_\epsilon).
\end{equation}

The term $d_t$ captures the persistent component of dividends, which we refer to as the asset’s fundamentals, while the term $v_t$ captures a transitory component of dividends. While dividends are observed each period, the underlying fundamentals are never revealed to investors.

The model follows an overlapping generations structure. Each period, a unit mass of identical investors is born. Investors born in period $t$ make an investment decision in that period. In period $t + 1$ they liquidate their investments, consume the proceeds, and pass their beliefs onto the new generation of investors. The assumption of overlapping generations leads investors to maximize their short horizon return expectations in their portfolio decisions. This myopic behavior is consistent with the empirical evidence we discuss in Section 1.4 and Appendix Section D.3. All investors have identical exponential utility, with a constant absolute risk aversion coefficient of $\gamma$.

We allow investors in the model to hold potentially misspecified fundamental and higher order beliefs. We do not take a stance on the form of this misspecification, but allow the survey data to speak to it. In our quantitative analysis in Section 3, we find that allowing for misspecification is important for matching survey data on investors’ expectations.

The equilibrium price of the risky asset is the market clearing outcome of investors’ risky asset demand and residual demand for the risky asset. Residual demand can be thought of as demand in the market not related to survey-based measures of beliefs. The market clearing condition is given by

\begin{equation}
0 = X_t + \psi_t,
\end{equation}

where $X_t$ is investors’ demand in period $t$, and $\psi_t$ is residual demand in period $t$. We next proceed to specifying details of investors’ beliefs, which determine their demand.
2.1.2 Assumptions about Investors’ Beliefs

We begin by making three assumptions about investors’ beliefs. Together, these assumptions form the core of how investors form expectations of future dividends and returns in the model.

**Assumption 1 (Fundamental Beliefs).** Investors perceive the parameters \((\rho, \sigma_v)\), the persistence of fundamentals and the transitory noise reflected in dividends each period, as \((\rho_S, \sigma_{S,v})\).

Assumption 1 captures the fact that investors have subjective beliefs about the parameters of the fundamentals process that may deviate from the true parameters governing the fundamentals process.\(^{12}\)

Investors in the model are identical, but they perceive belief disagreements with others. To make notationally clear that investors perceive disagreements with other investors ("the market"), we denote the market’s first order beliefs about fundamental persistence and transitory noise as \((\rho_M, \sigma_{M,v})\). By construction, \((\rho_M, \sigma_{M,v}) = (\rho_S, \sigma_{S,v})\).

Investors do not recognize that they share fundamental belief parameters with the market. Investors each know their own belief parameters to be \((\rho_S, \sigma_{S,v})\). However, their second order beliefs about the market’s belief parameters are denoted by \((\rho_{SM}, \sigma_{SM,v})\).

**Assumption 2 (Second Order Beliefs About Fundamental Beliefs).** Investors believe that all other investors perceive the parameters \((\rho, \sigma_v)\) as \((\rho_{SM}, \sigma_{SM,v})\).

**Assumption 3 (Additional Higher Order Beliefs).** Investors believe that other investors act as fundamental traders. That is, they believe that other investors forecast that all investors born in period \(t + 1\) will perceive the parameters \((\rho, \sigma_v)\) as \((\rho_{SM}, \sigma_{SM,v})\), and will see this perception as common knowledge.

Together, Assumptions 2 and 3 characterize investors’ higher order beliefs in the model. Assumption 2 indicates that investors believe that the market has different beliefs about the fundamentals process than they do. Assumption 3 captures that investors effectively represent the market as a fundamental trader, because they believe that other investors perceive that all future investors will share their beliefs.\(^{13}\)

We make an additional assumption regarding how investors reason about residual demand in the model. While this assumption is not essential to our results, it is helpful for illustrating them in an analytically clear way.

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\(^{12}\)We assume that investors hold fixed belief parameters that they do not update over time primarily for simplicity, but also because survey data indicate that investors make systematic mistakes in forecasting cash flows, as discussed in our literature review. Even when investors are able to learn about model parameters over time, realistic models of learning may still result in investors exhibiting substantial deviations from the full information rational expectations benchmark (e.g., see Johannes, Lochstoer and Mou (2016)).

\(^{13}\)Assumption 3 allows us to avoid the ‘infinite regress’ problem, where investors may want to forecast others’ forecasts of others’ forecasts of others’ forecasts, and so on and so forth (Townsend (1983)). In Section 4, we discuss how to interpret our results given that investors may engage in such higher level reasoning.
Assumption 4 (Beliefs about Residual Demand). Investors believe that residual demand follows a random walk, i.e., they believe that \( \psi_t \sim N(\psi_{t-1}, \sigma^2_{\psi}) \). Investors believe that other investors perceive residual demand as independent and identically distributed random noise each period, i.e., they believe that other investors perceive that \( \psi_t \sim N(0, \sigma^2_{\psi}) \).

With these assumptions in hand, we next turn to describing investors’ fundamental and return expectations.

### 2.1.3 Investors’ Fundamental Beliefs

Investors are Bayesian in forming their beliefs about fundamentals, \( d_t \). Using their beliefs about the parameters governing the dividend process and their observations of past dividends, they form their expectations of \( d_t \) by Kalman filtering. For ease of notation, investors’ beliefs about fundamentals in period \( t \) after observing the period \( t \) dividend, \( D_t \), are denoted as

\[
d^S_t \equiv \mathbb{E}_{S,t}(d_t),
\]

where \( \mathbb{E}_{S,t} \) is the subjective expectations operator. We follow the common assumption that a sufficient number of periods have passed such that investors are in a learning steady state. This means that investors’ Kalman gain – the weight they place on the observed dividend versus their prior when forming their beliefs about fundamentals – is constant each period. Lemma 1 outlines the process by which investors’ fundamental beliefs evolve.

**Lemma 1 (Fundamental Beliefs).** Assume that a sufficient number of periods have passed, such that investors are in a learning steady state. Then, investors’ beliefs about fundamentals \( d_t \) evolve according to the updating process

\[
d^S_t = (1 - \kappa_S) \rho_S d^S_{t-1} + \kappa_S D_t,
\]

where

\[
\kappa_S = \frac{\rho_S^2 \Sigma + \sigma^2_e}{\rho_S^2 \Sigma + \sigma^2_e + \sigma^2_{S,v}}, \quad \text{and} \quad \Sigma_S = (1 - \kappa_S) (\rho_S^2 \Sigma_S + \sigma^2_e).
\]

**Proof.** All proofs are provided in Appendix A.

The subjective Kalman gain, \( \kappa_S \), depends on investors’ perception of the relative importance of transitory noise versus persistent shocks in the dividend process, \( \sigma^2_{S,v}/\sigma^2_e \). If investors believe there is a substantial amount of transitory noise in dividends relative to fundamentals (e.g., larger \( \sigma^2_{S,v} \)), they put less weight on the most recently observed dividend and more weight on their prior in their updated belief each period. This updating behavior reflects the belief that dividends are a noisy signal of underlying fundamentals.\(^{14}\)

\(^{14}\)Introducing a higher \( \sigma^2_{S,v} \), which we do in our applications of the model, induces sluggishness in the belief-
We assume that all investors know and agree on the importance of persistent shocks to fundamentals, as captured by $\sigma^2$. Accordingly, a higher perceived amount of transitory noise is equivalent to a lower Kalman gain.

2.1.4 Investors’ Higher Order Beliefs and Return Expectations

Investors forecast asset returns by forecasting the beliefs of the market (i.e., all other investors). All investors have identical beliefs about fundamentals in period $t$, which we denote as $d^S_t$. However, because investors believe that others have different beliefs about the fundamentals process, they also mistakenly perceive a belief disagreement with the market.

To make clear that investors perceive a belief disagreement with other investors, we denote the market’s belief about fundamentals in period $t$ as $d^M_t$. This is always equal to $d^S_t$. However, investors’ second order belief about the market’s fundamental belief in period $t$ is $d^{SM}_t$. With this notation, we can write the perceived form that investors believe the equilibrium pricing rule takes.

**Lemma 2 (Perceived Form of the Pricing Rule).** Investors perceive that the form of the equilibrium pricing rule for the risky asset is

$$P_t = \frac{\rho_M}{1 - \rho_M} d^M_t + e \psi_t$$

where

$$\kappa_M$$ is the market’s Kalman gain,

and

$$e = \frac{1 - \sqrt{1 - 4 \left( 1 + \frac{\kappa_M \rho_M}{1 - \rho_M} \right)^2 \gamma^2 \sigma^2 \sigma^2_M + \sigma^2_e}}{2 \gamma \sigma^2_M}$$

Investors perceive the equilibrium pricing function as consisting of two components: the market’s fundamental valuation of the risky asset and residual demand.

Lemma 2 outlines the form that investors believe the pricing rule takes. However, when analyzing current prices and forecasting future prices, investors input their own second order beliefs into the pricing function.

**Proposition 1 (Perceived Prices).** Investors perceive the equilibrium price in period $t$ as

$$P_t = \frac{\rho_{SM}}{1 - \rho_{SM}} d^{SM}_t + e_S \psi^S_t,$$

where $e_S$ is investors’ subjective perception of the coefficient $e$, computed using $(\kappa_{SM}, \rho_{SM}, \sigma_{SM,e})$ in place of $(\kappa_M, \rho_M, \sigma_{M,e})$, and $\psi^S_t$ is investors’ subjective perception of residual demand.

The updating process of investors by lowering the Kalman gain $\kappa_S$. This is similar to models assuming investors have sticky expectations, e.g., Mankiw and Reis (2002), and subsequent literature that builds on their approach. Bouchaud et al. (2019) study the asset pricing implications of sticky expectations.
Proposition 1 captures investors’ perception of price formation. In particular, their perception of prices reflects their (incorrect) second order beliefs of the market’s fundamental beliefs and belief updating parameters.

The proposition also highlights that when observing the risky asset price, investors incorrectly infer that residual demand is the driver of the wedge between the asset price they observe and the price they expect given their higher order beliefs. That is, investors rationalize the price they observe with a subjective belief that residual demand is given by

$$\psi_t^S = \frac{P_t - \frac{\rho_{SM} d_t^S}{1 - \rho_{SM}}}{\epsilon_S}. \quad (9)$$

Investors do not learn that their higher order beliefs are incorrect because they always attribute their pricing mistakes to residual demand.\(^{15}\)

The cumulative return of the risky asset from period \(t\) to period \(t + h\), \(R_{t,t+h}\), is defined as the sum of the price change of the asset and all of the intermediate dividends of the asset:

$$R_{t,t+h} = P_{t+h} - P_t + \sum_{k=1}^{h} D_{t+k}.$$

Investors forecast returns by forecasting the dividends and future price of the asset. Investors’ forecasts of dividends in period \(t + k\) are given by \(\mathbb{E}_t(D_{t+k}) = \rho_S^k d_t^S\). They use the perceived pricing expression in Proposition 1 to forecast the future price of the risky asset. To do so, they also forecast what the market’s belief about fundamentals will be in period \(t + h\), using their higher order belief parameters \(\kappa_{SM}\) and \(\rho_{SM}\).

**Lemma 3 (Forecasts of the Market’s Fundamental Beliefs).** In period \(t\), investors forecast the market’s belief about fundamentals in period \(t + h\) as

$$\mathbb{E}_{S,t}(d_{t+h}^M) = (1 - \kappa_{SM}) \rho_{SM}^h d_t^S + \kappa_{SM} \frac{\rho_S ((1 - \kappa_{SM})^h \rho_{SM}^h - \rho_S^h)}{(1 - \kappa_{SM}) \rho_{SM} - \rho_S} d_t^S. \quad (10)$$

Lemma 3 can be understood as the result of investors’ perceptions of the market’s Bayesian updating. In period \(t + h\), a portion of the market’s belief reflects its period \(t\) forecast of fundamentals in period \(t + h\). But the market also updates its beliefs about fundamentals in response to observed dividends. Investors believe these dividends will evolve according to their beliefs about fundamentals, \(d_t^S\), and about persistence, \(\rho_S\). Using Lemma 3, we can derive investors’ return expectations from period \(t\) to \(t + h\).

**Proposition 2 (Investors’ Return Expectations).** Investors’ cumulative expected return for the
risky asset from period $t$ to period $t + h$ is given by

$$\mathbb{E}_{S,t}(R_{t,t+h}) = \mathbb{E}_{S,t} \left( P_{t+h} - P_t + \sum_{k=1}^{h} D_{t+k} \right)$$

$$= \frac{\rho_{SM}}{1 - \rho_{SM}} \left( \mathbb{E}_{S,t}(d_{t+h}^M) - d_t^S \right) + \frac{\rho_S (1 - \rho_{SM})^h}{1 - \rho_S} d_t^S, \quad (11)$$

where $\mathbb{E}_{S,t}(d_{t+h}^M)$ is defined as in Lemma 3.

Together, Lemma 3 and Proposition 2 illustrate how our model connects investors’ cash flow and return expectations. In particular, investors’ return expectations are determined by a combination of the cash flows they expect to observe in the future, and how they expect other investors to update their beliefs in response to those cash flows. We discuss return expectations, including what parameter values deliver the empirical evidence, in more detail after solving the rest of the model.

### 2.1.5 Investors’ Risky Asset Demand and Equilibrium Asset Prices

Investors use their return expectations in order to formulate their risky asset demand. Given their exponential utility, investors’ risky asset demand in period $t$ is given by

$$X_t = \frac{\mathbb{E}_{S,t}(R_{t,t+1})}{\gamma \mathbb{V}_{S,t}(R_{t,t+1})}, \quad (12)$$

where $\mathbb{E}_{S,t}(R_{t,t+1})$ and $\mathbb{V}_{S,t}(R_{t,t+1})$ can be derived as

$$\mathbb{E}_{S,t}(R_{t,t+1}) = \frac{(1 - (1 - \kappa_{SM}) \rho_{SM})(\rho_S d_t^S - \rho_{SM} d_t^{SM})}{1 - \rho_{SM}}, \quad (13)$$

$$\mathbb{V}_{S,t}(R_{t,t+1}) = \left( 1 + \frac{\kappa_{SM} \rho_{SM}}{1 - \rho_{SM}} \right)^2 (\sigma_e^2 + \sigma_{SM,v}^2) + \sigma_S^2 \sigma_\phi^2. \quad (14)$$

To close the model, we provide some minimal structure to residual demand. We represent residual demand as a mass $\theta$ of traders with exponential utility and constant absolute risk aversion of $\gamma$. Residual traders are fundamental traders who trade as if they will be the only investors in the market in period $t + 1$. Their valuation of the risky asset in period $t$ is denoted as $\phi_t$, and they perceive the variance of the risky asset’s returns as $\sigma_\phi^2$. We can then write residual demand as

$$\psi_t = \theta \frac{\phi_t - P_t}{\gamma \sigma_\phi^2}. \quad (15)$$

Imposing market clearing ($X_t + \psi_t = 0$) allows us to derive the market clearing price.
Proposition 3 (Equilibrium Asset Prices). The market clearing price of the risky asset is

$$P_t = B_d S_t + B_M d_{SM} + \phi_t,$$

where the coefficients $B(\cdot)$ and $B_M(\cdot)$ are functions of belief, higher order belief, and residual demand parameters.

Proposition 3 indicates that the prevailing equilibrium asset price is a linear function of three variables: investors’ fundamental beliefs, $d_S$, investors’ higher order beliefs about fundamentals, $d_{SM}$, and residual demand. Note that unlike investors’ perceived pricing function, higher order beliefs also enter into the true equilibrium price function. This is because each investor’s trading behavior is driven by their higher order beliefs, even though they incorrectly perceive other investors to be fundamental traders.

2.2 Connecting the Model with the Empirical Evidence

We next connect our model with the empirical evidence, and illustrate the role that higher order beliefs play in giving rise to the patterns we observe. We first analyze what types of higher order beliefs rationalize patterns in surveyed investors’ return expectations. We next turn to understanding how investors’ higher order beliefs influence the informational efficiency of asset prices.

2.2.1 Investors’ Return Expectations and Trading Motivations

We first present a simple result that illustrates the role of fundamental and higher order beliefs in investors’ return expectations.

Proposition 4 (Fundamental Belief Disagreements and Long Horizon Returns). At long horizons, investors’ return expectations converge to the difference in their valuation of the asset versus their perception of the market’s valuation:

$$\lim_{h \to \infty} E_{S,t}(R_{t,t+h}) \to \frac{\rho_s}{1-\rho_s} d^S_t - \frac{\rho_{SM}}{1-\rho_{SM}} d^{SM}_t.$$

Proposition 4 illustrates that investors’ long horizon return expectations reflect their beliefs about fundamentals, and how they perceive these fundamental beliefs to differ from the market’s beliefs. Perceived belief disagreements determine the level and sign of investors’ return expectations: the larger the perceived disagreement about valuations, the larger the magnitude of expected returns. Ultimately, regardless of the exact configuration of investors’ fundamental belief and higher order belief parameters, they expect cumulative returns in the long run to be $\frac{\rho_s}{1-\rho_s} d^S_t - \frac{\rho_{SM}}{1-\rho_{SM}} d^{SM}_t$. This term reflects the difference in the discounted sum of dividends investors expect versus the discounted sum of dividends they perceive the market expects. We refer to the quantity $\frac{\rho_s}{1-\rho_s} d^S_t - \frac{\rho_{SM}}{1-\rho_{SM}} d^{SM}_t$ as a valuation wedge.

The empirical evidence that we present is about the term structure of investors’ return expectations, not just about the level of their return expectations. In particular, the evidence
indicates that investors often report that they expect returns at intermediate horizons to be differently signed than short horizon returns, a pattern that is particularly pronounced following past news. We make two assumptions that deliver this hump-shaped pattern in return expectations in response to news.

**Assumption 5.**  Investors believe the market overestimates the persistence of fundamentals \( \rho_{SM} > \rho_S \).

**Assumption 6.**  Investors perceive the market’s Kalman gain as satisfying \( \kappa_{SM} < \frac{\rho_{S}\kappa_S}{\rho_{SM}} \).

Assumption 5 leads to hump-shaped return expectations when investors perceive belief disagreements with the market, as we outline in Lemma 4. Assumption 6 endogenously gives rise to perceived belief disagreements in response to news that drive hump-shaped return expectations, as outlined in Proposition 5. We formally present these ideas below.

**Lemma 4 (Hump-Shaped Cumulative Return Expectations).**  Given Assumption 5 \( \rho_{SM} > \rho_S \),

(i) If \( d^S_t > 0 \) and investors believe they have a more optimistic forecast of dividends in period \( t + 1 \) than the market \( \rho_SD^S_t > \rho_SMd^{SM}_t \), then investors expect initially positive returns followed by subsequently negative returns. That is, \( E_{S,t}(R_{t,t+1}) > 0 \), and there is an \( \bar{h} > 0 \) such that \( h > \bar{h} \) implies that \( E_{S,t}(R_{t+h,t+h+1}) < 0 \).

(ii) If \( d^S_t < 0 \) and investors believe they have a more pessimistic forecast of dividends in period \( t + 1 \) than the market \( \rho_SD^S_t < \rho_SMd^{SM}_t \), then investors expect initially negative returns followed by subsequently positive returns. That is, \( E_{S,t}(R_{t,t+1}) < 0 \), and there is an \( \bar{h} > 0 \) such that \( h > \bar{h} \) implies that \( E_{S,t}(R_{t+h,t+h+1}) > 0 \).

We can unpack Lemma 4 based on the assumptions required to deliver the results. We focus on case (i) in our discussion, but the logic for case (ii) is similar.\(^1\) If investors perceive they have a more optimistic forecast of period \( t + 1 \) dividends than the market \( \rho_SD^S_t > \rho_SMd^{SM}_t \), they expect the market to be positively surprised by dividends in period \( t + 1 \), and expect it to positively revise its beliefs upwards, making period \( t + 1 \) returns positive. Assumption 5 \( \rho_{SM} > \rho_S \) delivers the sign switch in return expectations that we find is common in survey data. Because investors believe they have more optimistic forecasts of period \( t + 1 \) dividends than the market, they believe that the market will observe higher than expected dividends in the future and revise its beliefs upwards. Under the assumption that \( \rho_{SM} > \rho_S \), investors believe that once the market’s beliefs about fundamentals adjust upwards, the market overestimates how long dividends will remain higher than average, and accordingly becomes too optimistic in its valuation. Accordingly, investors forecast that asset returns eventually become negative, as they forecast that the market will observe lower than expected dividends in the future.

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\(^1\)Applying the model to the real world, \( d^S_t < 0 \) (negative dividend expectations) can be thought of as expectations of negative interest rate differentials in currency markets, or below average dividend growth in equity markets.
Lemma 4 outlines specific conditions on perceived belief disagreements with the market through which hump-shaped return expectations arise. Given Assumption 6, such conditions arise endogenously in response to news.\textsuperscript{17} In particular, if investors initially perceive homogeneous priors with others, then following a positive dividend shock, investors believe that they update their beliefs more in response to the shock than the market, and $\rho_{SD}^i > \rho_{SM}^i d_{SM}^i$. Similarly, following a negative dividend shock, $\rho_{S}^i d_{S}^i < \rho_{SM}^i d_{SM}^i$, as investors believe that other investors do not fully incorporate the information in the negative shock. This also allows us to expand the results of Lemma 4 such that investors have hump-shaped return expectations in response to public news, just as we observe in the data.\textsuperscript{18}

Proposition 5 (Hump-Shaped Return Expectations in Response to News). Given Assumption 5 ($\rho_{SM} > \rho_{S}$) and Assumption 6 ($\kappa_{SM} < \frac{\rho_{SM}^i}{\rho_{SM}^i}$), on average, following a positive shock to dividends in period $t$, investors expect risky asset returns to be initially positive and subsequently negative. On average, following a negative shock to dividends in period $t$, investors expect risky asset returns to be initially negative and subsequently positive. That is, given a dividend shock $\epsilon_t$ in period $t$, in expectation, $\text{sign}(R_{t,t+1}) = \text{sign}(\epsilon_t)$, and there is an $\bar{h} > 0$ such that $h > \bar{h}$ implies that $\text{sign}(R_{t+h,t+h+1}) \neq \text{sign}(\epsilon_t)$.

Proposition 5 illustrates how the model endogenously generates hump-shaped return expectations. In particular, the perceived disagreements required to generate the hump-shaped return expectations in Lemma 4 emerge endogenously due to different confidence in dividends as a signal about underlying fundamentals. In Proposition 5, the specific interpretation of hump-shaped return expectations in response to news is that investors believe that the market will continue to update its beliefs in the same direction of past news in the periods immediately following a shock, due to a belief that other investors do not sufficiently update their beliefs upon the arrival of the shock (Assumption 6). However, due to a perception that other investors overestimate the persistence of fundamentals (Assumption 5), investors believe that the market will eventually overreact and become overly optimistic once it fully incorporates a past positive shock. Hence, following a positive shock, investors believe returns will eventually turn negative, when the market begins to observe dividends lower than their overly optimistic forecasts. By the same logic, investors expect returns to be initially negative and subsequently positive following a negative shock to dividends.

\textsuperscript{17}In addition to giving rise to perceived belief disagreements in response to news, by placing conditions on perceptions of the market’s Kalman gain, Assumption 6 also governs the speed with which investors believe that the market’s belief will converge towards their own when they perceive disagreements. When investors perceive the market’s Kalman gain as higher, they expect returns to be earned more quickly, as they perceive the market as learning more quickly from data. Similarly, investors expect returns to take longer to accrue when they perceive the market’s Kalman gain as lower.\textsuperscript{18}

Scheinkman and Xiong (2003), David (2008), Xiong and Yan (2010), and Sastry (2021) similarly feature different perceived informativeness of public information as a tool to generate belief disagreement. Gourinchas and Tornell (2004) applies this idea to explain persistence of currency excess returns in the direction of past interest rate news. A similar effect can be generated in other models featuring different forms of belief formation. For example, in Bastianello and Fontanier (2021a), investors that believe that they are the only investors using past price changes to infer information will similarly believe that other investors’ beliefs do not sufficiently respond upon the arrival of news.
Figure 3: Hump-Shaped Cumulative Return Expectations

Note: The figure illustrates hump-shaped cumulative return expectations in the model, as described in Proposition 5. Both blue lines in the plot correspond with investors' period $t$ expectations of cumulative returns from period $t$ to $t + h$, for different horizons $h$ following a positive dividend surprise in period $t$. Both lines in the plot correspond with investors expecting initially positive returns, which comes from a belief that they have a more optimistic forecast of period $t + 1$ dividends than the market following the dividend surprise (Assumption 6). The dark blue line corresponds with investors believing that the market has a more persistent belief about fundamentals than they do, $\rho_{SM} > \rho_S$ (Assumption 5). The dotted blue line corresponds with $\rho_{SM} = \rho_S$, which can be thought to reflect a standard differences-of-opinion model where investors believe that others agree with them about the persistence of the fundamentals process but have different beliefs about fundamentals. The dotted red line, labeled valuation wedge, corresponds with the long term cumulative expected returns perceived by investors (as described in Proposition 4).

Figure 3 illustrates the proposition, plotting investors’ period $t$ expectations of cumulative returns from period $t$ to $t + h$, for different horizons $h$. The solid blue line illustrates the forecasted cumulative returns of investors following a positive dividend surprise in period $t$, given Assumptions 5 and 6. Returns are expected to be initially positive and then become subsequently negative, as captured by the hump-shaped pattern of cumulative return expectations. The dotted blue line plots forecasted cumulative returns if we relax Assumption 5, but maintain Assumption 6. This corresponds with assuming that investors and the market have the same beliefs about the persistence of fundamentals. These return expectations correspond with expectations in standard differences-of-opinion models. In this case, returns never switch signs; expected returns are positive for all future horizons. In both cases, cumulative returns converge to the valuation wedge in the long run.

A striking feature of the survey evidence on return expectations and higher order beliefs is that investors often expect long term cumulative returns to be negative, perceiving other investors to be overly optimistic, but they also expect short term returns to be positive (or
vice-versa). That is, not only do investors expect that asset returns will have different signs at short and long horizons, but they also expect a positive short horizon return to completely reverse, and for the asset’s value to eventually decline from its present value. This belief is related to, but distinct from, the hump-shaped pattern of return expectations. It can be interpreted as a belief that an asset is mispriced, and will have negative long term cumulative returns, but that it will become more mispriced in the short term. In the model, this type of belief induces investors to trade in the direction of widening perceived mispricing, e.g., by purchasing an asset they perceive to be overvalued.

We define such motivations for trading – trading in a direction that exacerbates perceived mispricing, or equivalently, trading in a direction opposite long term cumulative expected returns – as destabilizing trading motives. Alternatively, we may also observe corrective trading motives when investors believe that short and long term cumulative returns are the same sign, and accordingly take positions in a direction that closes perceived mispricing.

**Corollary 1 (Corrective and Destabilizing Trading Motives).** Given Assumption 5 \((\rho_s > \rho_{SM})\), when \(d_s^S > 0\) and investors perceive a higher forecast of dividends than the market \((\rho_s d_s^S > \rho_{SM}d_{1}^{SM})\),

(i) if \(\rho_{SM} > \frac{\rho_s}{d_{1}^{SM} + \frac{\rho_s}{d_{1}^{SM}} d_s^S}\), investors have destabilizing trading motives, in the sense that they hold long positions in period \(t\) although they perceive the risky asset to be overvalued and expect cumulative returns to be negative for all periods \(t + h\) following some period \(t + h\) \((R_{t,t+h} < 0,\) but there is an \(h > 0\) such that \(R_{t,t+h} < 0\) \(\forall h > h)\);

(ii) if \(\rho_{SM} < \frac{\rho_s}{d_{1}^{SM} + \frac{\rho_s}{d_{1}^{SM}} d_s^S}\), investors have corrective trading motives, in the sense that they hold long positions in period \(t\), perceive the risky asset to be undervalued, and expect cumulative returns from period \(t\) to be positive in all future periods \((R_{t,t+h} < 0,\) \(\forall h > 0)\).

19When investors perceive the market’s belief of fundamental persistence as sufficiently higher than their own, they perceive the risky asset as overvalued, i.e., they perceive the market’s forecast of the sum of future dividends as higher than their own. When combined with a belief that the market will revise its valuation of the asset upwards \((\rho_{SM}d_{1}^{SM})\), investors that see the asset as overvalued believe it will become even more overvalued in the future, exactly corresponding with the survey evidence on investors’ higher order beliefs in the U.S. equity market.20 Alternatively, if investors do not perceive the market’s

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19A symmetric result holds when \(d_s^S < 0\) and \(\rho_s d_s^S < \rho_{SM}d_{1}^{SM}\).

20We note that the mechanism that generates destabilizing trade here - a belief that the market will revise its belief in the future in the direction of the current misvaluation - is distinct from the mechanism in models that rely on short-sale constraints. For example, in Scheinkman and Xiong (2003), with short sale constraints, agents are willing to pay a higher price than their fundamental valuation of an asset because holding the asset confers an option value associated with selling to a potentially more optimistic agent in the future. Closer to the mechanism here are models where investors predict sentiment, and may accordingly engage in destabilizing speculation (e.g., De Long et al. (1990) and Martin and Papadimitriou (2022)). In De Long et al. (1990), rational investors speculate in a destabilizing manner due to the presence of noise traders that follow positive-feedback trading strategies that buy when prices are high and sell when prices are low. In Martin and Papadimitriou (2022), sentiment is deter-
belief of fundamental persistence as sufficiently high, then they perceive the risky asset as undervalued, and they believe that the market will revise its beliefs upwards in a corrective manner in the future. Accordingly, the hump-shaped patterns in return expectations that we document can coincide with destabilizing or corrective trading motives, depending upon investors’ higher order beliefs.

Figure 4 illustrates investors’ potential corrective versus destabilizing trading motivations. The figure plots investors’ expected cumulative returns, as perceived in period $t$. The solid blue line illustrates investors with corrective trading motivations in period $t$. In period $t$, they perceive that the risky asset is undervalued, and believe returns in period $t + 1$ will be positive, as the market revises its valuation upwards. Their motives for trading in period $t$ are in the direction of correcting a perceived undervaluation of the risky asset. In contrast, the dashed red line illustrates investors with destabilizing trading motives in period $t$. They perceive the risky asset as overvalued in period $t$, but believe it will become more overvalued in period $t + 1$, as the market revises its beliefs upwards. Investors’ motives for trading in period $t$, therefore, are destabilizing, as they seek to trade in a direction that exacerbates their perceived valuation wedge.

The figure also illustrates that even when they have corrective trading motives in a given period, investors may have destabilizing trading motives in subsequent periods, and vice-versa. For example, investors may believe that the risky asset is undervalued in period $t$, and that its price will increase for the next two periods. Accordingly, in period $t$, they have corrective trading motives, where their trading corrects a perceived mispricing. If they believe the market will over-correct its beliefs in period $t + 1$ and overvalue the asset, and that this over-correction will continue in period $t + 2$, then investors’ trading motivations in period $t + 1$ will switch to being destabilizing. They will continue to hold long positions, despite seeing the asset as overvalued. This is precisely what is illustrated in the solid blue line in Figure 4. The dashed red line in Figure 4 illustrates the reverse pattern playing out. Investors have destabilizing trading motives in periods $t$ and $t + 1$, when they perceive the asset as overvalued and take long positions based on the forecast it will become more overvalued. But they have corrective motives in period $t + 2$, when they perceive the asset as overvalued but forecast that its returns will be negative, and accordingly take short positions.

We can combine the logic of Corollary 1 and Proposition 5 to understand whether investors have corrective or destabilizing trading motives following the arrival of news. Investors have corrective trading motives when they believe that the market’s valuation underreacts upon the arrival of news, e.g., following positive news, investors believe the market’s fundamental valuation is lower than theirs. Investors have destabilizing motives when they think the market’s valuation overreacts upon the arrival news, e.g., following positive

mined by the wealth-weighted belief of investors. Since individuals that are correct in hindsight become wealthier, sentiment is bullish following good news and bearish following bad news. Understanding this, investors have an incentive to speculate.
Figure 4: Corrective versus Destabilizing Trading Motives

Note: The figure illustrates corrective and destabilizing trading motivations in the model, as described in Corollary 1. Each line in the plot corresponds with investors’ period $t$ expectations of cumulative returns from period $t$ to period $t + h$, for different horizons $h$. The solid blue line corresponds with a corrective trading motivation in period $t$. Investors perceive a positive valuation wedge (positive long term cumulative returns), and that returns will be positive in period $t + 1$. The dashed red line corresponds with a destabilizing trading motivation in period $t$. Investors perceive a negative valuation wedge, but perceive that returns will be positive in period $t + 1$.

news, investors perceive the market’s valuation as higher than theirs, and they believe the market’s valuation will rise even further in subsequent periods.

Corollary 2 (Perceived Underreaction and Continued Overreaction). Denote
$$\bar{\rho} \equiv \frac{\rho_S \kappa_S}{1 + \frac{\rho_S \kappa_S}{\rho_{SM}}},$$
which is the value of $\rho_{SM}$ for which investors believe the risky asset will be correctly priced in period $t$ on average given a fundamental shock. Given Assumption 6 ($\kappa_{SM} < \frac{\rho_S \kappa_S}{\rho_{SM}}$),

(i) if $\rho_{SM} > \bar{\rho}$, on average, upon the arrival of a positive fundamental shock, investors believe that the market price overreacts to fundamental news in period $t$, and that long term cumulative returns are negative. However, they believe that the period $t + 1$ return will be positive (the market will continue to overreact), and they take long positions with a destabilizing motive;

(ii) if $\rho_{SM} < \bar{\rho}$, on average, upon the arrival of a positive fundamental shock, investors believe that the market price underreacts to fundamental news in period $t$, and long term cumulative returns are positive. Accordingly, they take long positions with a corrective motive.

Under both assumptions, investors expect asset returns to be initially positive ($R_{t,t+1} > 0$), and subsequently negative (there is an $\bar{h} > 0$ such that $h > \bar{h}$ implies that $R_{t+h-1,t+h} < 0$).

Taken together, the results highlight a set of rich implications regarding investors’ higher order beliefs and trading motivations. Proposition 5 indicates that the pattern of hump-shaped return expectations can be rationalized by a belief that other investors will persistently update their beliefs about future dividends in the direction of past information, before reversing course upon observing lower dividends than expected. However, Corollaries 1 and 2 highlight that these patterns of expectations correspond with ambiguous trading
motives on the part of investors. If investors believe that other investors strongly overestimate the persistence of fundamentals, they believe that market valuations initially overreact to news and hump-shaped return expectations correspond with destabilizing trading motivations. Alternatively, if investors believe that other investors only weakly overestimate the persistence of fundamentals, they believe that market valuations initially underreact to news, and they trade with corrective motivations.

2.2.2 Higher Order Beliefs and Equilibrium Asset Prices

The previous results don’t immediately speak to the actual impact that investors’ trading may have on asset prices in equilibrium; they only speak to investors’ return expectations and motivations for trading. The true impact that investors have on equilibrium asset prices may differ from their trading motivations if they have incorrect higher order beliefs or incorrect fundamental beliefs. Investors may have corrective motivations for trading, but have a destabilizing impact on asset prices. Alternatively, they may have destabilizing motivations for trading, but have a corrective impact on equilibrium asset prices.

To illustrate this point, we note that the equilibrium asset price in Proposition 3 can be re-written as

$$P_t = \frac{\nu}{\theta} E_{S,t} (R_{t,t+1}) + \phi_t,$$

where $\nu$ is a positive constant that is independent of investors’ fundamental beliefs, and $\theta$ is a constant that captures the relative importance of residual demand. From the proof of Lemma 4, we know that investors’ price impact is positive when they perceive a more optimistic forecast of the period $t+1$ dividend than the market ($\rho_S d_t > \rho_{SM} d_{SM}^t$). One way to see the potential stabilizing or destabilizing influence of investors’ higher order beliefs is to consider a stylized setting where investors have a vanishing influence, and asset prices are determined entirely by residual demand.

Example 2.1 (Corrective and Destabilizing Trading). Consider an economy where investors have a vanishing influence ($\theta \to \infty$). If investors perceive a more optimistic forecast of dividends than the market ($\rho_S d_t^S > \rho_{SM} d_{SM}^t$),

(i) if residual demand reflects optimistic valuations of the risky asset relative to its fundamental value ($\phi_t > \frac{\rho}{1-\rho} d_t$), then introducing a small mass of investors has a destabilizing impact on risky asset prices;

(ii) if residual demand reflects pessimistic valuations of the risky asset relative to its fundamental value ($\phi_t < \frac{\rho}{1-\rho} d_t$), then introducing a small mass of investors has a corrective impact on risky asset prices.

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21 We can derive Equation (17) by plugging Equations (12) and (15) into the market clearing condition $X_t + \psi_t = 0$, where $\nu = \frac{\sigma_2}{\left(1 + \frac{\sigma_{SM}^2}{\sigma_S^2}\right) \left(\sigma_2^2 + \sigma_{SM}^2\right)}$. 

31
In Example 2.1, when investors have higher order beliefs that other investors will predictably revise their beliefs upwards, they take positive positions. When residual demand reflects over-optimism, this trading behavior has a destabilizing effect, while it has a corrective effect when residual demand reflects pessimism. Importantly, we note that these corrective or stabilizing effects are independent of whether investors believe that they are trading in a destabilizing manner, which, per Corollary 2, depends on whether $\rho_{SM} > \bar{\rho}$, i.e., if investors’ higher order belief about others’ perceived fundamental persistence is sufficiently high.

The logic behind this example, which is that the higher order beliefs we find in the data may play a corrective role if undervaluation is the dominant feature of fundamental valuations, and a destabilizing role if overvaluation is dominant, is a helpful characterization of higher order beliefs in our model. A point to make note of, however, is that Example 2.1 captures the impact of investors when they are infinitesimal relative to residual demand. Investors are non-infinitesimal and have price impact. Their return expectations do affect asset prices, both in our empirical evidence and in our quantitative analysis. Accordingly, their aggressive trading in response to fundamental information does affect asset prices in important ways.

One example of the potential destabilizing price impact of investors comes from them over-correcting an undervalued asset. This happens if investors’ return expectations are sufficiently higher than the wedge between prices and valuations implied by residual demand, and if residual demand is sufficiently small (small $\theta$). Investors’ aggressive trading may lead to asset price overreaction even if residual demand reflects underreaction to news, if investors sufficiently underestimate the market’s Kalman gain.

The discussion highlights that investors’ own fundamental and higher order beliefs jointly play an important role in equilibrium asset prices. When considering the effects of investors’ higher order beliefs, it is also important to account for their own fundamental beliefs as well. The broader implication is that understanding the exact impact of investors’ trading depends on the interaction between investors’ fundamental valuations, investors’ higher order beliefs, and residual demand. We return to this point when discussing the results of the estimated model.

3 A Quantitative Analysis of Higher Order Beliefs

Section 2 illustrates that we can reconcile the evidence on hump-shaped return expectations with investors holding higher order beliefs that the market will persistently update its beliefs in the same direction as past fundamental news, before eventually reversing course. However, the results also highlight that the implications for investors’ trading motives and for asset prices are ex-ante ambiguous. The higher order beliefs implied by hump-shaped return expectations can correspond with corrective or destabilizing trading motives on the part of investors, depending upon their fundamental and higher order beliefs. The trading
behavior implied by investors’ hump-shaped return expectations may actually correct or destabilize prices, depending upon the prevailing fundamental beliefs in the economy.

In this section, we conduct a quantitative analysis in currency markets, in order to understand the influence of higher order beliefs on investors’ trading motivations and to explore the corresponding asset pricing implications. To do so, we first adapt the model presented in the previous section in order to take it to the data. We then discuss how we use survey data and asset price data to calibrate the model, before moving onto a quantitative analysis that illustrates the role of higher order beliefs.

3.1 Residual Demand in the Model

Before taking the model to the data, we impose some additional structure on how residual demand is formed. As before, we represent residual demand as a mass $\theta$ of fundamental traders. In period $t$, residual traders trade as if all investors in period $t + 1$ will share their beliefs. Residual traders are all Bayesian, and have exponential utility with constant absolute risk aversion $\gamma$. Their beliefs about fundamentals in period $t$ are $d_R^t$, and they believe the persistence of fundamentals and the transitory noise reflected in dividends each period to be $(\rho_R, \sigma_{R,v})$. While we know little ex-ante about residual demand, this representation provides a rich enough structure to match features of the empirically observed returns process when investors with beliefs that match survey data trade against residual demand.

These assumptions allows us to represent the residual valuation $\phi$ as $\phi = \frac{\rho_R}{1 - \rho_R} d_R^t$, and the residual traders’ perceived variance, $\sigma_{\phi}^2$, as $\sigma_{\phi}^2 = \left(1 + \frac{\kappa_R \rho_R}{1 - \rho_R}\right)^2 \sigma_{\epsilon}^2 + \sigma_{R,v}^2$, where $\kappa_R$ is the residual traders’ Kalman gain.

We also assume the limiting case where investors perceive the noise in residual demand is zero, i.e., they treat residual demand as deterministic. This simplifying assumption allows us to substitute out risk aversion, $\gamma$, and the perceived noise of residual demand, $\sigma_{\psi}^2$, from our estimation. We have limited ability to estimate these parameters in the data, and find that they have limited explanatory power for the moments we seek to match in the data.

3.2 Applying the Model to Currency Markets

The currency market model we take to the data can be interpreted as an analog to the main model where the risky asset price is the log exchange rate (rather than $P_t$), and where interest rate differentials play the role of dividends.

The model is a small open economy model, featuring a Home country and an infinitesimal Foreign country. The (endogenous) log exchange rate is $s_t$, expressed in units of home currency per unit of foreign currency. Each country offers a bond in zero net supply. The interest rate differential (the foreign interest rate minus the home interest rate), $i^d t$, is equal to an unobserved fundamental, $\iota t$, plus idiosyncratic Gaussian noise each period. Foreign currency excess returns are given by $s_{t+1} - s_t + i^d_t$. Investors in the model are identical to
the investors in the main model, except that their risky asset allocation can be thought of as borrowing in local currency and investing in foreign currency. We present a more detailed exposition of the model in Appendix Section B.

3.3 Calibrating the Model

We calibrate the model by matching model-implied moments pertaining to fundamental expectations, return expectations, and realized returns, with corresponding moments in the data. Below, we provide an overview of our approach to calibrating the model, which proceeds in four steps. We discuss how we obtain model-implied moments for a given choice of parameters in more detail in Appendix C.

Step 1: Estimating the Cash Flow Parameters

We estimate the cash flow parameters in the model by Maximum Likelihood estimation of the Kalman Filter associated with Equation (2). These parameters are the persistence of fundamentals, \( \rho \), the amount of noise in the persistent component of fundamentals each period, \( \sigma_{\epsilon} \), and the amount of transitory noise in cash flows relative to fundamentals each period, \( \sigma_v \). We estimate the system using data on interest rate differentials for G11 currencies from 1986-2019.

Step 2: Estimating Investors’ Cash Flow Belief Parameters

We assume that all investors in the model know and agree on the amount of noise in the persistent component of fundamentals each period, \( \sigma_{\epsilon} \). Accordingly, there are two subjective parameters that govern the evolution of investors beliefs about cash flows: \( \sigma_{S,v} \) (the amount of transitory noise relative to fundamentals that investors believe is embedded in realized cash flows each period) and \( \rho_S \) (investors’ beliefs about the persistence of the cash flow process). Given the parameter value \( \rho_S \), the parameter \( \sigma_{S,v} \) identifies investors’ Kalman gain, \( \kappa_S \). For clarity of interpretation, when reporting results, we report Kalman gains.

We follow Angeletos, Huo and Sastry (2021) in estimating these belief parameters by matching the model-implied impulse response function (IRF) of average forecast errors of fundamentals in response to shocks with the corresponding empirical IRF estimated with survey data.\(^{22}\) Given a shock to fundamentals in period \( t \), \( \epsilon_t \), the impulse response function of forecast errors to the shock is captured by the coefficients of regressions of the form

\[
D_{t+h} - \mathbb{E}_{S,t+h-k}(D_{t+h}) = a_h + \beta_h \epsilon_t + e_t,
\]

where \(\beta_h\) are the coefficients of interest.

\(^{22}\)The general idea of matching model-implied and empirical impulse response functions follows the approach proposed by Christiano, Eichenbaum and Evans (2005). Similar to us, Valente, Vasudevan and Wu (2021) implement this approach to compute beliefs about the interest rate differential process in order to calibrate a model of exchange rate determination.
We estimate the parameters by solving the objective function

$$\min_{\rho_S, \sigma_S, v} (\hat{\Theta}_{CF} - \Theta_{CF}(\rho_S, \sigma_S, v))^\prime \Omega_{\Theta_{CF}} (\hat{\Theta}_{CF} - \Theta_{CF}(\rho_S, \sigma_S, v)), \quad (19)$$

where $\Omega_{\Theta_{CF}}$ is a diagonal matrix containing the weights of the target moments, $\Theta_{CF}(\rho_S, \sigma_S, v)$ is a function that maps $\rho_S$ and $\sigma_S$ to a vector containing the model-implied impulse responses of forecast errors, and $\hat{\Theta}_{CF}$ is a vector containing empirical impulse responses of survey-based forecast errors in response to fundamental shocks. We use AR(1) innovations to interest rate differentials as our measure of shocks, and we use 4-quarter ahead interest rate differential forecasts from Consensus Economics to measure survey-based forecast errors in the data. We use impulse responses from $h = 4$ to $h = 20$ periods after a fundamental innovation, which provides 16 moments to estimate the two parameters.

The identification of $\sigma_S$ and $\rho_S$ comes from initially positive forecast errors and subsequently negative forecast errors of expectations in response to fundamental news in survey data. The Kalman gain of investors is decreasing in $\sigma_S$. Hence, following a positive shock, for larger values of $\sigma_S$, forecasters underestimate the level of fundamentals, and forecast errors are positive. Higher perceived persistence than the true persistence of the fundamentals process ($\rho_S > \rho$) means that following a positive fundamental shock, once their beliefs adjust to reflect the true value of fundamentals, forecasters believe that fundamentals will remain high for longer than they actually do. This results in forecast errors eventually becoming negative. As discussed in Angeletos, Huo and Sastry (2021), matching IRFs allows us to estimate $\sigma_S$ and $\rho_S$ by matching the sign and magnitude of errors made by forecasters.

**Step 3: Estimating Investors’ Higher Order Belief Parameters**

Investors’ higher order belief parameters are $\rho_{SM}$ and $\sigma_{SM, v}$ (which, as in the case of investors’ fundamental parameters, also determine perceptions of the market’s Kalman gain, $\kappa_{SM}$). As derived in Proposition 2, investors’ fundamental and higher order belief parameters pin down their return expectations.

We estimate investors’ higher order belief parameters by matching the impulse responses of their return expectations of different future horizons in response to a fundamental shock in period $t$, $\epsilon_t$, taking the previously estimated parameters as given. In particular, impulse response functions are captured by coefficients of regressions of the form

$$E_{S,t}(R_{t+h,t+h+k}) = a_h + \beta_h \epsilon_t + \epsilon_t, \quad (20)$$

where once again, $\beta_h$ are the coefficients of interest. We use 3-month ahead, 3- to 6-months ahead, and 6- to 12-month month ahead return expectations. We estimate the parameters

\footnote{For the impulse responses, weights are inversely proportional to the sample variances of the empirical impulse responses of fundamentals.}
by solving the objective function

$$\min_{\rho_{SM}, \sigma_{SM,v}} \left( \hat{\Theta}_{ER} - \Theta_{ER}(\rho_{SM},\sigma_{SM,v}) \right)' \Omega_{\Theta_{ER}} \left( \hat{\Theta}_{ER} - \Theta_{ER}(\rho_{SM},\sigma_{SM,v}) \right),$$

(21)

where $\Omega_{\Theta_{ER}}$ is a diagonal matrix containing the weights of the target moments, $\Theta_{ER}(\rho_{SM},\sigma_{SM,v})$ is a function that maps $\rho_{SM}$ and $\sigma_{SM,v}$ to a vector containing the model-implied impulse responses of return expectations, and $\hat{\Theta}_{ER}$ is a vector containing empirical impulse responses of survey-based return expectations.

Conditional on investors’ fundamental belief parameters, identification of $\rho_{SM}$ and $\sigma_{SM,v}$ comes from the hump-shaped pattern of return expectations, as discussed in Proposition 5. Initially positive expected returns following positive fundamental shocks emerge from high values of $\sigma_{SM,v}$ (or correspondingly, low values of $\kappa_{SM}$), while expectations of subsequent negative returns emerge from higher values of $\rho_{SM}$.

**Step 4: Estimating Residual Demand Parameters**

The last set of parameters to be estimated are the residual demand parameters, $(\theta, \rho_R, \sigma_R, \nu)$. We estimate these parameters by matching moments of the realized returns process, taking the previously estimated parameters as given.

We estimate impulse response functions of realized returns in response to fundamental shocks in period $t$, $\epsilon_t$, via regressions of the form

$$R_{t+h,t+h+1} = \alpha_h + \beta_h \epsilon_t + \epsilon_t,$$

(22)

where $\beta_h$ are the coefficients of interest. We estimate the model parameters by solving the objective function

$$\min_{\rho_R, \sigma_R, \nu, \theta} \left( \hat{\Theta}_R - \Theta_R(\rho_R,\sigma_R,\nu,\theta) \right)' \Omega_{\Theta_R} \left( \hat{\Theta}_R - \Theta_R(\rho_R,\sigma_R,\nu,\theta) \right),$$

(23)

where $\Omega_{\Theta_R}$ is a diagonal matrix containing the weights of the target moments, $\Theta_R(\rho_R,\sigma_R,\nu,\theta)$ is a function that maps $\rho_R$, $\sigma_R$, $\nu$, and $\theta$ to a vector containing the model-implied impulse responses of return expectations, and $\hat{\Theta}_R$ is a vector containing empirical impulse responses of returns to interest rate differential shocks. We match returns from $h = 1$ to $h = 30$ periods after the shock.\(^{24}\)

The parameters $\sigma_{R,v}$, which contributes to the sluggishness of asset price, and $\rho_R$, which contributes to asset price overreaction, are separately economically identified from one another. However, they are not separately economically identified from $\theta$, which is the mass of traders driving residual demand. The parameter $\sigma_{R,v}$ is identified by the empirically observed positive persistence of returns, while $\rho_R$ is identified by long term return reversals. Both parameters are only separately identified from $\theta$ by functional form assumptions.

\(^{24}\)Our motivation for using a different number of periods than in step 2 is primarily empirical. As captured in Figure 7, currency excess returns exhibit reversal up to 30 quarters following an interest rate differential shock.
Fundamental Parameters  \( \rho = 0.95 \quad \kappa \approx 1 \)

Fundamental Beliefs  \( \rho_S = 0.97 \quad \kappa_S = 0.44 \)

Higher Order Beliefs  \( \rho_{SM} = 0.99 \quad \kappa_{SM} = 0.41 \)

Residual Demand  \( \rho_R = 0.98 \quad \kappa_R = 0.12 \quad \theta = 0.05 \)

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<th>TABLE 5: ESTIMATED CURRENCY MODEL PARAMETERS</th>
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<tbody>
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<td>Note: The table presents estimated model parameters for developed market currencies. Fundamental parameters are estimated to match moments of the interest rate differential process. Fundamental belief parameters are estimated to match impulse response functions of forecast errors of interest rate differentials in response to innovations to interest rate differentials. Higher order belief parameters are estimated to match the responses of return expectations to innovations to interest rate differentials. Residual demand parameters are estimated to match moments of the true currency excess return process in response to interest rate differential shocks.</td>
</tr>
</tbody>
</table>

As an example to provide intuition on this point, it is possible for residual demand to contribute to sluggishness in asset prices because there is a large mass of slightly sluggish residual traders (higher \( \theta \), lower \( \sigma_{R,p} \)), or because there is a small mass of very sluggish residual traders (lower \( \theta \), higher \( \sigma_{R,s} \)). Accordingly, it is important that the residual demand parameters be understood in conjunction with one another.

### 3.4 Currency Market Results

#### 3.4.1 Model Estimation

Table 5 displays the estimated currency market parameters. The first row in the table shows the estimated fundamental parameters, \( \rho \) and \( \kappa \). We estimate the persistence \( \rho \) to be 0.95, and the objective Kalman gain \( \kappa \) to be approximately 1, indicating little evidence of a transitory component of interest rate differentials, consistent with the findings of Gourinchas and Tornell (2004).

The second row displays the estimated fundamental belief parameters of investors. We estimate \( \rho_S = 0.97 \) and \( \kappa_S = 0.44 \). These numbers indicate that investors in survey data overestimate the persistence of interest rate differentials, and that their Kalman gain is lower than the objective Kalman gain. That is, following a shock to interest rate differentials, it takes them multiple periods to incorporate the news, and they update their beliefs in the direction of the shock for multiple periods. Once again, these estimates are in line with previous findings (Candian and De Leo (2021), Valente, Vasudevan and Wu (2021)). Figure
Figure 5: Estimating Investors’ Fundamental Beliefs

Note: The figure plots impulse responses of investors’ interest rate forecast errors following a one standard deviation shock to interest rate differentials, as estimated in the data versus as captured by model parameters. The shaded area corresponds with 68% HAC confidence intervals.

5 plots forecast errors for 4-quarter ahead interest rate differential forecasts in the model versus in survey data from Consensus Economics. The figure indicates that the model does a reasonable job in capturing forecast errors of interest rate differentials. The low Kalman gain of investors is reflected by their initially positive forecast errors of interest rate differentials following an interest rate differential shock, indicating sluggishness in incorporating news. However, once they incorporate past news about higher interest rate differentials, investors tend to overestimate future interest rate differentials, due to their overestimation of the persistence of fundamentals. This is captured by the negative forecast errors of interest rate differentials approximately eight quarters after a shock.

The third row displays the estimated higher order belief parameters of investors. We estimate \( \kappa_{SM} = 0.41 \) and \( \rho_{SM} = 0.99 \). These numbers indicate that investors believe that the market under-infers information about future interest rate differentials (since \( \kappa_{SM} < \kappa_S \)), and also overestimates the persistence of interest rate differentials. This captures investors’ hump-shaped return expectations following shocks to interest rate differentials, as illustrated by Figure 6. The model does well in capturing investors’ return expectations, especially their 1-quarter ahead and 2- to 4-quarter ahead return expectations following a shock.

The last two rows of Table 5 display estimates of residual demand parameters, which are \( \kappa_R = 0.12 \), \( \rho_R = 0.98 \), and \( \theta = 0.05 \). The estimates suggest that residual demand can be thought of as a small mass of very sluggish traders that also delayed overreact to news about interest rate differentials. However, as we noted when describing parameter identification, the implications of these estimates are similar to having a larger mass of less sluggish investors. One possible interpretation is that residual demand may capture “real
Figure 6: Estimating Investors’ Higher Order Beliefs

Note: The figure plots coefficients of investors’ return expectations for 3-months ahead, 3- to 6-months ahead, and 6- to 12-months ahead regressed on a one standard deviation interest rate differential shock, as estimated in the data versus as captured by estimated model parameters. The bars correspond with 68% HAC confidence intervals.

money,” e.g., businesses that use currency markets to hedge and behave in an especially sluggish way.25

Figure 7 plots the model’s fit for currency excess returns in response to a shock to interest rate differentials, which are the targeted moments in estimating $\theta$, $\kappa_R$, and $\rho_R$ (step 4). The model captures the fact that for approximately twelve quarters following a positive shock to interest rate differentials, currency excess returns tend to be positive, and they subsequently become negative.26 While the model does a reasonable job, it is unable to capture some features of the data, for example the steepness of reversals that occur in currency excess returns.

3.4.2 Understanding Currency Market Returns

The red line in Figure 8 plots the deviations of exchange rates from fundamental values over time in response to a one standard deviation interest rate differential shock in the esti-

25In the context of our model, the sluggishness of residual demand comes via an expectations channel. However, it may also be the case that residual traders have correct expectations, but only infrequently revisit their foreign exchange exposures. Bacchetta and van Wincoop (2010, 2021) study infrequent portfolio rebalancing in currency markets, and suggest that it may help explain a number of puzzling facts about exchange rate behavior.

26The fact that currency excess returns are initially positive and subsequently negative following a shock to interest rate differentials is related to puzzling failures of the uncovered interest rate parity condition in currency markets. In particular, in time-series regressions, the interest rate differential for a currency positively predicts its near-term excess returns, as first documented by Fama (1984). Moreover, interest rate differentials for a currency also negatively predict the currency’s excess returns after 8-12 quarters, as documented and studied by Bacchetta and van Wincoop (2010) and Engel (2016). This has been described as the ‘predictability reversal puzzle’ in the literature.
Note: The figure plots the regression coefficients of currency excess returns in period \( t + k \) following a one standard deviation shock to interest rate differentials in period \( t \), as estimated in the data and as captured by the model. The shaded area corresponds with 68% HAC confidence intervals.

Figure 7: Estimating Residual Demand Parameters

Note: The figure plots the regression coefficients of currency excess returns in period \( t + k \) following a one standard deviation shock to interest rate differentials in period \( t \), as estimated in the data and as captured by the model. The shaded area corresponds with 68% HAC confidence intervals.

mated model. Upon the arrival of the interest rate differential shock, the exchange rate underreacts and is 2.9% percent lower than the fundamental value implied by the shock. Over time, investors in the model incorporate the news of higher interest rate differentials into their valuations, and currency excess returns are positive for a number of periods following the shock. Eventually, due to investors’ overestimation of the persistence of fundamentals, the exchange rate overshoots its fundamental value.

What is the impact of investors’ trading behavior? Using Equation (17), we note that at each point in time, we can decompose the exchange rate into residual demand and investors’ price impact. Figure 8 plots valuations implied by residual demand in a solid blue line, and investors’ price impact in a dashed blue line. Valuations implied by residual demand initially underreact more than the exchange rate does in the model. Upon an interest rate differential shock, residual valuations are 4.7% lower than the fundamental value implied by the shock. They underreact for three periods, before subsequently overreacting. In the overreaction stage, valuations implied by residual demand also overreact more than the exchange rate does. Investors play a stabilizing role in helping to correct initial underreaction and to also mitigate delayed overreaction, as can be seen by their price impact.

The corrective behavior of investors is driven by their hump-shaped return expectations. Because they expect initially positive returns in the periods following an interest rate differential shock, investors trade more aggressively, reducing the initial underreaction of exchange rates. Similarly, they also expect returns to eventually turn negative following a shock, as they perceive the market as overestimating the persistence of fundamentals. This drives them to take short positions precisely when the exchange rate delayed overreacts and
becomes overvalued.

Investors’ trading behavior plays a corrective role for exchange rates despite the fact that investors’ beliefs are incorrect. That is, while their trading behavior is correct, investors’ trading motivations are not always so. We note that following a positive interest rate shock, investors always perceive a negative valuation wedge, as they (incorrectly) perceive the markets’ valuation as overreacting to the shock. This perception occurs due to the strong overestimation of fundamental persistence that investors attribute to other investors’ beliefs ($\rho_{SM} > \rho_S$). In the data, such overestimation arises from a belief in sharp reversals occurring within a year following an interest rate differential innovation. The valuation wedge means that while investors’ aggressive trading behavior in response to fundamental shocks plays a corrective role for the exchange rate, their initial trading motives are actually destabilizing. However, the stabilizing role that investors play in reducing delayed overreaction to fundamental shocks is as intended; their negative price impact pushes the exchange rate in the direction of their long term cumulative return expectations.

3.4.3 Counterfactuals

To dig more deeply into the role that higher order beliefs may play in asset prices, we consider the behavior of exchange rates in a set of counterfactual calibrations. We present more details on the exact implementation of the counterfactual models in Appendix B.1.4.
Counterfactual 1. Correcting Higher Order Beliefs

In our calibrated model, investors’ higher order beliefs are incorrect. To understand the impact of these incorrect higher order beliefs, we study a counterfactual model where investors correctly perceive the beliefs of investors in the market and the process that residual demand follows.\textsuperscript{27} The solid blue line in Figure 9 plots how exchange rates deviate from fundamental values following an interest rate differential shock over time in the counterfactual model.

In the counterfactual model, the exchange rate \emph{slightly} underreacts in the period that an interest rate differential shock occurs, but it quickly overreacts in subsequent periods. The underreaction of the exchange rate in the initial period is only 0.46\% relative to the fundamental value of the shock. This result stems from the fact that in the counterfactual model, the initial underreaction of the market’s valuations, stemming from a low Kalman gain, is almost perfectly offset by the market’s overestimation of the persistence of interest rate differentials in the period of the shock. However, the exchange rate is 2.8\% overpriced one quarter following an interest rate differential shock, and steadily overreacts more, peaking at a 5.6\% overpricing five quarters following the shock. Following the shock, in the counterfactual model, the mispricing of the exchange rate is larger in magnitude than the mispricing in the estimated model for 35 quarters, at which point both models imply that exchange rates are priced close to correctly.

Why does correcting higher order beliefs in the counterfactual model lead mispricing to increase? The dashed blue line in Figure 9 plots investors’ valuations over time following an interest rate differential shock. The exchange rate closely tracks investors’ valuations in the counterfactual model. In the counterfactual model, investors realize that they occupy a very substantial share of the economy, and accordingly trade aggressively towards their beliefs. However, because investors’ exchange rate valuations overreact, due to their overestimation of the persistence of fundamentals, the exchange rate also overreacts. By correcting their higher order beliefs, investors’ price impact becomes over-correcting, due to investors’ fundamental belief biases.

Counterfactual 2. Correcting Fundamental and Higher Order Beliefs

The green line in Figure 9 plots the evolution of exchange rates under a second counterfactual, where, in addition to correcting investors’ higher order beliefs, we also correct their fundamental beliefs. That is, we consider the impact of replacing investors in the model with informed traders that have correct fundamental and higher order beliefs. In this second counterfactual model, mispricing almost entirely disappears. Contemporaneous with an interest rate differential shock, there is slight underreaction (0.13\%), stemming from the

\textsuperscript{27}A caveat is that in order to solve the counterfactual model, we assume that investors believe that all investors (including those comprising residual demand) maximize their 1-period ahead expected returns based on common knowledge of all investors’ beliefs. These assumptions do not quite correspond with correct higher order beliefs, as residual demand does not take into account investors’ beliefs.
underreaction of residual demand. In subsequent periods, the deviation of the exchange rate from its fundamental values is always less than 0.05%.

Analyzed in concert with the first counterfactual, this second counterfactual highlights the importance of the interaction of investors’ higher order beliefs and fundamental beliefs. While investors with correct higher order beliefs but incorrect fundamental beliefs overcorrect the underreaction in currency markets and lead the exchange rate to strongly overreact to news, informed speculators with correct higher order beliefs and fundamental beliefs would entirely correct mispricing in the exchange rate.

**Counterfactual 3. Errors in Forecasting Fundamentals**

Figure 9 also highlights that models that only focus on fundamental expectations may yield different conclusions about whether market prices overreact or underreact to information. Investors’ valuations in the figure correspond to the evolution of the exchange rate in a third counterfactual model: one in which there is no residual demand, and investors have correct higher order beliefs. This third counterfactual model is a model of errors in forecasting fundamentals, as is commonly studied in the literature. Variation in the exchange rate is driven solely by variation in the fundamental beliefs of investors. The counterfactual model suggests that the exchange rate primarily displays overreaction, whereas our model suggests that initial underreaction is also an important feature of exchange rate dynamics. This difference arises despite the fact that both models display similar patterns of predictable variation in returns in the form of momentum and reversal (initially positive and subsequently negative returns following positive past news).  

Investors’ higher order belief mistakes are the main driver of differences between this counterfactual model and our main model. Investors’ higher order belief mistakes push them to not trade fully towards their fundamental valuations, which leads our model’s quantitative predictions to substantially deviate from a model only reflecting investors’ fundamental valuations. Investors’ higher order belief mistakes lead residual demand to have additional influence on exchange rates versus a model of forecasting fundamentals. The results highlight the importance of accounting for higher order beliefs in understanding asset price variation.

**3.4.4 Taking Stock of the Quantitative Results**

The currency market results indicate that investors’ higher order beliefs and trading behavior appear to play a corrective role for exchange rates. Higher order beliefs help to mitigate initial underreaction and delayed overreaction of currencies in response to news. However, the model also indicates that investors do not trade as aggressively towards their

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28Momentum and reversal of this sort – time-series persistence in returns that reverses over longer horizons – is commonly observed not just in currency markets, but also in a variety of other asset classes, including equity indices, commodities, and sovereign bonds. See Moskowitz, Ooi and Pedersen (2012).
Figure 9: Correcting Higher Order Beliefs

Note: The figure plots the deviations of the exchange rate from its fundamental value in response to a one standard deviation shock to interest rate differentials for a variety of model specifications. The red line corresponds with the estimated model. The solid blue line corresponds with a counterfactual model where investors know the beliefs of other investors. The dashed blue line corresponds with investors’ valuations of the exchange rate. The green line corresponds with a counterfactual model where investors have correct fundamental and higher order beliefs.

fundamental beliefs as they would if they properly understood other investors’ beliefs. In a counterfactual world, where investors’ beliefs correctly reflected other investors’ beliefs, we might expect mispricing to generally increase, reflecting the fundamental belief biases of investors. Together, the results highlight that the impact of investors’ higher order beliefs depend upon the interactions of investors’ higher order beliefs and fundamental beliefs.

3.5 Higher Order Beliefs in Equity Markets

Our quantitative application focuses on currency markets, where data on fundamental forecasts and multi-horizon return expectations are readily available for the same types of investors (large financial institutions). We do not do a similar exercise for equity markets, where we have multi-period return expectations, but do not observe fundamental forecasts for the same types of investors. Nevertheless, our model and quantitative analysis for currency markets, taken in combination with the empirical facts presented in Section 1, shed light on equity market investors’ higher order beliefs and their potential asset pricing implications. The evidence broadly suggests that equity market investors’ higher order beliefs destabilize asset prices, contributing to the overreaction of stock markets in response to news.

We first note that the empirical evidence from investors’ return expectations and higher order beliefs indicates that equity market investors believe that other investors’ beliefs over-
react, and hence that market valuations overreact to news. The higher order optimism index, which captures a belief that other investors are overly optimistic about the stock market, is high precisely following positive stock market returns and positive GDP news. Additionally, a belief in market overreaction is also consistent with the explicitly stated beliefs of equity market investors in the Shiller / Yale ICF surveys. In response to the question “What do you think is the cause of the trend of stock prices in the United States in the past six months,” 43% of survey respondents choose the response “It is based on speculative thinking among investors or overreaction to current news.” Equity market investors’ beliefs in overreaction, taken in conjunction with their expectation of short-term returns to move in the same direction as past news, indicate destabilizing trading motivations.

How might these motivations play out in asset price behavior? The prevailing evidence suggests that they are likely to have a destabilizing effect on equilibrium stock market valuations and contribute to stock market overreaction. A long literature documents that stock price returns are excessively volatile relative to dividends (e.g., see LeRoy and Porter (1981) and Shiller (1981)), and that a high price dividend ratio predicts low stock returns over a 3- to 5-year horizon. One strand of literature emphasizes that, assuming constant risk premia, this evidence is consistent with overreaction of prices (e.g., Barberis et al. (2015), Nagel and Xu (2022a), Bordalo et al. (2020)). Over the sample for which we have survey data on stock market return expectations (1989 to 2021), we find evidence that can be similarly interpreted as overreaction, though the evidence is noisier given the short sample. Figure 10 plots the response of excess stock market returns in the quarters following a one standard deviation GDP innovation from 1989 to 2021, the period for which we have data from the Shiller/Yale ICF survey. We observe a strong contemporaneous response (3.0% return) in the quarter of GDP news, which continues into the next quarter (1.0% return). However, after about 10 quarters following the arrival of news, equity market returns begin to reverse, as an overreaction story would suggest. Cumulative returns become negative after 17 quarters, and bottom out a little more than five years after the shock.

Taken together, the survey and return evidence suggest that investors in the stock market may contribute to overreaction, buying into the market when they expect positive returns

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29 In response to the question, survey respondents may choose the response provided in the main text, the response “It properly reflects the fundamentals of the U.S. economy and firms,” they may fill in their own custom response, or they may choose to express no opinion. The 43% number understates the proportion of investors that report beliefs of overreaction or speculative behavior, as several respondents that choose to fill in their own custom response often indicate a view that stock prices are driven by others’ overreaction or speculation acting in conjunction with additional forces, such as monetary policy.

30 A different way to see this point is by noting that in the model, a belief that the market’s valuation overreacts stems from a belief that the market sufficiently overestimates the persistence of fundamentals ($\rho_{SM} > \hat{\rho}$). The magnitude of $\rho_{SM}$ and investors’ beliefs in others’ overreaction are tightly linked to the magnitude of negative returns investors expect in the downwards part of their hump-shaped return expectations. In Section 1, our evidence suggests that individual and institutional investors in equity markets expect even sharper reversals than currency market investors (who also believe in market overreaction), as seen in Figure 1. This indicates a belief in market overreaction. These expectations of sharper reversals are consistent with the high proportion of equity market investors that expect short term and long term cumulative returns to be of different signs and advise taking positions opposite their long term return expectations.
in the short term, even though they (correctly) expect those returns to eventually reverse over longer horizons due to other investors’ overreaction. The literature finds that analysts’ fundamental (earnings growth) expectations also appear to overreact to news. An open question is decomposing the importance of higher order beliefs for stock market fluctuations. Our paper provides a framework for such analysis, either with data on fundamental and return expectations for the same types of investors (which we do not have), or by making assumptions about the fundamental expectations of investors in our sample.\footnote{We find little evidence of a correlation between analysts’ earnings growth expectations and the stock market return expectations in our sample. Our model predicts a relationship between fundamental and return expectations, which also is an empirical feature of surveys that ask investors for both their fundamental and return expectations (Giglio et al. (2021)).}

4 Comparisons with Other Models

In this section, we discuss potential alternative explanations for the empirical results on return expectations based on existing theories. We also discuss our evidence in the broader context of the literature on higher order beliefs, and consider the implications of our results for modeling higher order beliefs.

4.1 Potential Alternative Explanations for the Results

We discuss a few sets of theories of asset price fluctuations that are common in the literature, and how they may be applied to explain our results on investors’ return expectations.
The first class of alternative theories we discuss follows the traditional paradigm that emphasizes that investors have rational expectations, and suggests that asset price fluctuations are driven by time-varying risk premia. We also discuss two types of alternative behavioral theories: theories of systematic errors in forecasting cash flows; and theories of mechanical return forecasting (e.g., return extrapolation), where investors have mechanical beliefs about the dynamics of the asset return process. While we broadly argue that the empirical evidence is most consistent with an explanation based on higher order beliefs, we cannot fully reject that mechanical return forecasting may also be at play. In any case, we argue that our framework provides useful insight in understanding the potential implications of mechanical return forecasting.

**Time-Varying Risk Premia**

Theories of time-varying risk premia assume full information rational expectations, with investors forming correct expectations of future cash flows and prices. In such theories, fluctuations in an asset’s price reflect fluctuations in the return required by investors in order to hold the asset. Under this paradigm, investors’ return expectations should reflect rational expectations of how required compensation for bearing risk will vary in the future. Higher return expectations indicate that investors demand substantial compensation for bearing risk, while low return expectations indicate that investors demand lower compensation for bearing risk.

In equity markets, our empirical evidence indicates that 1-quarter ahead return expectations are high in quarters following positive news, while 2-, 3-, and 4-quarter ahead return expectations are low following such news. In typical models of time-varying risk premia, good macroeconomic news tends to lower risk premia and expected returns, which is not consistent with the evidence we observe. Additionally, the sharp change in expected returns forecasts from 1-quarter ahead to 1-year ahead, where returns are often expected to change signs, indicates a belief in risk premia that are faster moving and more negatively autocorrelated at short horizons than typically assumed in standard models. Lastly, our evidence indicates that investors make systematic errors in forecasting future returns, whereas investors should make no such systematic mistakes if return expectations reflected required risk compensation.

While time-varying risk premia may play some role in return expectations, they seem

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32 See Barberis (2018) and Adam and Nagel (2022) for additional discussion of theories and evidence regarding systematic errors in forecasting cash flows and returns.

33 Prominent examples in the context of the aggregate stock market include the habit-formation model of Campbell and Cochrane (1999), where stock market fluctuations are driven by fluctuations in aggregate consumption relative to an external ‘habit’ level of consumption; the long-run risks model of Bansal and Yaron (2004), where stock market fluctuations are driven by fluctuations in a persistent and difficult-to-detect component of consumption growth; and the time-varying rare disasters model (Gabaix (2008), Wachter (2013)), where stock market fluctuations are driven by fluctuations in the probability of a catastrophic rare disaster. These models have also been expanded to understand price fluctuations in other asset markets, for example foreign exchange and bond markets.
unlikely to be the primary driver of the patterns we document, and the standard models of time-varying risk-premia make predictions about returns that differ from the observed patterns in the data.

**Errors in Forecasting Fundamentals**

A number of theories suggest that errors in forecasting fundamentals may be important for explaining asset price fluctuations. Notable recent papers on the topic focusing on stock market fluctuations include Bordalo et al. (2020), De La O and Myers (2021), and Nagel and Xu (2022). We note that errors in forecasting fundamentals are a key ingredient in our analysis. In particular, our analysis emphasizes that the interaction between higher order beliefs and errors in forecasting fundamentals may be crucial to understanding the trading behavior of investors.

In the absence of assumptions about higher order beliefs, however, theories of systematic errors in forecasting fundamentals do not make predictions about investors’ return expectations. Said differently, models that feature investors who make systematic errors in forecasting fundamentals can make strikingly different predictions than our findings, based on the implicit assumptions made about higher order beliefs. For example, investors that make systematic errors in forecasting fundamentals can also believe that all other investors share their beliefs and belief-updating processes. In this case, in the absence of time-varying risk premia, investors would expect returns to be constant over time; they would not have the hump-shaped pattern of return expectations that we find investors exhibit in our analysis. Hence, theories based on fundamental forecast errors alone cannot explain our results. They require the additional focus on higher order beliefs that we provide.

In terms of asset pricing implications, our results also highlight that higher order beliefs are important for understanding return dynamics in asset markets. Our quantitative calibration in currency markets suggests that while initial underreaction is an important feature of the data, a model that solely focuses on fundamental expectations obscures this fact, and would suggest that overreaction is the primary feature of currency market returns. In equity markets, on the other hand, our evidence suggests that higher order beliefs may contribute to overreaction in the same way that errors in forecasting fundamentals also do.

**Mechanical Beliefs about the Returns Process**

One theory of return expectations emphasizes mechanical return extrapolation, whereby investors that observe high past returns infer that subsequent future returns will be high.

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34 Some models do feature assumptions that tightly link expectations of future cash flows and expectations of future returns. For example, in the representative agent model of Jin and Sui (2022), cash flow and return expectations are high following high returns due to the fact that the price-dividend ratio in the model is mean-reverting. However, the model of Jin and Sui (2022) does not predict the form of expectations of high returns followed immediately by low returns (and vice-versa) that we find in the data and capture in our model.
without necessarily giving consideration to the underlying drivers of returns. By itself, this theory does not explain our empirical evidence on return expectations. We find in survey data that investors believe that returns will be high immediately following good news (correspondingly, high past returns), but that they also believe that returns will be low in subsequent quarters. To match the evidence, return extrapolation would also need to be augmented with an additional assumption, for example a belief that returns are negatively autocorrelated with past returns of intermediate horizons.

Our explanation based on higher order beliefs, that other investors persistently update their beliefs in the same direction in quarters following past news and that others overestimate the persistence of fundamentals, can be considered a potential micro-foundation for such beliefs about the dynamics of returns. This micro-foundation unifies investors’ beliefs about returns, their beliefs about fundamentals, and their higher order beliefs reported in survey data. Empirical work on the topic notes that investors’ surveyed fundamental and return expectations are correlated (Giglio et al. (2021)), a feature that our model matches.

Moreover, by linking fundamental beliefs with return expectations, our paper highlights that commonly observed features in the data, such as return extrapolation, may have an ambiguous effect on the informational efficiency of markets. If investors’ beliefs in fundamental persistence are sufficiently high, then the trading behavior implied by return extrapolation plays a destabilizing role, while it plays a corrective role otherwise. Prior models of return extrapolation largely focus on the destabilizing role (e.g., Barberis et al. (2015, 2018)), while we find evidence of a stabilizing role in currency markets.

Finally, and importantly, we note that the evidence discussed in Section 1.3 showing a tight link between the Higher Order Optimism index with investors’ return expectations, and the evidence discussed in Section 3.5 – that 43% of survey respondents indicate a belief that the six month trend in stock prices is “based on speculative thinking among investors or overreaction to news” – provide direct evidence in favor of our higher order beliefs-based-channel.

### 4.2 Models of Higher Order Beliefs

We finally discuss how our results more broadly fit within the context of the extant literature that studies higher order beliefs, which is primarily theoretical in nature. We first discuss how our model relates to models of asset pricing with heterogeneous beliefs, where higher order beliefs play an important role. We next discuss how to interpret our results given investors’ potential higher order reasoning.

#### 4.2.1 Incomplete Information and Differences-of-Opinion

Higher order beliefs are a focus in models where investors have heterogeneous beliefs. The literature on higher order beliefs primarily studies two classes of models: incomplete

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35[^35]: E.g., see Barberis et al. (2018) and Adam, Marcet and Beutel (2017) for models of return extrapolation.
information models, where heterogeneous beliefs arise due to investors having noisy information about fundamentals and not being able to observe other investors’ information; and differences-of-opinion models, where all investors are dogmatic in their beliefs, and “agree-to-disagree” with other investors.

The primary result from the early literature on higher order beliefs in incomplete information settings suggests that when investors have short investment horizons, there is an over-reliance on public signals, information is slow to be impounded into prices, and, conditional on past performance, asset prices drift in the same direction (Allen, Morris and Shin (2006)). Later work points out that reliably generating such drift requires additional ingredients, such as higher order belief disagreements, due to the fact that investors can use prices to extract others’ information (Banerjee, Kaniel and Kremer (2009)). Speculative trading behavior and return predictability in incomplete information models can only come from information that is orthogonal to public information; Barillas and Nimark (2017) argue in favor of such predictability in bond markets. In contrast, in our model, return predictability can (and does) stem from publicly available information. This is due to the fact that investors make mistakes in forecasting fundamentals, and they also make higher order belief mistakes about how other investors respond to public information.

The difference between our model and work on incomplete information along this line emerges from the fact that in incomplete information settings, investors want to know others’ beliefs, which convey useful information about fundamentals. This is in contrast to our model, where investors are dogmatic and do not believe other investors’ beliefs convey useful information about fundamentals. We offer some evidence in support of our model with regards to this assumption. Returning to the Shiller/Yale ICF survey data, we find that 70% of institutional investors and 76% of individual investors report that they “must be careful not to be influenced by [other investors],” whom they believe to be either excessively optimistic or pessimistic about the stock market. This evidence is inconsistent with the underlying assumptions of incomplete information models, broadly supports our approach of assuming dogmatic beliefs, and suggests further investigation of higher order beliefs from this perspective.

Our model is conceptually closer to differences-of-opinion models, in its assumption that investors hold dogmatic beliefs. However, in differences-of-opinion models, beliefs are generally assumed to be common knowledge; investors are assumed to know other investors’ beliefs, but to agree-to-disagree with them. In our setting, investors have incorrect higher order beliefs. We find this to be plausible, given the inherent difficulty in observing others’ beliefs, or in extracting them from prices given the noise in financial markets. Our quantitative exercises suggest that incorrect higher order beliefs contribute to errors in return expectations and contribute to asset price fluctuations. In this regard, our results support models where investors may hold incorrect higher order beliefs.\(^\text{36}\)

\(^{36}\text{Han and Kyle (2018) explicitly study investors that incorrectly believe they are more optimistic than other investors. Banerjee, Kaniel and Kremer (2009) make the assumption that lack of common knowledge of others’}\)
4.2.2 Higher Order Reasoning

In our model, we collapse investors’ higher order beliefs into beliefs about a single type of agent that trades based on their fundamental beliefs, which amounts to a strong assumption about the form of investors’ higher order beliefs. Interpreted literally, our model reflects a form of $K$-level thinking, where level-1 thinkers do not take into account other investors’ behavior, level-2 thinkers assume all other investors are level-1 thinkers, level-3 thinkers assume that all other investors are level-2 thinkers, and so on.\textsuperscript{37} Investors in our model can be thought of as level-2 thinkers. But in reality, investors may engage in higher levels of reasoning. How should our results be interpreted in light of this potential higher order reasoning?

In our analysis, investors’ higher order beliefs are extracted from their return expectations, which reflect their beliefs about other investors’ risky asset demand. In principle, a given investor’s risky asset demand should reflect all levels of reasoning they engage in (e.g., their fundamental beliefs, their beliefs about others’ beliefs, etc.). Return expectations capture the aggregated effect of all higher levels of beliefs held by investors. Without additional information or identifying assumptions, return expectations cannot be used to differentiate different levels of higher order reasoning.\textsuperscript{38} Because of this, taken outside the context of our model, our discussion and analysis of higher order belief parameters reflects the aggregated effect of all higher levels of reasoning done by investors. This aggregated effect can be thought of as a higher order belief wedge, akin to the same concept studied in noisy rational expectations models.\textsuperscript{39}

In terms of asset pricing implications, for maximizing their subjective expected returns, investors do not necessarily care about the exact impact of each level of other investors’ reasoning, only the aggregated effect, which is reflected in their return expectations. Hence, subject to the assumption that investors trade based on their return expectations (as reported in survey data), our analysis captures the asset pricing implications that are implied by higher order beliefs.

\textsuperscript{37}Other work has highlighted that $K$-level thinking may be helpful to explain financial markets facts, particularly when applied to the context of inferring information from prices. Eyster, Rabin and Vayanos (2019) find evidence that the presence of level-1 thinkers, whom they describe as cursed traders, may help explain financial market trading volume. Bastianello and Fontanier (2021\textsuperscript{a},\textsuperscript{b}) find evidence that a form of level-2 thinking, which they call partial equilibrium thinking, may contribute to underreaction, overreaction, and bubble-like behavior in financial markets.

\textsuperscript{38}A similar point is made by Angeletos and La’O (2013) in the context of sentiment-driven macroeconomic fluctuations.

\textsuperscript{39}The logic of only focusing on the aggregated effect of others’ higher order reasoning (higher order belief wedges) also helps to solve dynamic rational expectations models with dispersed information and strategic complementarities, where others may engage in infinite orders of reasoning (Huo and Takayama (2021)). In asset pricing contexts, higher order belief wedges can be considered as the wedges between investors’ return expectations and their fundamental valuations of an asset. Studies that consider higher order belief wedges in noisy rational expectations settings include Bacchetta and van Wincoop (2006, 2008) and Valente, Vasudevan and Wu (2021). In these models, given the noisy expectations equilibrium assumption, higher order belief wedges are exactly pinned down by the assumed information structure of the economy.
investors’ higher order beliefs, without taking a stance on the exact microfoundation of higher order beliefs and higher order reasoning that investors engage in. A set of interesting questions, which we do not explore, is how different levels of higher level reasoning explain the form of higher order beliefs, return expectations, and asset pricing patterns we find.

5 Conclusion

Across currency and equity markets, we find that investors report hump-shaped return expectations: when investors expect high 1-quarter ahead returns, they often report expectations of low returns in subsequent quarters. The pattern of hump-shaped return expectations is especially pronounced following the arrival of news. We show that the pattern of investors’ return expectations are strongly related to their higher order beliefs, as captured by a novel Higher Order Optimism index that we construct from survey data on investors’ higher order beliefs. While these results are not consistent with the predictions of standard models, we argue that they can be reconciled by investors holding higher order beliefs that other investors update their beliefs in the direction of past news for multiple periods, as well as a belief that other investors overestimate the persistence of fundamentals.

We formalize this form of higher order beliefs in a theoretical asset pricing model. Our model illustrates a rich set of implications for higher order beliefs implied by survey data on return expectations, and also provides a tractable way to measure features of investors’ higher order beliefs using survey data. In a quantitative application to currency markets, we find that the higher order beliefs of financial institutions have a corrective influence on exchange rates, despite the fact that fundamental and higher order beliefs implied by survey data exhibit systematic biases. We find suggestive evidence that the higher order beliefs of equity market investors may contribute to destabilizing equity market valuations and to the equity market’s overreaction to news.

More broadly, our analysis provides direction for future research. One point that our paper highlights as important for further analysis is the relationship between investors’ fundamental and return expectations. Our results highlight that investors’ fundamental and return expectations should be studied in concert with one another in order to understand the asset pricing implications of investors’ beliefs. Additionally, the insight that enables us to analyze higher order beliefs in financial markets – that people’s forecasts of equilibrium outcomes embed their higher order beliefs – also suggests that readily available survey data in other settings, for example on macroeconomic outcomes, may be helpful for understanding the role that higher order beliefs play in economics and finance more generally.

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A Proofs

Proof of Lemma 1

Proof. Given the linear-Gaussian environment, belief updating follows the standard Kalman Filter recursion method derived in Hamilton (2020):

\[ d_t^S = \rho S d_{t-1}^S + \kappa S, t-1 D_t \]

\[ \kappa_{S,t-1} = \frac{\rho^2 \Sigma_{S,t-1} + \sigma^2}{\rho^2 \Sigma_{S,t-1} + \sigma^2 + \sigma^2}, \text{ and } \Sigma_{S,t} = (1 - \kappa_{S,t-1}) (\rho^2 \Sigma_{S,t-1} + \sigma^2). \]

The Kalman gain, \( \kappa_{S,t} \) and conditional variance \( \Sigma_{S,t} \) converge to the steady state values reported in the main text.

Proof of Lemma 2

Proof. Investors conjecture that the perceived pricing rule is of the form

\[ P_t = a D_t + b d_{t-1}^M + e \psi_t. \]  (A.1)

They forecast the market’s demand for the risky asset in period \( t + 1 \) as

\[ X_{M,t} = \frac{\mathbb{E}_{M,t}(P_{t+1} + D_{t+1}) - P_t}{\gamma \sigma^2_M} \]  (A.2)

\[ = \frac{(1 + a) \mathbb{E}_{M,t}(D_{t+1}) + b d_t^M - P_t}{\gamma \left( (1 + a)^2 \left( \sigma^2 + \sigma^2_{M,V} \right) + e^2 \sigma^2 \right)}. \]  (A.3)

We know that

\[ d_t^M = (1 - \kappa_M) \rho_M d_{t-1}^M + \kappa_M D_t, \text{ and } \]

\[ \mathbb{E}_{M,t}(D_{t+1}) = \rho_M d_t^M. \]  (A.4)

Imposing the perceived market clearing condition \( (X_{M,t} + \psi_t = 0) \), re-arranging in terms of \( P_t \), and matching coefficients yields solutions for the coefficients.\(^{40}\)

\(^{40}\)The existence of a perceived pricing rule requires that risk aversion, \( \gamma \), satisfies \( \gamma < \sqrt{\frac{1}{4(1+a)^2 + 4e^2 \left( \sigma^2_M + \sigma^2 \right)}} \), i.e., agents in the economy are sufficiently risk tolerant to tolerate the risks associated with perceived residual noise. Additionally, akin to the equilibrium multiplicity that arises in overlapping generations models with noise traders or noisy asset supply, there are two potential solutions for the coefficient \( e \), indicating that the perceived price impact of residual noise can be self-fulfilling. We follow the majority of the literature by presenting the unique solution that is stable to perturbations about investors’ beliefs about equilibrium; in our setting, this corresponds with a low-volatility equilibrium, where noise trading has a relatively smaller price impact. See Spiegel (1998), Bacchetta and van Wincoop (2003), Banerjee (2011), Greenwood and Vayanos (2014), and Albagli (2015) for discussions and treatments of the issue. In theory, the stochastic nature of future perceived noise trader demand
\[ a = \frac{\kappa_M \rho_M}{1 - \rho_M}, \quad (A.6) \]
\[ b = \frac{(1 - \kappa_M)^2 \rho_M^2}{\rho_M}, \quad (A.7) \]
\[ e = \frac{1 \pm \sqrt{1 - 4(1 + a_M)^2 \gamma^2 \sigma^2_{\varphi}(\sigma^2_{\psi} + \sigma^2_{\psi, d})}}{2 \gamma \sigma^2_{\varphi}}. \quad (A.8) \]

We note that \( aD_t + bd_{t-1}^M = \frac{\rho_M}{1 - \rho_M} d_t^M \), corresponding with the solution presented in the main text.

\[ \Box \]

**Proof of Proposition 1**

*Proof.* This follows immediately from Assumption 2 and Lemma 2.

\[ \Box \]

**Proof of Lemma 3**

*Proof.* Investors’ period \( t \) expectation of \( d_t^M \) is given by

\[ \mathbb{E}_{S,t}(d_{t+h}^M) = \rho_{SM}(1 - \kappa_{SM}) \mathbb{E}_{S,t}(d_{t+h-1}^SM) + \rho_{SM} \kappa_{SM} \mathbb{E}_{S,t}(d_{t+h-1}^M). \quad (A.9) \]

We can similarly compute \( \mathbb{E}_{S,t}(d_{t+j}^M) \), for \( j = 1, 2, \ldots, t + h - 1 \). Iterating and plugging into Equation (A.9), we get that

\[ \mathbb{E}_{S,t}(d_{t+h}^M) = \rho_{SM}^h(1 - \kappa_{SM})^h d_t^SM + \sum_{j=0}^{h-1} (1 - \kappa_{SM})^j \rho_{SM}^j \kappa_{SM}^{h-j} d_t^S, \quad (A.10) \]

which simplifies to Equation (10).

\[ \Box \]

**Proof of Proposition 2**

*Proof.* Using investors’ perceived pricing formula, we can see that

\[ \mathbb{E}_{S,t}(P_{t+h}) = \frac{\rho_{SM}}{1 - \rho_{SM}} \mathbb{E}_{S,t}(d_{t+h}^SM) + e_S \mathbb{E}_{S,t}(\psi_{t+1}). \quad (A.11) \]

Lemma 3 provides an expression for \( \mathbb{E}_{S,t}(d_{t+h}^M) \). Given investors’ perception of \( \psi_t \) as perfectly persistent, \( \mathbb{E}_{S,t}(\psi_{t+1}) = \psi_t^S \). Plugging in investors’ perceived pricing formula, we can also serve a dual role of capturing the influence of investors’ higher order uncertainty (uncertainty regarding the beliefs of other investors) in reduced form. However, we find that it has limited usefulness in matching the data. In our calibrations, we study the limiting case where the variance of perceived noise goes to zero.
write the expected price change component for returns as
\[ \mathbb{E}_{S,t}(P_{t+h}) - P_t = \frac{\rho_{SM}}{1 - \rho_{SM}} \left( \mathbb{E}_{S,t} \left( d_{t+h}^M \right) - d_t^S \right). \] (A.12)

Lastly, we can write investors’ expected dividends as
\[ \mathbb{E}_t \left( \sum_{k=1}^{h} D_{t+h} \right) = \sum_{k=1}^{h} \rho_t d_t^S = \frac{\rho_S(1 - \rho^h_S)}{1 - \rho_S} d_t^S. \] (A.13)

**Proof of Proposition 3**

*Proof.* Investors’ expected returns come from setting \( h = 1 \) in Equation (11). Investors’ variance of expected returns is derived using the pricing formula from the proof of proposition 2, and noting that the sources of subjective uncertainty in next period’s price are from future dividends, \( D_t \), and residual demand. Accordingly, the variance expected returns is given by \((1 + a_S)^2 \mathbb{V}_{S,t}(D_t) + \sigma_S^2 \mathbb{V}_{S,t}(\psi_{t+1})\), yielding the expression given in the main text.

Imposing the market clearing condition \((X_t + \psi_t = 0)\), re-arranging terms, and matching coefficients yields
\[
B = \frac{(1 - (1 - \kappa_{SM})\rho_{SM})\rho_S \sigma^2_\phi}{\theta(1 - \rho_{SM}) \left( (1 + a_S)^2(\sigma_{SM,\rho}^2 + \sigma^2_\phi) + e^2_\phi \sigma^2_\phi \right)} \text{, and}
\]
\[
B_M = -\frac{(1 - (1 - \kappa_{SM})\rho_{SM})\rho_SM \sigma^2_\phi}{\theta(1 - \rho_{SM}) \left( (1 + a_S)^2(\sigma_{SM,\rho}^2 + \sigma^2_\phi) + e^2_\phi \sigma^2_\phi \right)} \text{,}
\] (A.14)\( (A.15)\)

where \( a_S = \frac{\kappa_{SM} \rho_{SM}}{1 - \rho_{SM}} \).

**Proof of Proposition 4**

*Proof.* We can re-write investors’ perceived price in period \( t \) as
\[
\frac{\rho_{SM}}{1 - \rho_{SM}} d_t^S + e_S \psi_t^S. \] (A.16)

Given investors’ belief that noise follows a random walk, we can re-write investors’ expected price change as
\[
\lim_{h \to \infty} \mathbb{E}_{S,t}(P_{t+h} - P_t) = -\frac{\rho_{SM}}{1 - \rho_{SM}}. \] (A.17)
Investors’ expected dividends from holding the asset from period \( t \) in perpetuity are

\[
\mathbb{E}_{S,t} \left( \sum_{k=1}^{\infty} D_{t+k} \right) = \sum_{k=1}^{\infty} \rho_S d^S_t = \frac{\rho_S}{1 - \rho_S} d^S_t.
\]

Hence, we can write investors’ expected returns as

\[
\frac{\rho_S}{1 - \rho_S} d^S_t - \frac{\rho_{SM}}{1 - \rho_{SM}} d^{SM}_t.
\] (A.18)

\[\square\]

**Proof of Lemma 4**

*Proof.* We present the proof for (i). The proof for (ii) follows by a symmetric argument.

First we show that investors expect initially positive returns. We can write investors’ one period expected return as

\[
\mathbb{E}_{S,t}(R_{t+1}) = \rho_S d^S_t + \frac{\rho_{SM}}{1 - \rho_{SM}} (\mathbb{E}_{S,t}(d^{SM}_{t+1}) - d^{SM}_t).
\] (A.19)

We know that

\[
\mathbb{E}_{S,t}(d^M_{t+1}) = \kappa_{SM} \rho_S d^S_t + (1 - \kappa_{SM}) \rho_{SM} d^{SM}_t.
\] (A.20)

We observe a positive one period return expectation when

\[
\mathbb{E}_{S,t}(R_{t+1}) > 0
\] (A.21)

\[
\iff \rho_S d^S_t + \frac{\rho_{SM}}{1 - \rho_{SM}} (\kappa_{SM} \rho_S d^S_t + (1 - \kappa_{SM}) \rho_{SM} d^{SM}_t - d^{SM}_t) > 0
\] (A.22)

\[
\iff \left( 1 + \kappa_{SM} \frac{\rho_{SM}}{1 - \rho_{SM}} \right) \rho_S d^S_t > (1 - (1 - \kappa_{SM}) \rho_{SM}) \frac{\rho_{SM}}{1 - \rho_{SM}} d^{SM}_t
\] (A.23)

\[
\iff \rho_S d^S_t > \rho_{SM} d^{SM}_t.
\] (A.24)

Next, we show that for some period in the future \( \tilde{h} \), for \( h > \tilde{h} \), investors forecast returns to be negative.

Using Proposition 2, we can write investors’ return expectations \( h \) periods ahead as

\[
\mathbb{E}_{S,t}(R_{t+h-1, t+h}) = \frac{(1 - (1 - \kappa_{SM}) \rho_{SM}) (1 - \kappa_{SM}) \rho_{SM} \kappa_{SM} \rho_S d^S_t + (1 - \kappa_{SM}) \rho_{SM} d^{SM}_t - \rho_S d^{SM}_t - \rho_{SM} (\rho_{SM} - \rho_S) \frac{\rho_{SM}}{1 - \rho_{SM}} d^S_t}{(1 - \kappa_{SM}) (1 - \rho_{SM}) (\rho_S - (1 - \kappa_{SM}) \rho_{SM}) (1 - \kappa_{SM}) (1 - \rho_{SM}) (\rho_S - (1 - \kappa_{SM}) \rho_{SM})}.
\] (A.25)

We next proceed by cases, based on whether \( \rho_S > (1 - \kappa_{SM}) \rho_{SM} \) or \( \rho_S < (1 - \kappa_{SM}) \rho_{SM} \).
Case 1: $\rho_S > (1 - \kappa_{SM})\rho_{SM}$

The denominator of Equation (A.25) is positive, since $0 < \kappa_{SM} < 1$, $0 < \rho_{SM} < 1$, and $\rho_S > (1 - \kappa_{SM})\rho_{SM}$ (by assumption). Hence, the claim requires that the numerator is positive. In the numerator, $1 - (1 - \kappa_{SM})\rho_{SM}$ is positive. So our claim requires that

$$(\rho_{SM} - \rho_S)(1 - \kappa_{SM})\rho_S^h d_t^S > (1 - \kappa_{SM})^h \rho_{SM}^h (\kappa_{SM}\rho_S d_t^SM + (1 - \kappa_{SM})\rho_{SM}d_t^{SM} - \rho_S d_t^{SM}).$$

(A.26)

Re-arranging terms, this is equivalent to

$$(\frac{(1 - \kappa_{SM})\rho_{SM}}{\rho_S})^h < \frac{\kappa_{SM}(\rho_S d_t^S - \rho_{SM} d_t^{SM}) + (\rho_{SM} - \rho_S) d_t^{SM}}{(\rho_{SM} - \rho_S)(1 - \kappa_{SM})d_t^S}.\quad (A.27)$$

Each of the terms on the right hand side is positive. Hence, the right hand side reduces to a positive constant. The term $\frac{(1 - \kappa_{SM})\rho_{SM}}{\rho_S}$ is less than 1 (since $\rho_S > (1 - \kappa_{SM})\rho_{SM}$ by assumption), and hence, the left hand side is decreasing in $h$. As $h \to \infty$, the left hand side converges to zero. Therefore, we can find an $\bar{h}$ such that for $h > \bar{h}$, the left hand side is less than right hand side, and the forecasted expected return is negative.

Case 2: $\rho_S < (1 - \kappa_{SM})\rho_{SM}$

The denominator of Equation (A.25) is negative, since $\rho_S < (1 - \kappa_{SM})\rho_{SM}$. Hence, our claim requires that the numerator is positive, which requires that

$$(\rho_{SM} - \rho_S)(1 - \kappa_{SM})\rho_S^h d_t^S < (1 - \kappa_{SM})^h \rho_{SM}^h (\kappa_{SM}\rho_S d_t^SM + (1 - \kappa_{SM})\rho_{SM}d_t^{SM} - \rho_S d_t^{SM}).$$

(A.28)

Re-arranging terms, this is equivalent to

$$(\frac{(1 - \kappa_{SM})\rho_{SM}}{\rho_S})^h > \frac{\kappa_{SM}(\rho_S d_t^S - \rho_{SM} d_t^{SM}) + (\rho_{SM} - \rho_S) d_t^{SM}}{(\rho_{SM} - \rho_S)(1 - \kappa_{SM})d_t^S}.\quad (A.29)$$

As in the previous case, the right-hand side is a positive constant. By assumption, the $\frac{(1 - \kappa_{SM})\rho_{SM}}{\rho_S} > 1$. Hence, we can find a $\bar{h}$ such that $h > \bar{h}$ implies that the inequality holds.

Proof of Proposition 5

Proof. On average, investors’ prior beliefs satisfy $d_{t-1}^S = d_{t-1}^{SM} = 0$. Upon a shock to dividends in period $t$, $\epsilon_t$, investors update their beliefs about fundamentals to $d_t^S = \kappa_S \epsilon_t$ and believe that the market updates its beliefs about fundamentals to $d_t^{SM} = \kappa_{SM} \epsilon_t$. Lemma 4 delivers the desired results as long as

$$\rho_S \kappa_S \epsilon_t > \rho_{SM} \kappa_{SM} \epsilon_t.\quad (A.30)$$

Re-arranging terms, this happens when $\kappa_{SM} < \frac{\rho_S \kappa_S}{\rho_{SM}}$, i.e., Assumption 6.
Proof of Corollary 1

Proof. From Proposition 4, investors’ long horizon cumulative returns are

\[ \frac{\rho_S}{1 - \rho_S} d^S_t - \frac{\rho_{SM}}{1 - \rho_{SM}} d^{SM}_t. \]  

(A.31)

Re-arranging terms, we observe that Equation (A.31) is positive when \( \rho_{SM} < \frac{\rho_S}{d^S_t d^{SM}_t + \rho_S d^S_t} \) and negative when \( \rho_M > \frac{\rho_S}{d^{SM}_t d^{SM}_t + \rho_S d^{SM}_t} \). When \( \rho_S d^S_t > \rho_{SM} d^{SM}_t \), returns are initially positive, so positive long horizon cumulative returns correspond with corrective trading motivations and negative long horizon cumulative returns correspond with destabilizing trading motivations.

Proof of Corollary 2

Proof. On average, investors’ prior beliefs satisfy \( d^S_{t-1} = d^{SM}_{t-1} = 0 \). Upon a shock to dividends in period \( t, \epsilon_t \), investors update their beliefs about fundamentals to \( d^S_t = \kappa_S \epsilon_t \) and believe that the market updates its beliefs about fundamentals to \( d^{SM}_t = \kappa_{SM} \epsilon_t \). The proof follows from plugging \( d^S_t \) and \( d^{SM}_t \) into Corollary 1 and using the results from Proposition 5.
B Currency Market Model

In Section 2, we outline an asset pricing model that we use to explore the role of higher order beliefs. We modify the model in order to apply it to global currency markets. Here, we provide a detailed exposition of the currency models that we use.

B.1 Model Setup

The model is a small open economy model, featuring a Home country and an infinitesimal Foreign country. The (endogenous) log exchange rate between the two countries in period $t$ is $s_t$, expressed in units of home currency per one unit of foreign currency.

Each country offers a bond in zero net supply. Investors may take short positions (borrow) or take long positions (lend) in each of the bonds. The interest rates of the bonds are given by $i_t$ (home) and $i_t^*$ (foreign). We denote the interest rate differential between the two countries as $i^d_t = i_t^* - i_t$. Interest rate differentials follow the process

$$i^d_t = \sigma_t + \nu_t,$$  \hspace{1cm} \text{(B.1)}

$$i_t = \rho i_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma^2).$$ \hspace{1cm} \text{(B.2)}

Because the Foreign country is infinitesimal, only the Home country investors matter for the bond market equilibrium. Each period, a unit mass of investors is born. Investors born in period $t$ make an investment decision. In period $t + 1$, they liquidate their positions and consume the proceeds, pass their beliefs onto the new investors born in the period, and die.

As in the main model, investors trade against residual demand in the market, which is demand not captured by surveyed investors’ beliefs. The market clearing condition is

$$X_t + \psi_t = 0,$$ \hspace{1cm} \text{(B.3)}

where $X_t$ is investors’ demand and $\psi_t$ is residual demand.

B.1.1 Investors’ Fundamental Beliefs

Investors hold fixed beliefs about the parameters governing the interest rate differential process, which they do not update over time, and which may differ from the true parameters. In particular, investors believe the parameters $(\rho, \sigma_\sigma)$, the persistence of fundamentals and the transitory noise reflected in interest rate differentials each period, to be $(\rho_S, \sigma_{S,\sigma})$.

Investors are Bayesian. Using their beliefs about the parameters governing the interest rate differential process and their observations of past interest rate differentials, they form their expectations of $i_t$ by Kalman filtering. For ease of notation, we write investors’ belief about fundamentals in period $t$ as

$$i^S_t \equiv \mathbb{E}_{S,t}(i_t),$$ \hspace{1cm} \text{(Fundamental Beliefs)}
We follow the common assumption that a sufficient number of periods have passed such that investors are in a learning steady state. As in Lemma 1

$$i_t^S = (1 - \kappa_S)\rho_{S_t^S} + \kappa_S i_{t-1}^S. \tag{B.4}$$

### B.1.2 Investors’ Higher Order Beliefs and Return Expectations

To forecast currency excess returns, each investor forecasts the fundamental beliefs of “the market,” i.e., all other investors. Investors represent the market as a Bayesian investor, and their second order belief regarding the market’s belief about fundamental persistence and transitory noise are $(\rho_{SM}, \sigma_{SM,v})$. In period $t$, they believe the market’s belief about fundamentals to be $i_{t-1}^{SM}$.

Investors attribute the wedge between the price they observe and the price they expect given their higher order beliefs to residual demand. They perceive residual demand to be $\psi_t^S$ and believe that it follows a random walk, such that

$$\psi_{t+1}^S = \psi_t^S + \epsilon_{\psi,t}, \text{ where } \epsilon_{\psi,t} \sim N(0, \sigma_{\psi}^2). \tag{B.5}$$

Investors believe that other investors believe residual demand to be perfectly transitory Gaussian noise with variance $\sigma_{\psi}^2$.

**Proposition B.1.1 (Perceived Exchange Rate).** Investors perceive the exchange rate as

$$s_t = a_S i_t^d + b_S i_{t-1}^{SM} + e_S \psi_t^S, \tag{B.6}$$

where

$$a_S = \frac{1 - (1 - \kappa_{SM})\rho_{SM}}{1 - \rho_{SM}}, \tag{B.7}$$

$$b_S = \frac{(1 - \kappa_{SM})\rho_{SM}^2}{1 - \rho_{SM}}, \tag{B.8}$$

$$e_S = \frac{1 - \sqrt{1 - 4a_S^2\gamma^2\sigma_{\psi}^2(\sigma_{SM,v}^2 + \sigma_{\psi}^2)}}{2\gamma\sigma_{\psi}^2}. \tag{B.9}$$

The cumulative return of the exchange rate from period $t$ to period $t+h$, $R_{t,t+h}$ is defined as the sum of change in the log exchange rate and all of the intermediate interest rate differentials: $R_{t,t+h} = s_{t+h} - s_t + \sum_{k=0}^{h-1} i_{t+k}^d$.

Investors forecast currency excess returns by forecasting changes in the exchange rate and forecasting future interest rate differentials. Investors’ forecast of the interest rate differential in period $t+k$ is given by $E_t(i_{t+k}^d) = \rho_{S_t^S}^k i_t^S$. They use the perceived price in Proposition B.1.1 to forecast the future exchange rate. To do so, they also forecast what they believe the markets’ belief about fundamentals will be in period $t+h$, which, following Lemma 3, can
be expressed as

\[
\mathbb{E}_{S,t}(t_{t+h}^M) = (1 - \kappa_{SM})^h \rho_{SM}^h t_t^{SM} + \kappa_{SM} \rho_{S} \left( (1 - \kappa_{SM})^h \rho_{SM}^h - \rho_{S}^h \right) t_t^{S}. \tag{B.10}
\]

We can then derive investors’ return expectations from period \( t \) to \( t + h \).

**Proposition B.1.2 (Investors’ Return Expectations).** Investors’ cumulative expected excess return for the home currency from period \( t \) to period \( t + h \) is given by

\[
\mathbb{E}_{S,t}(R_{t,t+h}) = \mathbb{E}_{S,t} \left( s_{t+h} - s_t + \sum_{k=0}^{h-1} i_{t+k} \right) = a_S \left( \rho_{S}^h t_t^S - i_t^d \right) + b_S \left( \mathbb{E}_{S,t}(i_{t+h-1}^M) - i_{t-1}^{SM} \right) + i_t^d + \frac{\rho_{S} - \rho_{S}^h}{1 - \rho_{S}} t_t^{S}. \tag{B.11}
\]

**B.1.3 Equilibrium Exchange Rate**

Investors maximize the objective function

\[
\max_{X_{S,t}} - \mathbb{E}_{S,t}(e^{-\gamma c_{t+1}})
\]

subject to

\[
c_{t+1} = X_{S,t}(-s_{t+1} + s_t + i_t^*) + (1 - X_{S,t})(1 + i_t). \tag{B.12}
\]

Solving Equation (B.12) yields the solution for investors’ demand,

\[
X_t = \frac{\mathbb{E}_{S,t}(R_{t,t+1})}{\gamma \sigma_S^2}. \tag{B.13}
\]

where \( \sigma_S^2 \) captures their subjective belief about the variance of returns, which is given by

\[
\sigma_S^2 = a_R^2 \left( \sigma_{SM,\psi}^2 + \sigma_{\psi}^2 \right) + \sigma_S^2 \sigma_{\psi}^2.
\]

Residual demand is represented as a mass \( \theta \) of Bayesian fundamental traders. Their beliefs about fundamentals in period \( t \) are denoted as \( i_t^R \), and they believe that persistence of fundamentals and transitory noise in interest differentials are \( \rho_R \) and \( \sigma_{R,\psi} \). Their demand can then be expressed as

\[
\psi_t = \frac{\theta a_R i_t + b_R i_{t-1}^R - s_t}{\gamma \sigma_R^2}
\]

where

\[
a_R = \frac{1 - (1 - \kappa_R)\rho_R}{1 - \rho_R}, \tag{B.15}
\]

\[
b_R = \frac{(1 - \kappa_R)\rho_R^2}{1 - \rho_R}, \tag{B.16}
\]
\[ \sigma_R^2 = a_R^2 (\sigma_e^2 + \sigma_{R,e}^2). \]  
(B.17)

Imposing market clearing \((X_t + \psi_t = 0)\) allows us to derive the market clearing exchange rate.

**Proposition B.1.3 (Equilibrium Exchange Rate).** The equilibrium pricing rule for the log exchange rate is

\[ s_t = A d_t + B_S (s_{t-1}^S + \rho_R s_{t-1}^R) + B_M s_{t-1}^{SM}, \]  
(B.18)

where

\[ A = \frac{a_R^2 (\sigma_{R,e}^2 + \sigma_e^2) (a_S (\kappa_S \rho_S - 1) + b_S \kappa_S + 1)}{\theta \left( a_S^2 \left( \sigma_{SM,e}^2 + \sigma_e^2 \right) + \epsilon_e^2 \sigma_R^2 \right)} + \frac{1 - (1 - \kappa_R) \rho_R}{1 - \rho_R}, \]  
(B.19)

\[ B_S = \frac{a_S a_R^2 (1 - \kappa_S) \rho_S^2 \left( \sigma_{R,e}^2 + \sigma_e^2 \right)}{\theta \left( a_S^2 \left( \sigma_{SM,e}^2 + \sigma_e^2 \right) + \epsilon_e^2 \sigma_R^2 \right)}, \]  
(B.20)

\[ B_R = \frac{(1 - \kappa_R) \rho_R^2}{1 - \rho_R}, \]  
(B.21)

\[ B_M = - \frac{a_R^2 b_S (1 - \kappa_S \rho_M) \left( \sigma_{R,e}^2 + \sigma_e^2 \right)}{\theta \left( a_S^2 \left( \sigma_{SM,e}^2 + \sigma_e^2 \right) + \epsilon_e^2 \sigma_R^2 \right)}. \]  
(B.22)

We can use Proposition B.1.3 to estimate the returns from period \(t\) to \(t + h\) as

\[ R_{t,t+h} = A (i^d_{t+h} - i^d_t) + B_S (s_{t+h-1}^S - s_{t-1}^S) + B_R (s_{t+h-1}^R - s_{t-1}^R) + B_M (s_{t+h-1}^{SM} - s_{t-1}^{SM}) + \sum_{k=0}^{h-1} i^d_{t+k}. \]  
(B.23)

**B.1.4 Counterfactuals**

In the main text, we report results for a set of counterfactuals. To estimate the counterfactuals, we write a slightly different version of the main model. This different version of the model nests each of the counterfactuals we run for different parametrizations. The structure of the economy and investors’ fundamental beliefs are the same as before. Investors correctly recognize the mass and the fundamental beliefs of traders that comprise residual demand. However, to be able to solve the model, we assume that investors believe that the residual traders are aware of and account for investors’ presence (which they do not).

**Perceived Pricing Rule**
Investors believe the equilibrium pricing rule is
\[
s_t = a_t + b_S i_t^S + b_R i_t^R. \quad \text{(B.24)}
\]
Investor type demand satisfies
\[
X_t = \frac{E(s_{t+1}) - s_t + i_t^d}{\gamma \sigma_j^2}, \quad \text{(B.25)}
\]
where \(\sigma_j^2 = (1 + a)^2 (\sigma_{\epsilon}^2 + \sigma_{\nu,j}^2)\). Imposing market clearing \((X_t + \psi_t = 0)\) yields the solutions for the coefficients.

\[
a = -\frac{((\kappa_S - 1) \rho_S + 1) ((\kappa_R - 1) \rho_R + 1) \left( \theta \sigma_{S,\nu}^2 + \sigma_{R,\nu}^2 + (\theta + 1) \sigma_{\epsilon}^2 \right)}{Z} \quad \text{(B.26)}
\]

\[
b_S = \frac{(\kappa_S - 1) \rho_S^2 ((\kappa_R - 1) \rho_R + 1) \left( \sigma_{R,\nu}^2 + \sigma_{\epsilon}^2 \right)}{Z} \quad \text{(B.27)}
\]

\[
b_R = \frac{\theta (\kappa_R - 1) \rho_R^2 ((\kappa_S - 1) \rho_S + 1) \left( \sigma_{S,\nu}^2 + \sigma_{\epsilon}^2 \right)}{Z} \quad \text{(B.28)}
\]

\[
Z = \theta (\rho_R - 1) ((\kappa_S - 1) \rho_S + 1) \sigma_{S,\nu}^2 + (\rho_S - 1) ((\kappa_R - 1) \rho_R + 1) \sigma_{R,\nu}^2 + \sigma_{\epsilon}^2 (-\theta + \rho_S (\theta + \theta \kappa_S (\rho_R - 1) + \rho_R (-\theta + \kappa_S - 1) + 1) + \theta \rho_R - \kappa_R \rho_R + \rho_R - 1)
\]

\[\text{(B.29)}\]

**True Equilibrium Pricing Rule**

In reality, investors and traders comprising residual demand use different coefficients for the pricing rule in Equation (B.24). Residual demand can be represented using the coefficients

\[a_R = \frac{1 + (1 - \kappa_R) \rho_R}{1 - \rho_R} \quad \text{(B.30)}\]

\[b_{RS} = 0 \quad \text{(B.31)}\]

\[b_{RR} = \frac{(1 - \kappa_R) \rho_R^2}{1 - \rho_R} \quad \text{(B.32)}\]

Investors use the coefficients \(\{a_S, b_{SS}, b_{RS}\}\). We provide more details on which coefficients deliver the appropriate counterfactuals in after presenting the model.

Investors’ demand functions in the true equilibrium are the same as before, except \(E_{S,t}(s_{t+1})\) is formed using their subjective coefficients. We can derive the counterfactual exchange rate rule as

\[
s_t = A_t i_t + B_S i_{t-1}^S + B_R i_{t-1}^R \quad \text{(B.33)}
\]
where

\[
A = \frac{\theta a_S^2 \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) (b_{SS} \kappa_S + b_{RS} \kappa_R + 1)}{\theta a_S^2 \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) + a_R^2 \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right)} + \frac{\theta a_S^2 \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) (a_R \kappa_R \rho_R + b_{SR} \kappa_S + b_{RR} \kappa_R + 1) + a_S a_R^2 \kappa_S \rho_S \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right)}{\theta a_S^2 \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) + a_R^2 \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right)}
\]  

(B.34)

\[
BS = -\frac{(\kappa_S - 1) \rho_S \left( a_R^2 \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right) \right) (a_S \rho_S + b_{SS}) + \theta a_S^2 b_{SR} \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right)}{\theta a_S^2 \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) + a_R^2 \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right)}
\]  

(B.35)

\[
BR = -\frac{(\kappa_R - 1) \rho_R \left( \theta a_S^2 b_{RR} \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) \right) + a_R^2 b_{RS} \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right) + \theta a_S^2 a_R \rho_R \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right)}{\theta a_S^2 \left( \sigma_{v,S}^2 + \sigma_{e}^2 \right) + a_R^2 \left( \sigma_{v,R}^2 + \sigma_{e}^2 \right)}
\]  

(B.36)

Applying the Model to Counterfactuals

For the first counterfactual, where higher order beliefs are approximately correct, we compute the coefficients \(\{a_S, b_{RS}, b_{SS}\}\) using the estimated fundamental beliefs of investors and residual demand traders, and the appropriate value for \(\theta\). The second counterfactual, where investors have correct fundamental and higher order beliefs, is computed by computing \(\{a_S, b_{RS}, b_{SS}\}\) with the true fundamental parameters substituted for investors’ parameters. The third counterfactual corresponds with a model where \(\theta = 0\), and \(\{a_S, b_{RS}, b_{SS}\}\) are computed using investors’ fundamental belief parameters.
C Model Implied Impulse Responses

As discussed in Section 3.3, to estimate the belief parameters of surveyed investors, we match the model-implied moments pertaining to fundamental expectations, return expectations, and realized returns with the corresponding empirical moments. Here we provide additional details.

C.1 Model-Implied IRFs of Cash Flow Forecast Errors

To compute the model-implied impulse response functions, we begin with the assumption that fundamentals and priors are at their long term means in period \( t-1 \), i.e., \( d_{t-1}^S = d_{t-1} = 0 \). We consider a shock to fundamentals in period \( t \), denoted as \( \epsilon_t \).

The objective expected value of the demeaned fundamental in period \( t+h \) is given by

\[
E_t(d_{t+h}) = \rho^h \epsilon_t. \tag{C.1}
\]

Investors’ beliefs in period \( t \) about the fundamental can be expressed as

\[
d_t^S = \rho^S d_{t-1}^S + \kappa_S \epsilon_t.
\]

For any choice of parameter values \((\sigma_S, \nu, \rho_S)\), or equivalently, \((\kappa_S, \rho_S)\), we can compute the objective expected value of investors’ beliefs at time \( t+h \) by the recurrence relation

\[
E_t(d_{t+h}^S) = \kappa_S \rho^h d_t + (1 - \kappa_S) \rho_S d_{t+h-1}^S, \tag{C.2}
\]

which reduces to

\[
E_t(d_{t+h}^S) = \kappa_S (\rho^h - (1-\kappa_S) \rho^h \rho^S) d_t + (1 - \kappa_S) \rho^h \rho_S d_{t+h-1}^S
\]

\[
= \kappa_S \rho^h - (1-\kappa_S) \rho^h \rho_S \rho^h \epsilon_t, \tag{C.3}
\]

We know that the period \( t+h \) forecast of the fundamental in period \( t+h+k \) is given by \( \rho_S^k d_{t+h}^S \), and accordingly the model-implied \( k \)-period ahead forecast error is

\[
FE_{t+h,t+h+k} = E_t(d_{t+h+k}) - E_{t+h}^S(d_{t+h+k})
= \rho^h \rho_S^k d_t - \rho_S^k \rho_S d_{t+h}
= \frac{\rho^h \rho_S^k - \rho^h (\rho^k + (\rho - \rho^k) \kappa_S) \rho_S + (1 - \kappa_S) \rho^h \rho_S \kappa_S \rho_{k+2}}{\rho - (1 - \kappa_S) \rho_S} \epsilon_t, \tag{C.4}
\]

Then, for any choice of parameters, we can use Equation (C.4) for different horizons to compute impulse response functions. The coefficient on \( \epsilon_t \) corresponds with the impulse response \( h \) periods in the future corresponding with a one standard deviation shock. We can use the computed impulse response functions to compute \( \Theta \) and in turn solve the objective function in Equation (19).

\footnote{This is the same computation as used in Lemma 3.}
C.2 Model-Implied Return Expectations

To compute model moments corresponding with model-implied return expectations, we once again start with the assumption that the demeaned fundamental and prior beliefs are at their long term means in period $t - 1$, i.e., $d_{t-1}^S = d_{t-1}^{SM} = d_{t-1}^R = 0$. We assume that there is a shock to the fundamental in period $t$, $\epsilon_t$, and that the period $t$ transitory shock is zero.

Given parameters about investors’ beliefs (estimated in Step 1 of the calibration procedure), and for a given set of higher order belief parameters about the market, we can compute the period $t$ using Proposition 1 as

$$P_t = a_S \epsilon_t.$$  \hfill (C.3)

Furthermore, we also know that $d_{t}^S = \kappa_S \epsilon_t$ and $d_{t}^{SM} = \kappa_{SM} \epsilon_t$. Investors’ period $t + 1$ expectations of $d_{t+1}^{SM}$ is given by

$$E_{S,t} \left( d_{t+1}^{SM} \right) = \rho_S \kappa_{SM} \left( \rho^h_S - (1 - \kappa_{SM})^h \rho^h_{SM} \right) d_t^S + (1 - \kappa_{SM})^h \rho^h_{SM} d_t^{SM}$$

$$= \kappa_{SM} \left( (1 - \kappa_{SM})^h \rho^h_{SM} + \frac{\kappa_S \rho_S (\rho^h_S - (1 - \kappa_{SM})^h \rho^h_{SM})}{\rho_S - (1 - \kappa_{SM})^h \rho^h_{SM}} \right) \epsilon_t. \hfill (C.5)$$

Given the belief parameters, we can also compute price expectations for period $t + h$ as

$$E_{S,t} (P_{t+h}) = E_{S,t} \left( \frac{\rho_{SM}}{1 - \rho_{SM}} \right) E_{S,t} \left( d_{t+h}^M \right). \hfill (C.6)$$

Finally, we can write the return expectations as

$$E_{S,t} (R_{t,t+h}) = E_{S,t} (P_{t+h}) - E_{S,t} (P_t) + \sum_{i=1}^{h} E_{S,t} (D_{t+i}) \hfill (C.7)$$

For a given choice of parameters, we can compute Equation (C.7) as the cumulative returns from period $t$ to period $t + h$ by substituting the relevant expressions.

C.3 Model-Implied Returns

To compute model moments corresponding with model-implied returns, we start with the assumption that in period $t - 1$, beliefs and higher order beliefs are at their long term means, i.e., $d_{t-1}^S = d_{t-1}^{SM} = d_{t-1}^R = 0$. Assume that in period $t$ there is a shock to fundamentals, such that $D_t = \epsilon_t$.

Given these assumptions, we know for $j \in \{ S, SM, R \}$ that $d_{t}^{j} = \kappa_{j} \epsilon_t$. For a given choice of parameters, this is sufficient to estimate the market clearing price in period $t$ using Equation (16). Following the computations for Equation (C.3), we can compute the objective

\footnote{Under the assumption of perfectly persistent perceived residual demand in the pricing formula, residual demand does not matter for return expectations, so we omit it here for simplicity. It is also possible to derive model-implied return expectations for different perceived persistence of residual demand, using a similar approach to the one that we take here.}
expected value of $d_{t+h}^j$ as

$$\mathbb{E}_t\left(d_{t+h}^j\right) = \kappa_j \frac{\rho^{h+1} - (1 - \kappa_j)^{h+1} \rho_j^{h+1}}{\rho - (1 - \kappa_j) \rho_j} \epsilon_t,$$

(C.8)

$$= \frac{\rho^{h+1} - (1 - \kappa_j)^{h+1} \rho_j^{h+1}}{\rho - (1 - \kappa_j) \rho_j} \epsilon_t.$$

(C.9)

Given belief parameters, we can compute the objective expected returns for period $t + h$ as

$$\mathbb{E}_t(R_{t,t+h}) = \mathbb{E}_t(P_{t+h}) - \mathbb{E}_t(P_t) + \sum_{i=1}^{h} \mathbb{E}_t(D_{t+i})$$

$$= B \left( \mathbb{E}_t(d_{t+h}^S) - d_t^S \right) + B_M \left( \mathbb{E}_t(d_{t+h}^{SM}) - d_t^{SM} \right)$$

$$+ \mathbb{E}_t(\phi_{t+h}) - \phi_t$$

$$+ \sum_{i=1}^{h} \mathbb{E}_t(D_{t+i}).$$

(C.10)
D  Data And Additional Empirical Analysis

D.1  Summary Statistics and Details of Return Expectations

Here, we present details on unconditional return expectations reported in survey data, and document that investors often report forecasts of returns that are different signs at short and long horizons. We then show additional evidence that this pattern is particularly pronounced following the arrival of news.

D.1.1  Shiller / Yale ICF Survey

Table D.2 reports statistics on the return expectations of individual and institutional investors in the Shiller survey data. The first two rows of the table report the mean and median of forecasters’ return expectations over the full sample. With the exception of individual investors’ mean 1-month return expectation (-0.1%), the mean and median return expectations are positive for all horizons. Return expectations are increasing in horizon, and are broadly consistent with investors expecting an annualized expected return of 4% to 7%. On average, investors report higher expected returns further in the future; for example, the mean and median 1-year and annualized 10-year return expectations are more than twice as large as the mean and median 6-month return expectations. That is, investors do not simply appear to report return expectations reflecting a constant risk premium for different future horizons.\(^{43}\)

The fourth row in the table reports the proportion of observations for a given horizon for which a forecaster reports a return expectation with a different sign than the sign of their reported 1-month return expectation. For example, the second column indicates that 28.8% of individual investor respondents report 3-month expected returns that have different signs than 1-month expected returns. The proportion of investors with differently signed return expectations than their 1-month expected returns is increasing in horizon and is broadly consistent for individual and institutional investors, ranging from approximately 30% for 3-month returns to approximately 50% for 1-year and 10-year expected returns. This is evidence that investors have hump-shaped return expectations. Nearly half the time, they report expectations of short horizon returns that are differently signed than their expectations of longer horizon returns.

The fifth row of the table reports the proportion of respondents that answered yes to questions (ii) or (iii), indicating either that they expect a substantial drop in stock prices eventually, but that they advise being heavily invested in stocks because they believe prices will rise for a while, OR that they expect a substantial rise in stock prices eventually, but that they advising being less invested in stocks because they believe prices will drop for a while.

\(^{43}\)The lower expectations for stock returns in the near-term versus the far-term are also consistent with the findings of Giglio et al. (2021), who report an average expected 1-year stock market return of 4.64% and average expected 10-year stock market return of 6.64% in a survey of Vanguard customers from February 2017 to June 2020.
The proportions are 60.1% for individual investors and 54.7% for institutional investors, which are in line with the proportion of investors with different sign short and long horizon return expectations.

The results from the fourth and fifth rows of Table D.2 are striking. They indicate that more than half the time, investors report expectations of short- and long horizon returns that have opposite signs, and advise taking over- or under-weight positions in the market that are opposite to their long horizon return expectations.

The sixth row of Table D.2 reports the median number of days to the market trough (for investors that answered “yes” in response to question (ii) indicating that they believed the market would underperform before rebounding) or the market peak (for investors that answered “yes” in response to question (iii) indicating that they believe market would outperform before reverting). Because investors that answer yes to questions ii and iii advise taking positions opposite to their long term return expectations, the numbers reported in the sixth row can be thought of as an upper bound for the median frequencies at which surveyed investors rebalance their portfolios. We compute a value of 242 days for individual investors and 202 days for institutional investors; these numbers suggest an upper bound of a little more than two quarters for investors’ rebalance frequencies, and suggest that investors’ short horizon return expectations influence their trading behavior, even when investors’ short and long horizon return expectations appear to differ.

D.1.2 Trailing 1-Month Returns and Hump-Shaped Returns in Equity Markets

Table D.1 reports results from regressions of return expectations on 30-day trailing returns. Individual investors report a pronounced hump-shape in their return expectations in response to one standard deviation 30-day trailing returns, with coefficients of 87 basis points, 80 basis points, -31 basis points, and -35 basis points for 1-month, 3-month, 3- to 6-month, and 6- to 12-month return expectations. Institutional investors also report a pronounced hump, with coefficients of 36 basis points, 24 basis points, -37 basis points, and -38 basis points. The evidence strongly supports the presence of hump-shaped return expectations. Taken in concert with the evidence presented in the main text, the regression evidence in Table D.1 indicates that equity market investors’ hump-shaped returns particularly emerge in response to recent past news.
<table>
<thead>
<tr>
<th></th>
<th>Individual Investors</th>
<th></th>
<th>Institutional Investors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 Month</td>
<td>3 Month</td>
<td>6 Month</td>
<td>1 Year</td>
</tr>
<tr>
<td>Mean Expected Return</td>
<td>-0.1%</td>
<td>0.7%</td>
<td>1.6%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Median Expected Return</td>
<td>0.5%</td>
<td>1.0%</td>
<td>2.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Different Sign than 1M</td>
<td>0.0%</td>
<td>28.8%</td>
<td>44.6%</td>
<td>50.1%</td>
</tr>
<tr>
<td>Different Advice than LT</td>
<td>60.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median Days to Peak/Trough</td>
<td>242 Days</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table D.2: Multi-Horizon Return Expectations: Shiller/Yale ICF Survey**

*Note:* The table presents statistics on return expectations reported in the Shiller/Yale ICF Survey Data. The first two rows of the table report the mean and median of return expectations over different return horizons reported in surveys. The first five columns of the table correspond with expectations reported by individual investors, while the last five columns correspond with expectations reported by institutional investors. The fourth row in the table reports the proportion of observations for which the sign of return expectations at a given horizon differs from the sign of return expectations reported for 1-month expected returns. The fifth row in the table reports the proportion of survey respondents indicating either (ii) that they advise being heavily invested in stocks despite eventually expecting a substantial drop in stock prices, because they believe short horizon returns will be high OR (iii) that they advise being less invested in stocks despite eventually expecting a substantial rise in stock prices, because they believe short horizon returns will be low. The sixth row reports the median number of days that survey respondents report until their expectation of the market peak (if they responded yes to (ii)) or market trough (if they responded yes to (iii)).
<table>
<thead>
<tr>
<th></th>
<th>$E_{S,t} (r_{t,t+1})$</th>
<th>$E_{S,t} (r_{t,t+3})$</th>
<th>$E_{S,t} (r_{t+3,t+6})$</th>
<th>$E_{S,t} (r_{t+6,t+12})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Investors</td>
<td>0.87</td>
<td>0.80</td>
<td>-0.31</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.13)</td>
<td>(0.08)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Institutional Investors</td>
<td>0.36</td>
<td>0.24</td>
<td>-0.37</td>
<td>-0.38</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.17)</td>
<td>(0.13)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>All Shiller Respondents</td>
<td>0.43</td>
<td>0.27</td>
<td>-0.35</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.14)</td>
<td>(0.10)</td>
<td>(0.14)</td>
</tr>
</tbody>
</table>

**Table D.1: Shiller/Yale ICF Expected Returns in Response to Past Month Returns**

*Note:* The table reports results from regressions of survey respondents’ return expectations over different horizons on trailing 30-day returns. $E_{S,t}(r_{t+h,t+h+k})$ represents a survey respondent’s $k$-month return expectation for $h$ months in the future. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation change in the independent variable. Standard errors are clustered by quarter and reported in parentheses.

### D.1.3 FX4casts Currency Return Expectations

Table D.3 reports statistics on currency return expectations in the FX4casts survey data. Positive numbers indicate positive returns to borrowing in USD and investing in foreign currency. The first five columns in the table report statistics corresponding with 1-, 3-, 6-, 12-, and 24-month expected excess returns. The last four columns, labeled $E_{S,t}(r_{t+h,t+h+k})$, report statistics corresponding with the period $t$ expectation of returns from period $t + h$ to period $t + h + k$, where $h$ and $k$ are in months. The first two rows of the table report the mean and median of expected returns. Across all horizons, expected excess returns are close to zero.

The last two rows of Table D.3 report the proportion of observations for which expected returns of a given horizon have a different sign than the 1- and 3-month expected returns. 3-, 6-, 12-, and 24-month return expectations have different signs than 1-month return expectations 12.1, 20.3, 24.2, and 34.3% of the time. 6-, 12-, and 24-month expected returns have different signs that 3-month expected returns 13.9%, 20.4%, and 30.9% of the time. The values are even more striking when we look at the last four columns. The 1- to 3-month ahead return expectations (the 2-month expected returns for 1 month in the future) have different signs than 1-month return expectation 30.7% of the time. The 3- to 6-month, 6- to

---

44The data on 1-month return expectations begin only in 2008, whereas 3-month return expectations are available over the full sample.
12-month, and 12- to 24-month ahead return expectations have different signs than 1-month return expectations 30.7%, 30.7%, 35.7%, and 51.2% of the time, and different signs than the 3-month return expectations 24.0%, 37.2%, and 50.2% of the time.

As in surveys of equity market expectations, survey respondents forecasting currency excess returns often report expected returns at short horizons and long horizons with different signs, providing additional evidence of the consistency of patterns of return expectations across markets.
<table>
<thead>
<tr>
<th></th>
<th>1 Month</th>
<th>3 Month</th>
<th>6 Month</th>
<th>12 Month</th>
<th>24 Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Expected Return</td>
<td>-0.3%</td>
<td>-0.1%</td>
<td>-0.1%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Median Expected Return</td>
<td>-0.3%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>-0.1%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Different Sign than 1M</td>
<td>0.0%</td>
<td>12.1%</td>
<td>20.3%</td>
<td>24.2%</td>
<td>34.3%</td>
</tr>
<tr>
<td>Different Sign than 3M</td>
<td>12.1%</td>
<td>0.0%</td>
<td>13.9%</td>
<td>20.4%</td>
<td>30.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{S,t}(r_{t+1,t+3})$</td>
<td>-0.2%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.9%</td>
<td></td>
</tr>
<tr>
<td>$E_{S,t}(r_{t+3,t+6})$</td>
<td>-0.1%</td>
<td>0.0%</td>
<td>0.2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{S,t}(r_{t+6,t+12})$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{S,t}(r_{t+12,t+24})$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE D.3: Multi-Horizon Return Expectations of Currency Excess Returns**

*Note:* The table presents statistics on return expectations over different horizons for various currencies versus the USD over different horizons, where a negative number indicates positive returns to borrowing in foreign currency and investing in USD. The first two rows report the mean and median of return expectations over different return horizons reported in surveys. The first five columns correspond with cumulative return expectations for 1, 3, 12, and 24 month return expectations. The last four columns, labeled $E_{S,t}(r_{t+h,t+h+k})$, correspond with expectations of returns from $h$ to $h+k$ months in the future. The last two rows in the panel report the proportion of observations for which the sign of the average return expectation of a given return horizon differs from the average expectation of the reported 1 and 3 month return expectations of the same period. The sample consists of survey responses from August 1986 through December 2019, except for calculations involving 1- and 24-month return expectations, for which the sample starts in January 2008.
D.2 Higher Order Optimism Indices

In this section, we present additional evidence on the Higher Order Optimism indices described and studied in Section 1.3.

First, Figure D.1 plots the Higher Order Optimism indices constructed using individual and institutional investor survey responses. The two indices are strongly correlated with one another (0.49), and the indices also each exhibit strong correlations with the pooled Higher Order Optimism index plotted in Figure 2 (correlation of 0.87 for the individual investor index and 0.83 for the institutional investor index). While the individual and institutional investor indices are correlated with one another, they also do display some differences from one another. For example, during the tech bubble, a greater proportion of institutional investors believe that others are overly optimistic than individual investors. In 2007, in the lead up to the financial crisis, a greater proportion of individual investors believed others to be overly optimistic than institutional investors.

![Figure D.1: Higher Order Optimism Indices](image)

**Note:** The figure plots the Higher Order Optimism index, which, in a given month, is measured as the difference between the proportion of investors in the Shiller surveys reporting a belief that other investors are overly optimistic about the stock market’s prospects and the proportion of investors reporting a belief that other investors are overly pessimistic about the stock market’s prospects. The figure plots separate indices constructed using survey responses from individual and institutional investors.

Second, we separately compute and plot the proportion of investors in the Shiller surveys reporting a belief that other investors are overly optimistic and the proportion of investors in the Shiller surveys reporting a belief that other investors are overly pessimistic. Figure D.2 plots the proportion of investors that believe others to be overly optimistic about the stock market. On average, a little more than 50% of investors believe other investors to be overly optimistic, and the time-series for individual and institutional investors are 0.43 correlated. Figure D.3 plots the proportion of investors that believe others to be overly pessimistic about the stock market. A little more than 50% of investors believe others to
be overly pessimistic, and the time-series for individual and institutional investors are 0.27 correlated.

**Figure D.2: Higher Order Optimism**

*Note:* The figure plots the proportion of investors in the Shiller surveys reporting a belief that other investors are overly optimistic about the stock market’s prospects in a given month. The figure plots results separately for individual and institutional investors, and also plots results pooling together all survey respondents.

**Figure D.3: Higher Order Pessimism**

*Note:* The figure plots the proportion of investors in the Shiller surveys reporting a belief that other investors are overly pessimistic about the stock market’s prospects in a given month. The figure plots results separately for individual and institutional investors, and also plots results pooling together all survey respondents.

Third, we analyze the relationship between the Higher Order Optimism indices and measures of past news. Table D.4 displays correlations of the Higher Order Optimism indices, constructed to be quarterly, with lagged AR(1) GDP growth innovations and lagged
<table>
<thead>
<tr>
<th></th>
<th>Individual</th>
<th>Institutional</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Past Quarter GDP Growth</td>
<td>0.33</td>
<td>0.42</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.10)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Past Quarter Returns</td>
<td>0.54</td>
<td>0.48</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.08)</td>
<td>(0.10)</td>
</tr>
</tbody>
</table>

**Table D.4: Higher Order Optimism Indices and News**

*Note:* The table displays the correlations of quarterly AR(1) GDP growth innovations and stock market returns with the next quarter’s Higher Order Optimism indices, which, in a given quarter, are measured as the difference between the proportion of investors in the Shiller surveys reporting a belief that other investors are overly optimistic about the stock market’s prospects and the proportion of investors reporting a belief that other investors are overly pessimistic about the stock market’s prospects. Newey-West standard errors are reported in parentheses.

Quarterly stock market returns. The individual, institutional, and pooled Higher Order Optimism indices are 0.33, 0.42, and 0.40 correlated with lagged GDP growth innovations, and 0.54, 0.48, and 0.55 correlated with the previous quarters’ stock market news. That is, investors’ beliefs in other investors’ over-optimism tend to precisely follow strong past market returns and strong macroeconomic news. The evidence is consistent with a belief that other investors overreact to the arrival of news.
D.3 Return Expectations and Investor Behavior

An important question is whether the measures of survey-based return expectations that we use actually relate to investor behavior. And to the extent that there is a relationship, given that our results suggest that investors exhibit hump-shaped return expectations over future horizons, a relevant question is which horizon return expectations appear relevant for investors’ trading behavior. In both equity and currency markets, we find that return expectations – and particularly short horizon return expectations – are highly positively related to measures of investors’ asset demand.

D.3.1 Equity Markets

The first way we evaluate these questions is using data on fund flows. We obtain data on fund flows into funds benchmarked to the S&P500 from Morningstar Direct. We construct two measures of fund flows: the first measure is monthly flows into open-end funds and ETFs benchmarked to the S&P500, normalized by the net asset value (NAV) of those funds in the previous month. The second is a measure of daily flows, computed as daily flows into funds normalized by the daily NAV of such funds. We compute rolling 10-(week)day and 21-day sums of this daily measure (corresponding broadly with 2-week to 4-week flows). The sample for the monthly measure runs from 2000-2019, while the sample for the daily measure runs from 2007 to 2019.

We first run a contemporaneous regression of our monthly flow measure on return expectations of different horizons. The regression is of the form

\[
\text{flow}_t = \alpha + \beta_1 \bar{E}_t(r_{t+1}) + \beta_2 \bar{E}_t(r_{t+3}) + \beta_3 \bar{E}_t(r_{t+6}) + \beta_4 \bar{E}_t(r_{t+12}) + \epsilon_t, \quad (D.1)
\]

where \(\text{flow}_t\) is monthly flows into S&P500 benchmarked mutual funds in month \(t\), \(\bar{E}_t(r_{t+h})\) is the average expectation reported in month \(t\) for returns from month \(t + h\) to month \(t + h + k\). Independent and dependent variables are standardized to have zero mean and unit standard deviation so that coefficients in the regressions can be interpreted as conditional correlation coefficients.

The first column of Table D.5 reports results from the regression, pooling observations across individual and institutional investors together. For 1-month, 1- to 3-month ahead, 3- to 6-month ahead, and 6- to 12-month ahead returns, the coefficients in the regression are 0.18 (\(t\)-statistic of 2.5), 0.15 (\(t\)-statistic of 1.7), 0.19 (\(t\)-statistic of 2.0), and 0.15 (\(t\)-statistic of 1.9). The second column and third columns in the table reports the same coefficients, separating the sample into individual and institutional investors. The coefficients for individual

---

45Previous work does suggest a connection, more broadly, between survey data on return expectations and investor behavior. For example, Greenwood and Shleifer (2014) find that fund flows are highly correlated with survey-based measures of return expectations in equity markets. Stavrakeva and Tang (2020) find that forecasted exchanged rate returns, as captured by Consensus Economics, correspond with investors’ futures positions in the CFTC Traders in Financial Futures and Commitments of Traders reports.
Table D.5: S&P 500 Monthly Flows and Shiller Survey Return Expectations

Note: The table reports results from regressions of the form \( \text{flows}_t = \alpha + \beta_1 \bar{E}_t(\text{rt}_{t+1}) + \beta_2 \bar{E}_t(\text{rt}_{t+1,t+3}) + \beta_3 \bar{E}_t(\text{rt}_{t+3,t+6}) + \beta_4 \bar{E}_t(\text{rt}_{t+6,t+12}) + \epsilon_t \), where flows\(_t\) are monthly flows into S&P 500 benchmarked funds normalized by the net asset value of the funds in the previous month, and \( \bar{E}_t(\text{rt}_{t+h,t+h+k}) \) are the average return expectations reported in period \( t \) for period \( t + h \) to \( t + h + k \) in the Shiller / Yale ICF survey data. The first column in the table reports results for a pooled sample of individual and institutional investor respondents to the Shiller / Yale ICF surveys, and the second and third columns report results separating the individual and institutional investor respondents. The sample runs from 2000 through 2019. t-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Individual</th>
<th>Institutional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{E}<em>t(\text{rt}</em>{t+1}) )</td>
<td>0.18</td>
<td>0.44</td>
<td>0.11</td>
</tr>
<tr>
<td>(2.50)</td>
<td>(6.56)</td>
<td>(1.62)</td>
<td></td>
</tr>
<tr>
<td>( \bar{E}<em>t(\text{rt}</em>{t+1,t+3}) )</td>
<td>0.15</td>
<td>0.52</td>
<td>0.07</td>
</tr>
<tr>
<td>(1.74)</td>
<td>(7.11)</td>
<td>(0.98)</td>
<td></td>
</tr>
<tr>
<td>( \bar{E}<em>t(\text{rt}</em>{t+3,t+6}) )</td>
<td>0.19</td>
<td>0.33</td>
<td>0.06</td>
</tr>
<tr>
<td>(2.00)</td>
<td>(5.34)</td>
<td>(0.76)</td>
<td></td>
</tr>
<tr>
<td>( \bar{E}<em>t(\text{rt}</em>{t+6,t+12}) )</td>
<td>0.15</td>
<td>0.24</td>
<td>0.14</td>
</tr>
<tr>
<td>(1.91)</td>
<td>(4.30)</td>
<td>(1.80)</td>
<td></td>
</tr>
</tbody>
</table>

Investors, in the same order, are 0.44 (t-statistic of 6.6), 0.52 (t-statistic of 7.1), 0.33 (t-statistic of 5.3), and 0.24 (t-statistic of 4.3), while the coefficients for institutional investors are 0.11 (t-statistic of 1.6), 0.07 (t-statistic of 1.0), 0.06 (t-statistic of 0.8), and 0.14 (t-statistic of 1.8). The evidence indicates that return expectations, especially for individual investors, are related to contemporaneous fund flows. The contemporaneous relationship is especially strong for short horizon return expectations, but is also present when looking at longer horizon return expectations as well.

We next run regressions of 10-day ahead and 21-day ahead flows on reported return expectations, where we use individual level observations (rather than monthly averages, as above). In particular, we measure flows from the day following the reported forecast date, until 10- or 21-days later. Regressions are of the form

\[
\text{flows}_{S,t+t+j} = \alpha + \beta_1 \bar{E}_{S,t}(\text{rt}_{t+1}) + \beta_2 \bar{E}_{S,t}(\text{rt}_{t+1,t+3}) + \beta_3 \bar{E}_{S,t}(\text{rt}_{t+3,t+6}) + \beta_4 \bar{E}_{S,t}(\text{rt}_{t+6,t+12}) + \epsilon_{S,t+t+j},
\]  

(D.2)

where \( \text{flows}_{S,t+j} \) are the flows into S&P500-benchmarked funds in the \( j \) days following forecast observation \( S \) in period \( t \), and \( \bar{E}_{S,t+r_{t+h,t+h+k}} \) is the return expectation for returns from month \( t + h \) to \( t + h + k \) reported in survey observation \( i \). Standard errors in the regressions are clustered by month. Relative to the monthly regressions, these regressions have the advantage of tightly linking return expectations with flows occurring in the immediately
The first column of Table D.6 reports results for the regression using 10-day ahead flows as the independent variable, pooling observations across individual and institutional investors together. For 1-month, 1- to 3-month ahead, 3- to 6-month ahead, and 6- to 12-month ahead return expectations, the coefficients in the regression are 0.12 (t-statistic of 2.6), 0.11 (t-statistic of 2.2), -0.00 (t-statistic of -0.1), and -0.03 (t-statistic of -1.8). The second and third columns of the table report results separating individual and institutional investors. For individual investors, the coefficients for 1-month, 1- to 3-month ahead, 3- to 6-month ahead, and 6- to 12-month ahead return expectations are 0.17 (t-statistic of 2.18), 0.14 (t-statistic of 1.86), -0.01 (t-statistic of -0.36), and -0.01 (t-statistic of -0.87). For institutional investors, the respective coefficients are 0.06 (t-statistic of 2.16), 0.05 (t-statistic of 1.30), 0.00 (t-statistic of 0.00), -0.05 (t-statistic of -1.54).

The fourth column of the table reports results for the regression using 21-day ahead flows as the independent variable, once again pooling observations across individual and institutional investors together. For 1-month, 1- to 3-month ahead, 3- to 6-month ahead, and 6- to 12-month ahead returns, the coefficients in the regression are 0.10 (t-statistic of 1.9), 0.10 (t-statistic of 2.1), 0.01 (t-statistic of 0.4), and -0.02 (t-statistic of -1.2), and the results are broadly similar for similar for individual and institutional investors. The evidence once again suggests an especially strong relationship between investors’ short horizon return expectations and fund flows.

Finally, we analyze how investors’ futures positions are related to return expectations in the Shiller / Yale ICF survey data. We obtain weekly data on the positions of investors in S&P500 equity index futures from the Traders in Financial Futures report, published weekly by the CFTC since 2010. The report classifies investors into four different categories, based on self-reported business purposes reported to the CFTC: futures dealers, levered funds (who can be thought of as hedge funds), institutional asset managers, and other. Hazelkorn, Moskowitz and Vasudevan (2022) show that dealers largely appear to take the other side of other investors’ demands in equity index futures markets. They show that the positions of futures dealers are negatively related to fund flows and investor demand for equity market exposure, while the futures positions of hedge funds and institutional asset managers are positively related to fund flows and investor demand for equity market exposure. Following the previous literature using these data, we consider an investor type’s net positioning for a given week as the number of long contracts minus the number of short contracts, normalized by open interest.

To capture the relationship between survey-based return expectations and investor future positions, for forecast observation $i$ in period $t$, we run regressions of the form

$$\text{Net Positioning}_{j,t+1} = \alpha_j + \beta_1 \mathbb{E}_{S,t}(r_{t,t+1}) + \beta_2 \mathbb{E}_{S,t}(r_{t+1,t+3}) + \beta_3 \mathbb{E}_{S,t}(r_{t+3,t+6}) + \beta_4 \mathbb{E}_{S,t}(r_{t+6,t+12}) + \epsilon_{j,t+1}, \quad (D.3)$$
<table>
<thead>
<tr>
<th></th>
<th>10-Day Rolling Flows</th>
<th>21-Day Rolling Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Individual</td>
</tr>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{t,t+1})$</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(2.60)</td>
<td>(2.18)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{t+1,t+3})$</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(2.16)</td>
<td>(1.86)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{t+3,t+6})$</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(-0.10)</td>
<td>(-0.36)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{t+6,t+12})$</td>
<td>-0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(-1.77)</td>
<td>(-0.87)</td>
</tr>
</tbody>
</table>

**Table D.6: S&P 500 Flows and Shiller Survey Return Expectations**

*Note:* The table reports results from regressions of the form $\text{flow}_{S,J+1} = \alpha + \beta_1 \mathbb{E}_{S,t}(r_{t,t+1}) + \beta_2 \mathbb{E}_{S,t}(r_{t+1,t+3}) + \beta_3 \mathbb{E}_{S,t}(r_{t+3,t+6}) + \beta_4 \mathbb{E}_{S,t}(r_{t+6,t+12}) + \epsilon_{S,t+1}$, where $\text{flow}_{S,J+1}$ are the flows into S&P500-benchmarked funds in the $j$ days following forecast observation $S$ in period $t$, and $\mathbb{E}_{S,t}(r_{t+h,t+h+k})$ is the return expectation for returns from month $t + h$ to $t + h + k$ reported in survey observation $S$ in the Shiller / Yale ICF survey data. The first three columns correspond with $j = 10$ rolling business days in the calculation of flows, and the last three columns correspond with $j = 21$ rolling weekdays in the calculation of flows. The table reports statistics for regressions pooling individual and institutional investors together, as well as statistics separating the two samples. Standard errors are clustered by month and $t$-statistics are reported in parentheses.
where $\text{Net Positioning}_{j,t+1}$ are the futures positions reported for investor type $j$ in the period following the forecast, and $E_t(r_{i,t+h:t+h+k})$ are the return expectations by forecaster $i$ for period $t+h$ to $t+h+k$ reported in the Shiller / Yale ICF survey data. The independent and dependent variables are standardized to have zero mean and unit standard deviation, so that coefficients can broadly be interpreted as conditional correlation coefficients. Standard errors are clustered by month, and $t$-statistics are reported in parentheses.

Table D.7 reports regression results pooling together individual and institutional investors’ expectations in the Shiller survey data. The first column reports results where the dependent variable is dealer net positioning. The coefficient for 1-month return expectations is $-0.12$ ($t$-statistic of $-3.34$), and the coefficient for 1- to 3-month return expectations is $-0.08$ ($t$-statistic of $-0.08$), while the $t$-statistic for return expectations from 3- to 6-months ahead and 6- to 12-months ahead is approximately zero. The regression evidence captures that equity market demand, as measured by (negated) dealers’ futures positioning, is strongly related to short horizon return expectations. The next three columns capture the relationship between investors classified as levered funds (who can be thought of as hedge funds), institutional asset managers, and other. The columns indicate that the positioning of institutional asset managers most closely reflects investors’ short horizon return expectations, with coefficients of $0.15$ ($t$-statistic of $3.36$) and $0.06$ ($t$-statistic of $1.41$) for 1-month and 1-to 3-month ahead return expectations. Interestingly, levered funds’ futures positioning, while not related to short horizon return expectations in the survey data, appears to be negatively related to 3- to 6-month and 6- to 12-month return expectations with coefficients $-0.05$ on both ($t$-statistics of $-2.05$ and $-1.91$).

Tables D.8 and D.9 report the regression results, broken down separately into the individual and institutional investor samples. The two tables yield largely the same conclusions as Table D.7 – equity market demand, as captured by negated dealers’ futures positions, appears to be highly related to short horizon return expectations; the source of that demand seems to come primarily from institutional asset managers. As highlighted by Hazelkorn, Moskowitz and Vasudevan (2022), such patterns are consistent both with fund managers using futures in order to increase or decrease their equity exposure in response to inflows or outflows from retail investors, as well with fund managers making strategic asset allocation decisions. In either case, it appears that investors’ short horizon return expectations are particularly important for determining equity market demand captured in futures positions.

Taken all together, the results provide strong evidence that investors’ return expectations, as captured by the Shiller / Yale ICF survey data, are strongly related to demand for equity market exposure, as captured by measures of fund flows and investors’ futures positioning. Moreover, it seems that investors’ short horizon return expectations are especially related to measures of equity market demand, while their longer horizon expectations are less important.
<table>
<thead>
<tr>
<th>$\mathbb{E}<em>S(t</em>{i,t+1})$</th>
<th>Dealer</th>
<th>Levered</th>
<th>Institutional</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.12</td>
<td>0.02</td>
<td>0.15</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(-3.34)</td>
<td>(0.51)</td>
<td>(3.36)</td>
<td>(0.81)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>S(t</em>{i,t+1,t+3})$</td>
<td>-0.08</td>
<td>0.02</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(-1.87)</td>
<td>(0.86)</td>
<td>(1.46)</td>
<td>(1.29)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>S(t</em>{i,t+3,t+6})$</td>
<td>-0.01</td>
<td>-0.05</td>
<td>-0.01</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(-0.38)</td>
<td>(-2.05)</td>
<td>(-0.41)</td>
<td>(2.51)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>S(t</em>{i,t+6,t+12})$</td>
<td>0.00</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(-0.16)</td>
<td>(-1.91)</td>
<td>(1.74)</td>
<td>(0.96)</td>
</tr>
</tbody>
</table>

**Table D.7: Shiller Survey Return Expectations and Futures Market Positioning**

*Note:* The table reports results from regressions of the form $\text{Net Positioning}_{j,t+1} = \alpha + \beta_1 \mathbb{E}_S(t_{i,t+1}) + \beta_2 \mathbb{E}_S(t_{i,t+1,t+3}) + \beta_3 \mathbb{E}_S(t_{i,t+3,t+6}) + \beta_4 \mathbb{E}_S(t_{i,t+6,t+12}) + \epsilon_{S,j,t+1}$, where $\text{Net Positioning}_{j,t+1}$ are the futures positions reported for investor type $j$ in the period following the forecast, and $\mathbb{E}_S(t_{i,t+h,t+h+k})$ are the return expectations by forecaster $S$ for period $t + h$ to $t + h + k$ in the Shiller / Yale ICF survey data. The sample consists of a pooled sample of individual and institutional investor respondents to the Shiller / Yale ICF surveys. The futures positioning data are from the Traders in Financial Futures report from 2010 onwards. Standard errors are clustered by month. $t$-statistics are reported in parentheses.
### Table D.8: Individual Return Expectations and Futures Market Positioning

Note: The table reports results from regressions of the form

\[ \text{Net Positioning}_{j,t+1} = \alpha + \beta_1 \text{ES}_t(r_{t,t+1}) + \beta_2 \text{ES}_t(r_{t+1,t+3}) + \beta_3 \text{ES}_t(r_{t+3,t+6}) + \beta_4 \text{ES}_t(r_{t+6,t+12}) + \epsilon_{j,t+1}, \]

where Net Positioning\(_{j,t+1}\) are the futures positions reported for investor type \(j\) in the period following the forecast, and \( \text{ES}_t(r_{t+h,t+k}) \) are the return expectations by forecaster \(S\) for period \(t+h\) to \(t+h+k\) in the Shiller / Yale ICF survey data. The sample consists of individual and institutional investor respondents to the Shiller / Yale ICF surveys. The futures positioning data are from the Traders in Financial Futures report from 2010 onwards. Standard errors are clustered by month. \(t\)-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Dealer</th>
<th>Levered</th>
<th>Institutional</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{ES}<em>t(r</em>{t,t+1}))</td>
<td>-0.15</td>
<td>-0.01</td>
<td>0.19</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(-2.64)</td>
<td>(-0.35)</td>
<td>(2.96)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>(\text{ES}<em>t(r</em>{t+1,t+3}))</td>
<td>-0.12</td>
<td>-0.01</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(-1.79)</td>
<td>(-0.24)</td>
<td>(1.73)</td>
<td>(2.06)</td>
</tr>
<tr>
<td>(\text{ES}<em>t(r</em>{t+3,t+6}))</td>
<td>-0.05</td>
<td>-0.09</td>
<td>0.03</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(-1.10)</td>
<td>(-2.25)</td>
<td>(0.66)</td>
<td>(2.51)</td>
</tr>
<tr>
<td>(\text{ES}<em>t(r</em>{t+6,t+12}))</td>
<td>-0.04</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-1.19)</td>
<td>(0.26)</td>
<td>(0.81)</td>
<td>(0.52)</td>
</tr>
</tbody>
</table>
### Table D.9: Institutional Return Expectations and Futures Market Positioning

Note: The table reports results from regressions of the form \( \text{Net Positioning}_{j,t+1} = \alpha + \beta_1 \mathbb{E}_{S,t}(r_{t+1}) + \beta_2 \mathbb{E}_{S,t}(r_{t+1,t+3}) + \beta_3 \mathbb{E}_{S,t}(r_{t+3,t+6}) + \beta_4 \mathbb{E}_{S,t}(r_{t+6,t+12}) + \epsilon_{S,j,t+1} \), where Net Positioning\(_{j,t+1}\) are the futures positions reported for investor type \( j \) in the period following the forecast, and \( \mathbb{E}_{S,t}(r_{t+h,t+h+k}) \) are the return expectations by forecaster \( S \) for period \( t + h \) to \( t + h + k \) in the Shiller / Yale ICF survey data. The sample consists of institutional investor respondents to the Shiller / Yale ICF surveys. The futures positioning data are from the Traders in Financial Futures report from 2010 onwards. Standard errors are clustered by month. \( t \)-statistics are reported in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Dealer</th>
<th>Levered</th>
<th>Institutional</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{E}<em>{S,t}(r</em>{t+1}) )</td>
<td>-0.10</td>
<td>0.05</td>
<td>0.11</td>
<td>-0.02</td>
</tr>
<tr>
<td>(2.40)</td>
<td>(1.28)</td>
<td>(2.01)</td>
<td>(-0.46)</td>
<td></td>
</tr>
<tr>
<td>( \mathbb{E}<em>{S,t}(r</em>{t+1,t+3}) )</td>
<td>-0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>(1.30)</td>
<td>(1.13)</td>
<td>(0.47)</td>
<td>(-0.19)</td>
<td></td>
</tr>
<tr>
<td>( \mathbb{E}<em>{S,t}(r</em>{t+3,t+6}) )</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.61)</td>
<td>(-0.50)</td>
<td>(-1.32)</td>
<td>(1.22)</td>
<td></td>
</tr>
<tr>
<td>( \mathbb{E}<em>{S,t}(r</em>{t+6,t+12}) )</td>
<td>0.02</td>
<td>-0.10</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>(0.65)</td>
<td>(-2.78)</td>
<td>(1.84)</td>
<td>(0.80)</td>
<td></td>
</tr>
</tbody>
</table>
D.3.2 Currency Markets

In currency markets, we analyze how investors’ futures positions are related to return expectations reported in survey data. We obtain weekly data on the positions of futures investors from the CFTC for seven currency pairs traded on the Chicago Mercantile Exchange (CME): the AUD/USD, CAD/USD, EUR/USD, JPY/USD, NZD/USD, CHF/USD, and GBP/USD. We obtain data from two reports published by the CFTC. The first is the Traders in Financial Futures report, which we use in equity markets, published weekly since 2010. The second is the legacy Commitment of Traders report, available for some currencies starting in 1992, which is a similar report that categorizes traders into the more coarse categories of “non-commercial” and “commercial,” where investors of the former category holds positioning largely opposite the latter category. The non-commercial category can be thought of as speculative trading hedge funds and commodity trading advisors, so we accordingly label them as “Speculators” (e.g., see Brunnermeier, Nagel and Pedersen (2008) and Moskowitz, Ooi and Pedersen (2012)). We note that a large amount of currency trading occurs in over-the-counter (OTC) markets, so futures positions only capture a small portion of investors’ positioning. Nevertheless, previous work has found these reports useful in capturing the behavior of currency market investors (e.g., see Brunnermeier, Nagel and Pedersen (2008), Moskowitz, Ooi and Pedersen (2012), Stavrakeva and Tang (2020), and Kremens (2020)).

We construct monthly observations, by merging end-of-month return expectations data from FX4casts with positions reported from the end of trading on the first Tuesday of the following month.\footnote{Survey questions for FX4casts are usually sent out on the last Friday of the survey month, with responses collected during Friday and the following Monday and Tuesday.} Once again, following the previous literature using these data, we consider an investor type’s net positioning for a given week as the number of long contracts minus the number of short contracts held by the investor type, normalized by open interest.

To capture the relationship between survey-based return expectations and investor future positions, for currency $i$ in period $t$, we run regressions of the form

\[
\text{Net Positioning}_{j,i,t+1} = \alpha_{j,i} + \beta_1 \mathbb{E}_t(r_{i,t+3}) + \beta_2 \mathbb{E}_t(r_{i,t+3,t+6}) + \beta_3 \mathbb{E}_t(r_{i,t+6,t+12}) + \epsilon_{j,i,t+1}, \tag{D.4}
\]

where $\text{Net Positioning}_{j,i,t+1}$ are the net futures positions reported for investor type $j$ for currency $i$ in the period following the forecast, and $\mathbb{E}_t(r_{i,t+h,t+h+k})$ are the return expectations for currency $i$ for period $t + h$ to $t + h + k$ implied by the consensus FX4casts survey. The independent and dependent variables are standardized to have zero mean and unit standard deviation, so that coefficients can broadly be interpreted as conditional correlation coefficients. Standard errors are HAC-panel standard errors, and $t$-statistics are reported in parentheses.

Table D.10 reports results from the regressions. We focus on regressions where dealer po-
sitioning, levered fund positioning, and speculator positioning are the dependent variables (the coefficients in the other regressions are all statistically insignificant). The first column reports regression results where the dependent variable is dealer net positioning, and the second column reports results from regressions where the dependent variable is levered fund net positioning. Both regressions have statistically significant coefficients for 3-month return expectations. The coefficients are -0.20 \((t\text{-statistic of -4.71})\) for dealer net positioning and 0.17 \((t\text{-statistic of 4.20})\) for levered fund net positioning. The coefficients on 3- to 6-month ahead and 6- to 12-month ahead return expectations are statistically insignificant. The last column reports results where speculator positioning is the dependent variable; the coefficient for 3-month return expectations is 0.19 \((t\text{-statistic of 5.34})\), while coefficients for 3- to 6-month ahead and 6- to 12-month ahead return expectations are statistically insignificant.

The results are consistent with myopic trading behavior in currency markets, with hedge funds trading on short-term return expectations, and dealers taking the other side of their positioning. Moreover, taken alongside the results reported in Table 3 in the main text, the results are consistent with the documented trading behavior of hedge funds. Table 3 indicates that short-term return expectations are high when past returns have been high, and when interest rate differentials for a currency have increased. Hedge funds are documented to take long futures positions in a currency precisely when its interest rate differential versus the USD is high relative to past history (Brunnermeier, Nagel and Pedersen (2008)), and when it has had recent past positive returns (Moskowitz, Ooi and Pedersen (2012)). Accordingly, the return expectations evidence broadly align with well-documented trading strategies of hedge funds.
### TABLE D.10: FX4casts Return Expectations and Futures Market Positioning

<table>
<thead>
<tr>
<th></th>
<th>Dealer</th>
<th>Levered</th>
<th>Institutional</th>
<th>Other</th>
<th>Speculators</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{i,t+3})$</td>
<td>-0.20</td>
<td>0.17</td>
<td>0.05</td>
<td>0.03</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(-4.71)</td>
<td>(4.20)</td>
<td>(1.12)</td>
<td>(0.73)</td>
<td>(5.34)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{i,t+3,t+6})$</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.06</td>
<td>0.05</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(-0.37)</td>
<td>(-1.02)</td>
<td>(0.77)</td>
<td>(-1.56)</td>
</tr>
<tr>
<td>$\mathbb{E}<em>{S,t}(r</em>{i,t+6,t+12})$</td>
<td>0.01</td>
<td>-0.03</td>
<td>0.00</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(-0.37)</td>
<td>(0.08)</td>
<td>(0.63)</td>
<td>(0.66)</td>
</tr>
</tbody>
</table>

**Note:** The table reports results from regressions of the form $\text{Net Positioning}_{j,i,t+1} = \alpha_i + \beta_1 \mathbb{E}_{S,t}(r_{i,t+3}) + \beta_2 \mathbb{E}_{S,t}(r_{i,t+3,t+6}) + \beta_3 \mathbb{E}_{S,t}(r_{i,t+6,t+12}) + \epsilon_{j,i,t+1}$, where Net Positioning$_{j,i,t+1}$ are the futures positions reported for investor type $j$ for currency $i$ in the period following the forecast, and $\mathbb{E}_{t}(r_{i,t+h:t+h+k})$ are the return expectations for currency $i$ for period $t+h$ to $t+h+k$ implied by FX4casts. The first four columns correspond with regressions using data from the Traders in Financial Futures report from the CFTC. The last column corresponds with non-commercial (“Speculator”) positioning in the Commitment of Traders report from the CFTC. Standard errors HAC-panel standard errors. $t$-statistics are reported in parentheses.