Speculating on Higher-Order Beliefs

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Abstract

Higher-order beliefs—beliefs about others' beliefs—may be important for trading behavior and asset prices but have received little systematic empirical examination. Examining more than 20 years of evidence from the Robert Shiller Investor Confidence surveys, we find that investors' higher-order beliefs provide substantial motivations for nonfundamental speculation—taking a stock market position that conflicts with one's valuation of the market. To explore the equilibrium implications, we construct a model that matches the survey evidence and highlights that investors' higher-order beliefs amplify stock market overreaction and excess volatility. (*JEL* G12, G40)

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Higher-order beliefs—beliefs about others' beliefs—are important in many models in economics and finance. As emphasized by a body of work starting with Keynes (1936), higher-order beliefs may be particularly important for investor behavior and financial market fluctuations. This is because investors have strong profit incentives to trade based on predictions of others' beliefs rather than their own valuations, potentially leading asset prices to deviate substantially from fundamental values. Despite their potential importance, higher-order beliefs have received little systematic empirical examination.

In this paper, we examine survey data on investors' higher-order beliefs, and consider the corresponding asset pricing implications. Our data come from the Robert Shiller Investor Confidence survey, which directly elicits investors' perceptions of other investors' beliefs. The survey also reports investors' stock market return expectations, which embed additional information about higher-order beliefs. This is because return expectations reflect an investor's forecasts of other investors' future demand, which, in turn, depend on her forecasts of their beliefs. Guided by the evidence, we construct a model that illustrates the potential importance of higher-order beliefs for stock market fluctuations. Below, we summarize our results.

Our headline finding is that in the U.S. stock market, nonfundamental speculation—taking a position in the stock market that conflicts with one's valuation of the market—is pervasive. Higher-order beliefs play a crucial role in giving rise to nonfundamental speculation.[1]

We examine the Shiller survey, which, for more than 20 years, has asked individual and institutional investors about their stock market expectations. Of particular relevance, the survey asks investors if they perceive other investors to be overly optimistic (or pessimistic) about the U.S. stock market, as well as if they perceive the stock market to be over- or undervalued. The majority of respondents report that others have mistaken beliefs, with the direction of their responses aligned with their perceived valuation of the stock market.

When investors report that others are overly optimistic, they also report expectations of high short-term (1- to 3-month-ahead) returns followed by low longer-term returns. A natural interpretation is that investors forecast others will become even more optimistic in the near term, fueling short horizon returns. These expectations provide strong incentives for nonfundamental speculation. Consistent with such speculation, investors recommend positions aligned with their short-term expectations, and misaligned with their valuations. Moreover, investors' short-term return expectations have strong explanatory power for asset managers' stock market positioning.

Exploring the source of investors' higher-order beliefs, we find that these beliefs and the accompanying nonfundamental speculation emerge in response to macroeconomic news. For example, following positive macroeconomic news, investors report beliefs that others have become increasingly optimistic and markets have become overvalued. At the same time, they

[1] This is speculation in the sense of Keynes (1936) and Harrison and Kreps (1978).

report expectations of higher short-term returns followed by lower long-term returns, consistent with a belief in momentum and reversal. The evidence indicates that investors believe that others overreact to fundamental news upon its arrival and will continue to overreact in the near term. This belief induces investors to engage in nonfundamental speculation.

Evaluating investors' short-term return expectations, we find that nonfundamental speculation is unprofitable on average. A monthly rebalanced market timing strategy taking long and short positions in proportion to the average reported 1-month-ahead return expectation earns a Sharpe ratio of -0.31. This poor performance aligns with the documented negative relationship between surveyed return expectations and realized returns (e.g., Greenwood and Shleifer 2014).

Our empirical results pose a challenge for existing models. Most notably, models without an explicit focus on higher-order beliefs cannot simultaneously explain investors' return expectations, valuations, and perceptions of others' relative optimism and pessimism.

To interpret the evidence and understand its implications for asset prices, we construct a model that reveals that higher-order beliefs induce asset price overreaction and excess volatility. The model features a riskless asset and a risky asset (the stock market) that pays a publicly observed dividend each period, which is drawn from persistent but unobserved fundamentals. The economy is populated by two types of investors: speculators, whose beliefs match the survey data; and arbitrageurs, who jointly behave as a mass of traders with rational expectations. Each investor receives a noisy but unbiased private signal about fundamentals. In the spirit of differences-of-opinion models, each investor believes that others' signals are uninformative conditional on their own. The average investor's belief about fundamentals is consistent with rational expectations, so all excess price movements come from higher-order beliefs.

Consistent with the survey evidence, speculators expect other investors to overreact to news and for the risky asset to be overvalued when fundamentals are positive. We model this expected overreaction as arising from higher-order beliefs about the persistence of fundamentals. These higher-order beliefs may be second-order beliefs, namely, a belief that other investors overestimate the persistence of fundamentals. Or they may be of an even higher-order, for example, a belief that other speculators believe that others overestimate the persistence of fundamentals.

Corresponding with their trading being unprofitable, speculators' higher-order beliefs are mistaken. They induce speculators to consistently expect more overreaction than they observe in asset prices. When fundamentals are positive, speculators see the risky asset as overvalued. But their higher-order beliefs lead them to incorrectly infer that others received attenuated signals about fundamentals, and accordingly to anticipate further overreaction. They engage in nonfundamental speculation, and buy into the overvalued risky asset, in the process causing its overvaluation. This process amplifies overreaction and excess volatility in the risky asset price. Prices revert over time, as speculators' expectations of increasing optimism do not manifest.

In addition to capturing the survey evidence, the model illustrates the asset pricing impact of higher-order beliefs along other dimensions. It reveals that all else equal, higher-order belief mistakes lower in the belief hierarchy have a stronger effect on asset prices; for example, mistaken second-order beliefs lead to more overreaction and volatility than mistaken third-order beliefs. And it suggests interactions between fundamental and higher-order belief mistakes, such as higher-order beliefs' potential amplification of the effect of fundamental belief overreaction on asset prices.

Our paper brings evidence to a primarily theoretical literature on higher-order beliefs in asset pricing.[2] Prior work can be viewed as falling into one of two traditions: noisy rational expectations models, where rational investors face frictions that prevent them from observing others' beliefs (Singleton 1987; Allen, Morris, and Shin 2006; Bacchetta and Van Wincoop 2006, 2008; Makarov and Rytchkov 2012; Kasa, Walker, and Whiteman 2014; Cespa and Vives 2015; Nimark 2017), or differences-of-opinion models, where investors know and disagree with other investors' valuations (Harrison and Kreps 1978; Harris and Raviv 1993; Kandel and Pearson 1995; Scheinkman and Xiong 2003; Banerjee and Kremer 2010).

Relative to both, our paper presents evidence and seeks to model higher-order beliefs in a manner consistent with the evidence. Our paper also highlights survey evidence that may be useful for future work, namely, questions about investors' higher-order beliefs, as well as data on the term structure of expected returns, which help pin down investors' beliefs about other investors' future beliefs.

Our modeling approach contributes to the literature on errors in strategic reasoning in finance. Previous models have studied investors who neglect the information content of prices (Eyster, Rabin, and Vayanos 2019) and neglect that other investors may learn about fundamentals from prices (Bastianello and Fontanier 2024, 2025), with implications for volume and prices.[3] We find that mistakes in investors' higher-order beliefs can rationalize investors' return expectations and unprofitable nonfundamental speculation.

Our paper also relates to a literature on nonfundamental speculation, where investors willingly buy into overvalued assets. Prior work has documented numerous instances of nonfundamental speculation (e.g, McKay 1841; Kindleberger 1978; Temin and Voth 2004; Brunnermeier and Nagel 2004), with prominent theoretical work rationalizing such behavior (De Long et al. 1990; Harrison and Kreps 1978; Scheinkman and Xiong 2003; Abreu and Brunnermeier 2002, 2003; Martin and Papadimitriou 2022). While the literature focuses on episodes where informed investors (e.g., hedge funds) may have profited, we find evidence

[2] Other papers that empirically examine higher-order beliefs include Egan, Merkle, and Weber (2014), who find that investors' beliefs about others' return expectations affect investment decisions, and Coibion et al. (2021), who test noisy information models using a survey of firm managers in New Zealand.

[3] Andre, Schirmer, and Wohlfart (2024) provide related evidence.

that nonfundamental speculation is a pervasive and unprofitable feature of the U.S. stock market.

Finally, our paper is related to the literature in finance using survey data to study market participants' beliefs, surveyed by Adam and Nagel (2023). A sizeable literature has studied fundamental and return expectations. Work on return expectations includes Vissing-Jorgensen (2003), Bacchetta, Mertens, and Van Wincoop (2009), Greenwood and Shleifer (2014), Amromin and Sharpe (2014), Barberis et al. (2015, 2018), Adam, Marcet, and Beutel (2017), Giglio et al. (2021), and Nagel and Xu (2023), while work on fundamental expectations includes Chen, Da, and Zhao (2013), Bordalo et al. (2024), De La O and Myers (2021), and Nagel and Xu (2022).[4] Our results bridge together fundamental and return expectations via higher-order beliefs.

1 Evidence from Survey Data

We study U.S. stock market expectations as reported by individual and institutional investors in the Robert Shiller Investor Confidence survey (Shiller 2000). The Shiller survey is unique in providing a long time series where investors are simultaneously asked about their higher-order beliefs, stock market valuations, and return expectations over multiple horizons. In Internet Appendix C.1, we discuss the implementation details of the survey. Individual respondents are likely to have high income and be wealthy, and institutional respondents manage large portfolios. While likely not representative of the full investor population, survey respondents are an important class of investors. We study the sample from July 2001 through April 2023.

In our analysis, we find that investors often believe that other investors hold incorrect stock market valuations, but find it profitable to speculate in the direction of these incorrect valuations. We also find that investors report a belief that the stock market overreacts to news upon its arrival, and report expectations consistent with a belief in momentum and reversal. We discuss these results in the context of existing models, which match some of the evidence, but not all of it.

1.1 Survey questions of interest

The particular questions from the Shiller survey that we examine are (with potential responses in parentheses):

(i) **Questions about higher-order beliefs**

[4]Prior work has largely focused on return expectations at a fixed horizon (e.g., 1-year ahead). We focus on the future path of expected returns (see also Gandhi, Gormsen, and Lazarus 2023).

(a) *Many people are showing a great deal of excitement and optimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them (True; False; No opinion).*

(b) *Many people are showing a great deal of pessimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them (True; False; No opinion).*

(ii) **Questions regarding stock market returns, valuations, and behavior**

(a) *How much of a change in percentage terms do you expect [for the Dow Jones index] in the following 1 month? 3 months? 6 months? 1 year? 10 years?*

(b) *Stock prices in the United States, when compared with measures of true fundamental value or sensible investment value are (Too low; Too high; About right; Do not know).*

(c) *Although I expect a substantial drop in stock prices in the U.S. ultimately, I advise being relatively heavily invested in stocks for the time being because I think that prices are likely to rise for a while (True, False, No opinion; if True, indicate best guess for date of peak).*

(d) *Although I expect a substantial rise in stock prices in the U.S. ultimately, I advise being less invested in stocks for the time being because I think that prices are likely to drop for a while (True, False, No opinion; if True, indicate best guess for date of bottom).*

(e) *If the Dow dropped 25% over the next 6 months, I would guess that the succeeding 6 months, the Dow would: (Increase (Give percent), Decrease (Give percent), Stay the same, No opinion).*

(iii) **Questions about the drivers of higher-order beliefs**

(a) *What do you think is the cause of the trend of stock prices in the United States in the past 6 months (It properly reflects the fundamentals of the U.S. economy and firms; It is based on speculative thinking among investors or overreaction to current news; Other (write-in); No opinion)?*

Of particular note, answers to question (i.a) and (i.b) directly provide information regarding investors' higher-order beliefs. However, the phrasing of the questions means they are open to multiple interpretations.[5] We present cross-sectional and time-series evidence that when investors respond that many others are overly optimistic and they must be careful not to be influenced by them, they also report that the stock market is overvalued and that they expect low long-term stock market returns. Hence, investors appear to interpret these questions as asking:

(i.a) *Many other investors are overly optimistic about the stock market's prospects.*

[5]There is particular ambiguity about the meaning of the second part of the questions, specifically "I must be careful not to be influenced by them." Based on the evidence that we present, investors seem to indicate that they believe others' valuations may be overly optimistic or pessimistic, but nevertheless that others' valuations still enter into their investment decisions given that they affect short-term market returns.

(i.b) *Many other investors are overly pessimistic about the stock market's prospects.*

While there may be some noise associated with assigning this interpretation, it is consistent with responses to other questions on average and is informative about investors' views. This is especially the case given the long time-series evidence relative to other surveys.

1.2 Summary statistics

Table 1 reports summary statistics for the survey responses. For all questions except for questions (ii.a) and (ii.e), the table reports the proportion of respondents that gave a particular answer. For questions (ii.a) and (ii.e), the table reports expected returns averaged across respondents. The table reports statistics separately for individual and institutional respondents.

Focusing on the first two rows, 59% of individual investors report that many others are overly optimistic and 61% report that many others are overly pessimistic (for institutional investors, the proportions are 53% and 62%). That is, the majority of respondents report that other investors have incorrect beliefs.[6]

In the third and fourth rows, 34% of individual investors report that they expect the stock market to eventually drop but recommend being overweight, and 34% report that they expect the market to eventually rise, but recommend being underweight (the numbers are both 32% for institutional investors). The claims are mutually exclusive; hence, the evidence indicates that a majority of investors expect short- and long-term returns to be differently signed.[7] This, in turn, suggests the presence of nonfundamental speculative motivations, that is, to take stock market positions opposite one's long-term return expectations.

The fifth row displays investors' reported return expectations over different horizons (labeled total), and the sixth row reports return expectations in excess of the corresponding maturity U.S. Treasury-bill rate (labeled excess). Individual and institutional investors report small return expectations for the next month, with an expectation of more positive returns for 3 to 6 months ahead. Total return expectations for 12 months ahead are 3.6% on average for individual investors and 4.9% on average for institutional investors; average excess return expectations are 2.1% and 3.4%.[8]

Regarding prices vis-à-vis fundamentals, 11% of individual investors indicate that stock

[6]The results also indicate that several respondents simultaneously indicate a belief that many others are overly optimistic and overly pessimistic; this can be seen by the fact that the sum of the proportions of investors reporting that others are overly optimistic and pessimistic is greater than 100%. This does not affect our analysis.

[7]Respondents largely recognize the claims' mutual exclusivity. Only 7.0% of institutional investors and 7.4% of individual responses are *True* to both questions (ii.c) and (ii.d) at the same time. Among individual investors, 69% that respond *True* to (ii.c) respond *False* to (ii.d) and 73% that report *True* to (ii.d) report *False* to (ii.c); these numbers are 70% and 72% for institutional investors.

[8]The small average expected excess return suggests that investors expect a small equity risk premium or (not exclusively) that some investors may implicitly subtract the risk-free rate when reporting return expectations. In our analyses, we use expected excess returns, but the results are not sensitive to using total returns instead.

valuations are low relative to fundamentals, 38% say they are high, 41% say they are about right, and 11% express no opinion (these numbers are 18%, 31%, 48% and 3% for institutional investors). Regarding recent stock market trends, of individual investor responses, 22% indicate that market movements properly reflect fundamental news, while 53% indicate that they reflect speculative thinking and overreaction by other investors; these numbers are 27% and 38% for institutional investors.

Lastly, given the scenario of a 25% drop in the stock market in the next 6 months, on average, individual investors expect returns in the subsequent 6 months to be +13.5% (institutional respondents expect +16.1%). This is consistent with investors believing that large market declines reflect overreaction that will revert in the intermediate term.

1.3 Higher-order beliefs and perceived valuations

Next, we examine responses to questions (i.a) and (i.b), regarding other investors' optimism and pessimism. We provide evidence of respondents interpreting the questions as asking whether others are *overly* optimistic or *overly* pessimistic about the stock market.

We compute a perceived *Overvaluation* variable by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-à-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). Higher values of *Overvaluation* correspond with higher stock market prices relative to fundamentals. We regress *Overvaluation* on two measures of higher-order beliefs: *Higher-Order Optimism* and *HO pessimism*. We construct these variables by mapping the responses to questions (i.a) and (i.b) (True; False; No Opinion) to the values (1; -1; 0). The *HO optimism* variable is increasing in agreement with the statement that other investors are overly optimistic, while the *HO pessimism* variable is increasing in agreement with the statement that other investors are overly pessimistic.

Table 2 reports the regression results. Columns 1 to 3 report results pooling together individual and institutional investor responses. The first two columns report results from regressions using survey response-level observations. With month fixed effects (column 2), the regressions capture cross-sectional comparisons, for example, whether an investor that believes others are more optimistic also is more likely to believe the stock market is overvalued. Column 3 reports results from time-series regressions, using cross-sectional monthly averages of the variables. These capture whether, in time periods where investors believe others are more optimistic, they also believe that the market is more overvalued (which we expect). All variables are standardized to have zero mean and unit standard deviation, so that the coefficients can broadly be interpreted as correlations.

Panels A, B, and C reports results where *HO optimism*, *HO pessimism*, and *HO belief* (defined as *HO optimism - HO pessimism*) are the independent variables, respectively. We find consistent evidence of a strong relationship in the expected direction. We focus on panel C, which incorporates responses both about others' optimism and about others' pessimism. The

	Individual				Institutional			
	True	False	No Opin.		True	False	No Opin.	
(i.a) Others overly optimistic about stocks	59%	27%	15%		53%	31%	16%	
(i.b) Others overly pessimistic about stocks	61%	22%	17%		62%	22%	16%	
(ii.c) Expect eventual drop but overweight	34%	52%	14%		32%	54%	14%	
(ii.d) Expect eventual rise but underweight	34%	49%	17%		32%	53%	15%	
	1M	3M	6M	12M	1M	3M	6M	12M
(ii.a) Percent expected return (total)	-0.1%	0.4%	1.4%	3.6%	0.0%	0.6%	2.0%	4.9%
Percent expected return (excess)	-0.2%	0.1%	0.7%	2.1%	0.0%	0.3%	1.3%	3.4%
	Low	High	Right	No Op.	Low	High	Right	No Op.
(ii.b) Stock prices vs. fundamental value are...	11%	38%	41%	11%	18%	31%	48%	3%
	Value	Overreac.	Other	No Op.	Value	Overreac.	Other	No Op.
(iii.a) Cause of 6-month market trend	22%	53%	21%	3%	27%	38%	34%	2%
(ii.e) Expected 6M return after 25% drop	13.5%				16.1%			
Total number of responses	8921				7262			

Table 1: Summary statistics for the Shiller survey

The table reports summary statistics for the survey responses to the Shiller surveys, reporting statistics separately for individual and institutional investor respondents. For questions (i.a,b), (ii.b,c,d), and (iii.a), the table reports the proportion of survey respondents in the sample that gave a particular answer in response to a given question. For questions (ii.a) and (ii.e), the table reports the average h-month-ahead expected return reported by respondents. For (ii.a), we return expectations both as provided on the survey (labeled total) and in excess of the h-month U.S. Treasury-bill rate (labeled excess). Questions and potential responses are presented in abbreviated form in the table; they are presented in their full form in Section 1.1.

coefficients are 0.27 in the response-level regression with no fixed effects, 0.23 in the response-level regression with time fixed effects, and 0.61 in the monthly time-series regression, with R^2 values of .08, .16, and .38.

For the rest of our empirical analysis, we focus on *HO belief*. In Table 2, columns 4–6 report regression results for individual investors and columns 7–9 report results for institutional investors. The results are similar across the samples. For the rest of the paper, we report pooled results in the main text, and provide breakdowns by investor type in the Internet Appendix; all results are similar across investor types.

The evidence indicates a strong relationship between the *HO belief* and *Overvaluation* measures; when investors report that they think others are more optimistic, they are substantially more likely to report that the stock market is overvalued. To better understand investors' valuations and higher-order beliefs, Figure 1 plots the quarterly averages of investors' higher-order beliefs and valuations over time.

Focusing on *HO belief* in the top panel of the figure, we see that it exhibits peaks and troughs related to the broader macroeconomy, which we contextualize using open-ended responses from the survey.[9] *HO belief* exhibits a trough in late 2002 and early 2003, with respondents discussing the Iraq war and geopolitical uncertainty; in 2008–2009, corresponding with the Great Financial Crisis; in late 2011, corresponding with global sovereign debt concerns; in Q1 2016, coinciding with concerns about oil prices and slowing Chinese growth; and in Q2 2022, corresponding with inflation, supply chain issues, and the Ukraine war. Meanwhile, *HO belief* exhibits peaks in Q2 2007; in Q2 2013, with discussion of quantitative easing; and is elevated in the 2013 to 2021 period of low interest rates and strong market performance.

The bottom panel of the figure plots *Overvaluation*, which rises and falls at similar periods as *HO belief*, but is more persistently elevated in the later period of low interest rates and rising valuations. In Section 1.6, we analyze the dynamics of *HO belief* and *Overvaluation*, and find that they systematically vary with macroeconomic news.

1.4 *HO belief* and return expectations

Next, we study the relationship between return expectations and perceptions of others' optimism. We regress investors' excess return expectations of different horizons (multiplied by 100) on *HO belief*. Panel A of Table 3 presents the results.

The first four columns report results for time-series regressions where observations are cross-sectional averages of responses in a given month. The coefficient on *HO belief* for 1-month return expectations is 1.82, indicating that a unit increase in *HO belief* corresponds with a 1.82% higher expected return for the following month. The coefficients decline with horizon; the coefficients on 3-, 6-, and 12-month return expectations are 1.22, 0.20, and -2.26.

[9] These responses come from respondents that mark "Other" in response to question (iii.a), regarding the driver of stock prices, as well as a "General Comments" section of the survey.

	A. x=HO optimism								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
HO optimism	0.24	0.20	0.64	0.18	0.13	0.57	0.30	0.26	0.51
	(0.02)	(0.01)	(0.10)	(0.02)	(0.01)	(0.08)	(0.02)	(0.02)	(0.10)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
N	15,949	15,949	259	8,785	8,785	259	7,164	7,164	259
R^2	.06	.14	.41	.03	.13	.32	.09	.20	.26

	B. x=HO pessimism								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
HO pessimism	-0.12	-0.10	-0.28	-0.10	-0.08	-0.22	-0.14	-0.12	-0.16
	(0.01)	(0.01)	(0.13)	(0.02)	(0.01)	(0.11)	(0.01)	(0.01)	(0.11)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
N	15,949	15,949	259	8,785	8,785	259	7,164	7,164	7,164
R^2	.01	.12	.08	.01	.12	.05	.02	.15	.03

	C. x=HO belief:=HO optimism - HO pessimism								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Pooled			Individual			Institutional		
HO belief	0.27	0.23	0.61	0.21	0.17	0.53	0.34	0.29	0.51
	(0.02)	(0.01)	(0.11)	(0.02)	(0.01)	(0.09)	(0.02)	(0.01)	(0.11)
Time FE	No	Yes	NA	No	Yes	NA	No	Yes	NA
N	15,949	15,949	259	8,785	8,785	259	7,164	7,164	259
R^2	.08	.16	.38	.05	.14	.28	.11	.21	.26

Table 2: Higher-order beliefs and perceived valuations

The table reports results from regressions of a perceived *Overvaluation* measure constructed from the Shiller surveys on *Higher-order belief* variables constructed from the surveys. The *Overvaluation* measure is constructed by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-à-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). The *HO optimism* and *HO pessimism* measures are constructed by mapping the responses to questions (i.a) and (i.b) regarding other investors' optimism and pessimism (True; False; No Opinion) to the values (1; -1; 0). Columns 1–3 pool together observations across the individual and institutional investor samples, and columns 4–6 and 7–9 separately report results for the two samples. The unit of observation for columns 3, 6, and 9 is the monthly cross-sectional average of the variables; Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observations for columns 1, 2, 4, 5, 7, and 8 are survey responses; Driscoll-Kraay standard errors (12 lags) for coefficients are reported in parentheses.

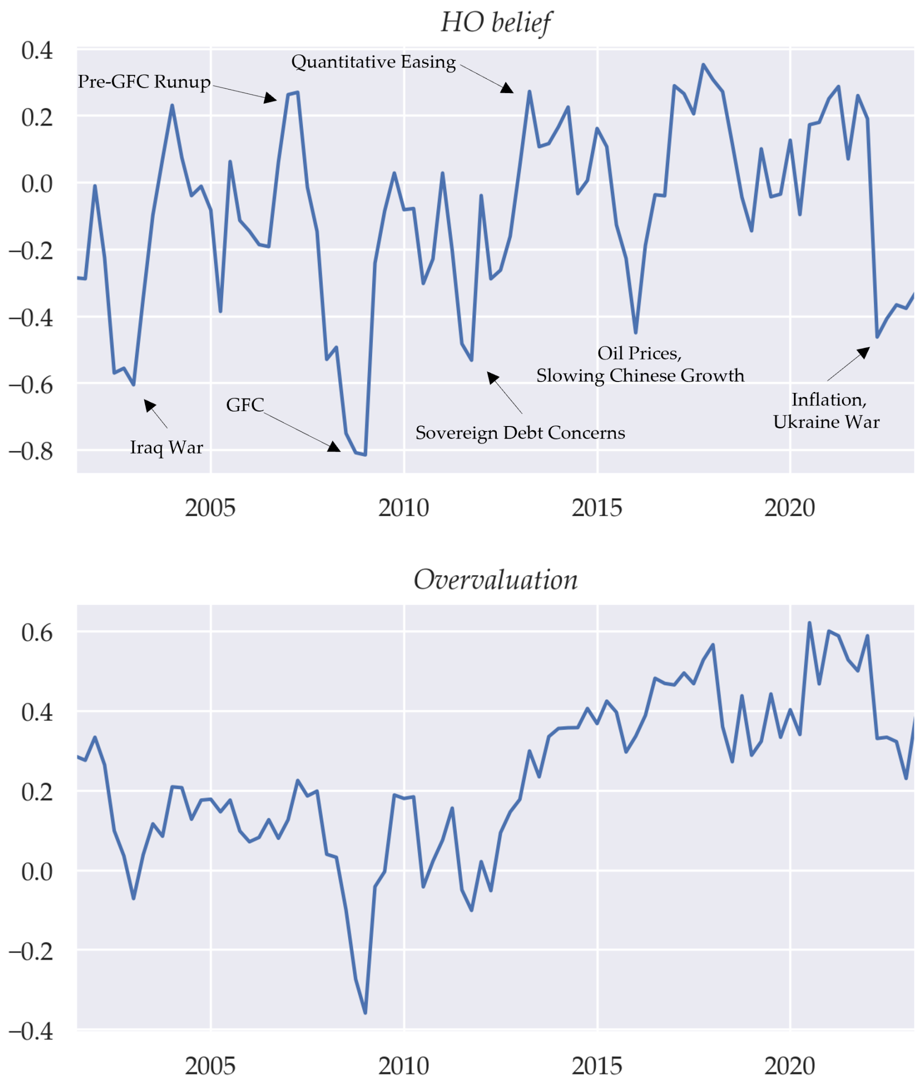

Figure 1: Higher-order beliefs and perceived valuations

The figure plots the time-series quarterly averages of a *Higher-order belief* measure and a perceived *Overvaluation* measure. The *Overvaluation* measure is constructed by mapping the responses to question (ii.b) regarding perceptions of stock market valuations vis-à-vis fundamentals (Too low; Too high; About right; Do not know) to the values (-1; 1; 0; 0). The *HO optimism* and *HO pessimism* measures are constructed by mapping the responses to questions (i.a) and (i.b) regarding other investors' optimism and pessimism (True; False; No Opinion) to the values (1; -1; 0).

Alt text: Line graphs displaying the time series of the quarterly averages of the *HO belief* and *Overvaluation* measures.

Table C.8 in the Internet Appendix verifies that these results hold for both *HO optimism* and *HO pessimism*.

The results reveal that on average, when investors report that others are overly optimistic (and that the market is overvalued), they expect positive returns over the next month and subsequent reversion. A natural interpretation is that investors expect others to become even more optimistic in the short term. These expectations provide motivations for nonfundamental speculation, where investors may take overweight positions in the stock market even when they see it as overvalued, because they perceive it will continue to rise before correcting.

The last four columns in panel A of Table 3 report results for cross-sectional regressions using month-level observations and including month fixed effects. The coefficients for 1-, 3-, 6-, and 12-month-ahead returns are 0.06, -0.30, -1.00, and -1.76. The results indicate that in cross-sectional comparisons, an investor that holds a stronger belief that others are overly optimistic does not necessarily believe that short-term returns will be higher than an investor with a weaker belief, though they do expect worse long-term performance.

We provide additional validation for the time-series results using investors' responses to questions asking whether they expect the stock market to reach a peak (trough) in the short run though they expect it to decline (rise) in the long run. We construct *Short-term peak* and *ST trough* variables by mapping responses to questions (ii.c) and (ii.d) (True; False; No Opinion) to the values (1, -1, 0). We run time-series regressions of *ST peak* and *ST trough* on *HO belief* and *Overvaluation*, with cross-sectional monthly averages as the unit of observation.

Panel B of Table 3 reports the results. The first two columns report results where the independent variable is *HO belief*. The coefficient on *HO belief* is 0.44 for *ST peak* (R^2 of .26) and -0.32 for *ST trough* (R^2 of .20). These results validate the relationship between beliefs regarding others' optimism and beliefs that markets will continue to rise before declining, using a qualitative elicitation method. The questions used to construct *ST Peak* and *ST Trough* also indicate investors' recommendations to be overweight stocks even though they expect an eventual decline in stocks, or underweight despite expecting an eventual rise. Hence, they provide further evidence of nonfundamental speculation induced by higher-order beliefs.

The last two columns in panel B of Table 3 reports regression results where the independent variable is *Overvaluation*. We observe a similarly strong relationship between *ST peak* and *Overvaluation* (coefficient of 0.73, R^2 of .35), though a weaker relationship between *ST trough* and *Overvaluation* (coefficient of -0.11, R^2 of .01).

1.5 Nonfundamental speculation

The evidence indicates that investors have strong incentives for nonfundamental speculation, for example, to "ride the bubble." An important question is whether the expectations data actually capture investors' trading behavior, and in particular, whether investors speculate based on their short-term expectations.

A. Term structure of expected cumulative returns								
	Time series				Cross-sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
HO belief	1.82	1.22	0.20	-2.26	0.06	-0.30	-1.00	-1.76
	(0.26)	(0.43)	(0.63)	(1.06)	(0.04)	(0.06)	(0.09)	(0.12)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
N	259	259	259	259	10, 957	10,957	10,957	10,957
R^2	.21	.05	.00	.05	.00	.00	.02	.04

B. Short-term peaks and troughs				
	ST peak	*ST trough*	*ST peak*	*ST trough*
HO belief	0.44	-0.32		
	(0.08)	(0.04)		
Overvaluation			0.73	-0.11
			(0.12)	(0.15)
Time FE	NA	NA	NA	NA
N	259	259	259	259
R^2	.26	.20	.35	.01

Table 3: Higher-order beliefs and return expectations

Panel A of the table reports results from regressions of cumulative return expectations on the *Higher-order belief* variable constructed from the Shiller surveys, pooling together observations across individual and institutional investors. Each column, labeled $\mathbb{E}_t(R_{t,t+k})$ represents cumulative return expectations in month t for returns from month t to month $t+k$. The unit of observation in the first four columns is the monthly cross-sectional average of survey responses. Newey-West standard errors (12 lags) for coefficients are reported in parentheses. The unit of observation for the last four columns are individual survey responses. Driscoll-Kraay standard errors (12 lags) of coefficients are reported in parentheses. Panel B of the table reports regressions of *Short-term peak* and *ST trough* on the *HO belief* and *Overvaluation* measures. The variable *ST* peak is constructed from question (ii.c), which asks whether investors expect markets to eventually fall but reach a peak in the near-term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The variable *ST* trough is constructed from question (ii.d), which asks whether investors expect markets to eventually rise but reach a trough in the near-term future, by mapping the responses (True; False; No Opinion) to the values (1; -1; 0). The units of observation in the regressions are monthly cross-sectional averages of the variables. Newey-West standard errors (12 lags) of coefficients are reported in parentheses.

We provide evidence of nonfundamental speculation, finding that the equity futures market positions of buy-side investors (asset managers, hedge funds, etc.) track short horizon return expectations. Futures positions reflect the equity exposures of funds in aggregate, which may reflect individual investors' and fund managers' expected returns.

We obtain weekly data from 2006 onward on the positions of investors in Dow Jones Industrial Average (DJIA) and S&P 500 equity index futures from the Traders in Financial Futures report from the Commodity Futures Trading Commission. The report presents the number of long and short contracts held in aggregate by investors classified into one of four categories based on self-reported business purposes: futures dealers, levered funds (i.e., hedge funds), institutional asset managers, and other. We analyze both DJIA and S&P 500 futures positions, since the Shiller survey asks investors about their expectations for the DJIA, but the S&P 500 is the more commonly tracked market index.

We construct *Net positioning* as the number of short minus long contracts held by dealers, normalized by open interest. Futures contracts are in zero net supply, and dealers meet the futures demand of other investors, so *Net positioning* measures long demand for market exposure by buy-side investors (Hazelkorn, Moskowitz, and Vasudevan 2023). We run contemporaneous regressions of the level of *Net positioning* on the cross-sectional average of investors' return expectations. We also run the regression in changes, regressing the quarterly change in the short minus long contracts held by dealers (normalized by lagged open interest) on changes in the cross-sectional average of return expectations. We standardize the dependent variables to have zero mean and unit standard deviation

Table 4 reports the results. Panel A reports results from the levels regressions. When the dependent variable is DJIA futures positions, the coefficient on 1-month return expectations is 0.51 in the univariate regression (standard error of 0.09, R^2 of .28), and 0.44 in the multivariate regression (standard error of 0.20). The univariate coefficient on 3-month return expectations is 0.37 (standard error of 0.09, R^2 of .21) and the multivariate coefficient is 0.02 (standard error of 0.26). The coefficients for 6-month return expectations are 0.19 (standard error of 0.10) and 0.09 (standard error of 0.18), while the coefficients for 12-month return expectations are 0.02 (standard error of 0.09) and -0.04 (standard error of 0.07). The results from the changes regressions presented in panel B follow a similar pattern. When the dependent variable is S&P 500 futures positions, the evidence similarly indicates that 1- and 3-month return expectations are strongly related to investors' futures positions, while longer-term 6- and 12-month return expectations are only weakly related to them.

To further explore the relationship between futures positions and investors' stated higher-order beliefs, we regress (changes in) *Net positioning* on (changes in) the *HO belief* and *Overvaluation* variables. We report the results in Table C.11 in the Internet Appendix. Consistent with the previous results, we find evidence of nonfundamental speculation, with investors buying into the stock market when they perceive others to be overly optimistic.

	DJIA futures					S&P 500 futures				
A. Levels regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.51				0.44	0.33				0.11
	(0.09)				(0.20)	(0.17)				(0.21)
$\mathbb{E}_t(R_{t,t+3})$		0.37			0.02		0.36			0.11
		(0.09)			(0.26)		(0.09)			(0.26)
$\mathbb{E}_t(R_{t,t+6})$			0.19		0.09			0.30		0.15
			(0.10)		(0.18)			(0.09)		(0.22)
$\mathbb{E}_t(R_{t,t+12})$				0.02	-0.04				0.14	0.04
				(0.09)	(0.07)				(0.06)	(0.06)
R^2	.28	.21	.08	.00	.29	.11	.20	.22	.11	.24
N	69	69	69	69	69	69	69	69	69	69

	DJIA futures					S&P 500 futures				
B. Changes regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.34				0.44	0.38				0.35
	(0.10)				(0.16)	(0.10)				(0.19)
$\mathbb{E}_t(R_{t,t+3})$		0.17			-0.13		0.29			-0.03
		(0.12)			(0.23)		(0.09)			(0.22)
$\mathbb{E}_t(R_{t,t+6})$			0.08		0.01			0.19		0.11
			(0.12)		(0.18)			(0.12)		(0.15)
$\mathbb{E}_t(R_{t,t+12})$				0.06	0.05				0.09	0.02
				(0.07)	(0.08)				(0.09)	(0.09)
R^2	.14	.05	.01	.01	.15	.17	.13	.08	.03	.19
N	68	68	68	68	68	68	68	68	68	68

Table 4: Return expectations and investor futures positions

The table reports results from regressions of investors' futures positions on return expectations. Our measure of futures positions is *Net positioning*, defined as the number of short minus long futures contracts held by futures dealers in aggregate, normalized by open interest. Data are from the Traders in Financial Futures report. Return expectations are the average return expectations in a given period from the Shiller survey. Observations are quarterly levels in panel A ("Level regressions"). In panel B ("Changes regressions"), observations are quarterly changes in return expectations and the change in short minus long futures contracts held by dealers, normalized by lagged open interest. The first four columns in the table report results where futures positions are those of dealers in Dow Jones Industrial Average (DJIA) futures. The last four columns report results where futures positions are those of dealers in S&P 500 futures. Newey-West standard errors (four lags) of coefficients are reported in parentheses.

One interpretation of the evidence in this section is that the survey data reflect buy side investors' expectations, which are accordingly reflected in futures positions. Under this interpretation, investors' short-term return expectations lead them to engage in nonfundamental speculation. While our evidence is consistent with such an interpretation, we are also cautious, in that we cannot link the identity of survey respondents with their trades.

1.6 What drives higher-order beliefs?

Given the observed time-series relationship between investors' reported higher-order beliefs and return expectations, we explore the drivers of these beliefs. We find macroeconomic news to be a key driver, with positive news increasing *HO belief* and short-term return expectations while decreasing long-term return expectations.

We use two measures of macroeconomic news. The first is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, a composite index of 10 leading macroeconomic indicators.[10] The second is quarterly AR(1) innovations in discussion of recessions in the *Wall Street Journal*, from Bybee et al. (2024).[11] The first measure corresponds with positive macroeconomic news, and the second with negative news.

We regress quarterly changes in the cross-sectional averages of *HO belief*, *Overvaluation*, and return expectations of different horizons on the measures of macroeconomic news. We standardize the independent variables, and changes in *HO belief* and *Overvaluation*, to have zero mean and unit standard deviation. The coefficients for return expectations can be interpreted as the change in expected returns (in percentage points) corresponding with a one standard deviation innovation to the independent variable, and the coefficients for the other dependent variables are correlation coefficients.[12]

Figure 2 plots the regression coefficients. With innovations to leading economic indicators as the independent variable, the coefficient on 1-month return expectations is 0.44, indicating that a one standard deviation innovation corresponds with a 44-basis-point (bp) higher return expectation for the next month. The coefficients for 3-, 6-, and 12-month return expectations are 0.21, -0.30, and -0.55, indicating that investors lower their return expectations for the next year contemporaneous with positive news. Innovations to the leading economic indicators index are 0.48 and 0.49 correlated with changes in *HO belief* and *Overvaluation*. With innovations to recession attention as the independent variable, the coefficients for 1-, 3-, 6-, and 12-month return expectations are -0.49, -0.27, 0.05, and 0.26, indicating expectations of strong

[10] The data are provided as an index, and we construct innovations in the percent change in the index. We lag observations by 1 month to ensure that innovations are in investors' information sets. We also report results in the Internet Appendix using coincident business cycle indicators.

[11] Bybee, Kelly, and Su (2023) and Bybee et al. (2024) find that discussions of recessions have substantive explanatory power for risk premiums and for future macroeconomic outcomes. We use an updated series from the authors containing data through January 2021.

[12] In the Internet Appendix, we also report results running the regressions in levels, with similar results.

negative short-term performance that will revert in the future. Innovations to recession attention are -0.42 and -0.28 correlated to *HO belief* and *Overvaluation*. These results suggest that macroeconomic news may drive substantial variation in investors' reports that others are overly optimistic and that markets are overvalued.

We can interpret the evidence as follows: in quarters with positive macroeconomic news, the stock market appreciates. The contemporaneous quarterly return associated with a one-standard-deviation shock to leading indicators is 1.68%. Investors perceive that in the following month, returns will be 49-bp higher, but that in the subsequent 11 months, this short-term return will entirely revert, and further, that returns will be lower by nearly about a quarter of the contemporaneous response to the news. That is, the evidence is consistent with the 1.68% return reflecting overreaction to news, with investors seeing the initial reaction more than 25% larger than justified by fundamentals.

The evidence and interpretation of perceived overreaction is augmented by other survey responses. Unconditionally, 53% of individual investors and 38% of institutional investors report that the cause of the 6-month stock market trend is overreaction and speculative thinking by other investors.[13] Additionally, when asked how they expect the market to perform following a 25% drop in the next 6 months, investors expect reversals of 13.5% to 16.1% on average, consistent with investors perceiving that the market overreacts.

1.7 Relationship to evidence on extrapolation

Prior work has examined investors' 6- to 12-month return expectations and found that they are extrapolative; investors expect past market performance to persist in the future (e.g., Greenwood and Shleifer 2014). Our evidence indicates that investors' short horizon return expectations (1 and 3 months) may be extrapolative, consistent with prior work; but that investors' 6-month, and especially 12-month expectations may be slightly contrarian, at apparent odds with the extant evidence.

We study this point in detail in Internet Appendix C.2 and provide a brief summary here. We conjecture that differences in reported expectations across surveys may be due to survey design. Unlike most other surveys, the Shiller survey asks investors their return expectations at multiple horizons. Investors may not formulate precise period-by-period forecasts in their minds. When asked about returns at multiple horizons, for the 1- and 3-month horizons, investors may report the short horizon expectations relevant for their portfolio choice; and report longer horizon contrarian expectations for 6- and 12-month returns. But when asked only about 6-month or only about 12-month return expectations, investors may report the short horizon return expectations relevant for their portfolio choice. That is, the omission of

[13] The true proportions are a bit higher; several respondents select "other" and choose to fill in custom responses that indicate a view that stock prices are driven by others' overreaction or speculation acting in conjunction with additional forces, such as monetary policy.

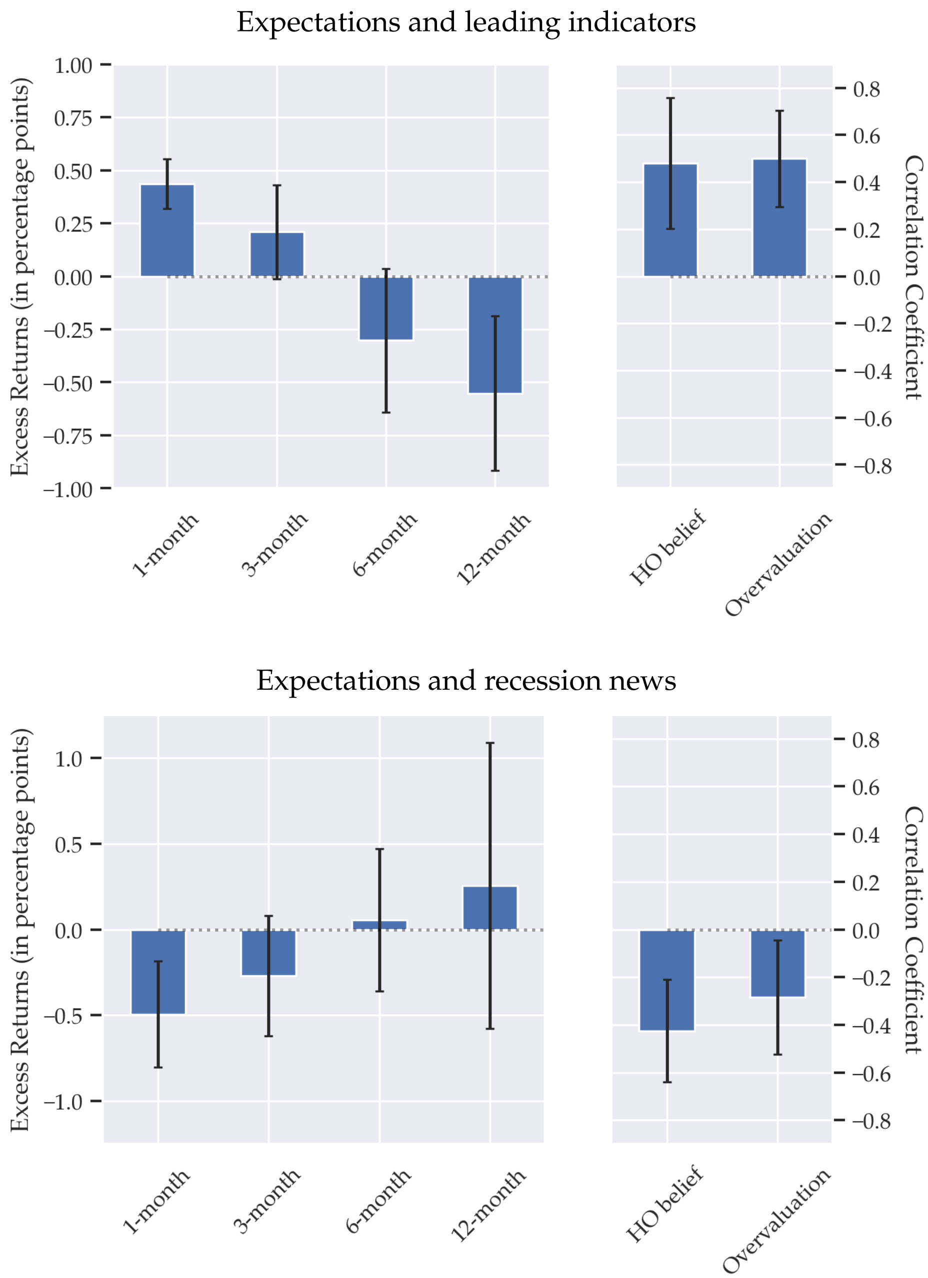

Figure 2: Macroeconomic news, return expectations, and higher-order beliefs

The figure plots coefficients from contemporaneous regressions of changes in quarterly average 1-, 3-, 6-, and 12-month excess return expectations, *HO belief*, and *Overvaluation* on measures of macroeconomic news. The measure of macroeconomic news in the first panel is AR(1) innovations in the quarterly average of the Conference Board Leading Economic indicators index, which is a composite index of 10 leading macroeconomic indicators. The measure of macroeconomic news in the second panel is AR(1) innovations in attention paid to recession news in the *Wall Street Journal* from Bybee et al. (2024). The independent variables, *HO belief* and *Overvaluation*, are scaled to have zero mean and unit standard deviation, and return expectations are multiplied by 100. Standard errors are Newey-West standard errors (four lags). The figure also plots plus and minus two standard errors for the estimated coefficients.

Alt text: Bar graph depicting the coefficients from regressions of return expectations, *HO belief*, and *Overvaluation* on macroeconomic news measures, with 95% confidence intervals.

	Pooled	Individual	Institutional
Coefficient	-0.36	-0.26	-0.25
	(0.22)	(0.21)	(0.19)
Mkt timing Sharpe	-0.31	-0.29	-0.23

Table 5: Return expectations and realized returns

The top row of the table displays coefficients from a regression of 1-month realized returns on investors' return expectations for the same period. Newey-West standard errors (12 lags) of coefficients are reported in parentheses. The last row of the table reports the Sharpe ratio of a market timing strategy that takes long and short positions in the stock market in proportion to the average respondent's return expectations for the next month.

questions about expectations at different horizons may lead to differences in reported beliefs.

We support this interpretation with two additional analyses. First, we show that return expectations from the American Association of Individual Investors (AAII) Investor Sentiment Survey, studied in Greenwood and Shleifer (2014), are highly correlated with short horizon return expectations reported in the Shiller survey; but lowly correlated with the longer horizon expectations. Second, we present evidence of extrapolative short horizon return expectations and contrarian longer horizon expectations in currency market surveys that solicit expectations at multiple horizons, which suggests that these may be more general features of expectations.

1.8 Return expectations and realizations

Our evidence suggests that investors' decisions are driven by their short horizon return expectations. Table 5 illustrates the performance associated with 1-month return expectations. The first row displays coefficients from regressions of realized returns on 1-month return expectations; coefficients are negative for the pooled, individual, and institutional samples. The table displays the Sharpe ratios of market-timing strategies that take long and short positions in the market in proportion to the average return expectation in the pooled, individual, and institutional samples; the Sharpe ratios are -0.31, -0.29, and -0.23, respectively. While the sample is limited in length, the evidence suggests that investors' short horizon return expectations are often wrong and that short-term speculation is unprofitable. Moreover, the evidence is in line with results found in survey data of longer time samples that investors' return expectations negatively predict future returns.

1.9 Summary and implications for theory

We can summarize the evidence presented in this section as follows:

(i) **(Perceived overreaction).** Investors perceive that the stock market overreacts to fundamental news; with positive news, investors believe that the market becomes overvalued and others become overly optimistic, and with negative news, they believe the market becomes undervalued and others become overly pessimistic.

(ii) **(Perceived time-series momentum and reversal).** Investors forecast that the stock market exhibits momentum and reversal in response to fundamental news.

(iii) **(Nonfundamental speculation).** When investors perceive others to be overly optimistic and markets to be overvalued, they forecast short-term returns to be high and long-term returns to be low. Speculators seek to buy into an overvalued stock market. They similarly forecast short-term returns to be low and long returns to be high when they perceive others to be overly pessimistic.

These results indicate that nonfundamental speculation is a pervasive feature of the U.S. stock market. Investors believe in overreaction-driven momentum and reversal, but buy into overvalued markets due to forecasts of potential short-term profits. Moreover, while previous work finds that nonfundamental speculation is profitable for informed investors in certain episodes, it appears unprofitable for the investors in our survey.

Below, we discuss our results in the context of existing models, which may help explain some, but not all of them.

Higher-order uncertainty. Previous work on higher-order beliefs has placed particular focus on higher-order uncertainty, namely, uncertainty about whether other investors agree with one's beliefs. Higher-order uncertainty has been used in models that predict underreaction to news and subsequent price drift (e.g., Allen, Morris, and Shin 2006; Banerjee, Kaniel, and Kremer 2009) or alternatively in models in which rational arbitrageurs engage in nonfundamental speculation due to uncertainty regarding other arbitrageurs' awareness of mispricing (Abreu and Brunnermeier 2002, 2003). The latter models are closer to our evidence, and may help explain it. But they require exogenous mispricing to arise and persist and are inconsistent with the poor average performance of nonfundamental speculation we find.

Return extrapolation. As discussed, our results relate to return extrapolation, where investors' return expectations correlate with past returns. However, existing models of return extrapolation do not address multiperiod return expectations or perceptions of others' beliefs. Beliefs in others' return extrapolation can lead to nonfundamental speculation (e.g., De Long et al. 1990, where rational speculators trade against extrapolators). However, speculators in De Long et al. (1990) would not find others overly optimistic contemporaneous with positive news shocks, and their nonfundamental speculation is profitable, unlike our evidence.

Errors in forecasting fundamentals. Prior work, highlighted in our literature review, has suggested that investors make systematic errors in forecasting assets' fundamentals. Without assumptions about higher-order beliefs, theories of fundamental belief mistakes can yield vastly different predictions about return expectations. For instance, if investors with mistaken fundamental beliefs assume others share their beliefs, they always expect constant returns absent time-varying risk premiums, contradicting our evidence.

A belief in other investors making errors in forecasting fundamentals can explain the evidence.[14] This features in the model we present in the next section. However, this is not the only configuration of the model that matches the evidence, as we discuss further.

Investor sentiment. A substantial literature examines *investor sentiment*, defined by Baker and Wurgler (2007) as "a belief about future cash flows and investment risks that is not justified by the facts." This may encompass errors in forecasting both fundamentals and returns. Sentiment does not directly address why investors buy into overvalued markets, whether due to excessive optimism about fundamentals or forecasts of other investors' future behavior. In our model, we discuss the role that each may play and their potential interactions.

Short-sale constraints. Dynamic models of short-sale constraints also predict nonfundamental speculation (Harrison and Kreps 1978; Scheinkman and Xiong 2003; Duffie, Garleanu, and Pedersen 2002). While short-sale constraints likely contribute, they only apply to overvalued markets with overly optimistic investors, and cannot match our evidence of nonfundamental speculation following negative news and with others perceived to be pessimistic.

Time-varying risk premiums. Expectations of time-varying returns are traditionally attributed to time-varying risk compensation demanded by investors (e.g., Campbell and Cochrane 1999; Bansal and Yaron 2004; Gabaix 2008; Wachter 2013). Models in this spirit typically predict countercyclical return expectations, inconsistent with our results. They also cannot explain investors' perceived belief disagreements.

2 Model of Nonfundamental Speculation

We present a stylized asset pricing model that interprets the empirical results, clarifies the relationship between return expectations and higher-order beliefs, and illustrates that higher-order beliefs amplify asset price overreaction and excess volatility.

As an expositional note, in the model, for convenience, we discuss investors expecting high returns and perceiving overvaluation corresponding with positive fundamentals. However, the model is symmetric and produces undervaluation and low return expectations when fundamentals are negative, consistent with the survey evidence.

[14] Martin and Papadimitriou (2022) propose a differences-of-opinion model where investors that are correct in hindsight become wealthier, causing the representative agent's belief to become more optimistic following good news. Internalizing this, investors may engage in nonfundamental speculation, potentially explaining part of the evidence.

2.1 Model setup

2.1.1 Model environment.

There is a risky asset (the stock market) and a riskless asset. The payoff of the riskless asset is normalized to zero. The risky asset pays a dividend D_t each period, where D_t evolves according to the process

$$\begin{aligned} D_t &= \mathbf{d}_t + v_t, \text{ where } v_t \sim N(0, \sigma_v^2), \text{ and} \\ \mathbf{d}_t &= \rho \mathbf{d}_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \sigma_\epsilon^2) \text{ and } \rho \in (0,1). \end{aligned} \tag{1}$$

The term $\mathbf{d}_t$ captures the persistent component of dividends, which we refer to as the asset's fundamentals, while v_t captures a transitory component of dividends. While dividends are observed each period, the underlying fundamentals are never revealed. The riskless asset is in perfectly elastic supply and the risky asset is in zero net supply.[15]

The model follows an overlapping generations structure. Each period, a unit mass of individually infinitesimal investors is born, indexed by $i \in [0,1]$. Investors born in period t make an investment decision in that period. In period $t+1$, they liquidate their investments, consume the proceeds, and pass their beliefs onto the newly born investor i. The assumption of overlapping generations is common in work on higher-order beliefs (e.g., Allen, Morris, and Shin 2006), and accentuates the importance of short-term price movements for traders. All investors have exponential utility, with risk aversion $\gamma > 0$.

In period t, in addition to observing the publicly announced dividend, D_t, each investor i also receives a private signal,

$$\begin{aligned} s_t^i =& s_t + \phi_t^i, \text{ where} \\ s_t =& \mathbf{d}_t + \eta_t, \\ \eta_t \sim& N(0, \sigma_\eta^2), \text{ and } \phi_t^i \sim N(0, \sigma_\phi^2). \end{aligned} \tag{2}$$

Each investor's private signal contains a common component that is informative about fundamentals, s_t, as well as idiosyncratic noise, ϕ_t^i. We later provide additional structure on how investors treat these signals in forming their higher-order beliefs.

There are two types of investors: a mass $\theta \in (0,1)$ of speculators, indexed by $i \in [0,\theta)$, and a mass $(1-\theta)$ of arbitrageurs, indexed by $i \in [\theta, 1]$. Investors of each type share beliefs about the parameters governing the economy with others of the same type.

Both investor types receive identical information about the risky asset's fundamentals and process it in the same way, but differ in their higher-order beliefs. Speculators are our primary focus, and have (mistaken) higher-order beliefs that align with those of the survey re-

[15]The assumption of zero net supply is not critical but focuses our analysis. In the Internet Appendix we outline a specification where the risky asset is in positive supply. As is standard, the level of supply determines the asset's risk premium. Positive supply also induces a persistent bias in speculators' return expectations.

spondents we study. Arbitrageurs have correct higher-order beliefs; the average arbitrageur's beliefs match rational expectations.

Each investor i's demand is given by

$$Q_t^i = \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})}, \tag{3}$$

where $\mathbb{E}_t^i(\cdot)$ and $\mathbb{V}_t^i(\cdot)$ are the subjective expectations and variance operators respectively, and P_t is the price of the risky asset in period t, determined by the market clearing condition $0 = \int_0^1 Q_t^i di$.

2.1.2 Investors' fundamental beliefs.

All investors are Bayesian in forming their beliefs about fundamentals, $\mathbf{d}_t$. Using their beliefs about the dividend process and their observations of past dividends, they form their expectations of $\mathbf{d}_t$ by Kalman filtering. We follow the common assumption that a sufficient number of periods have passed such that investors are in a learning steady state. This means that investors' Kalman gain—the weight they place on new information that arrives in period t versus their prior in forming their fundamental beliefs—is constant each period.

Before presenting the exact formulation of investors' belief updating, we make an assumption about how investors process their own and other investors' signals.

Assumption 1 (Differences-of-opinion) *The noise term in investor i's private signal, ϕ_t^i, is an idiosyncratic interpretation that i imputes to the informative component of s_t. Investors treat other investors' private signals as being uninformative about fundamentals conditional on their own. When updating their beliefs about $\mathbf{d}_t$, investor i treats their private signal s_t^i as if it has variance σ_η^2.*

The assumption that investors treat others' signals as uninformative follows in the spirit of 'differences-of-opinion' models (e.g., Harris and Raviv 1993; Kandel and Pearson 1995; Banerjee and Kremer 2010).[16] Given Assumption 1, investor i perceives that the average signal received by other investors, $s_t^{-i} \equiv \mathbb{E}_t^i(\int_0^1 s_t^j dj)$, is a biased signal about fundamentals.[17] With this assumption in hand, Lemma 1 outlines how investors' fundamental beliefs evolve.

Lemma 1 (**Fundamental beliefs**) *In steady state, investor i's beliefs about fundamentals, $\mathbf{d}_t$, evolve according to the updating process*

$$\mathbf{d}_t^i \equiv \mathbb{E}_t^i(\mathbf{d}_t) = (1 - \kappa_1 - \kappa_2)\rho \mathbf{d}_{t-1}^i + \kappa_1 D_t + \kappa_2 s_t^i, \tag{4}$$

[16] Differences-of-opinion mean that our model also matches cross-sectional belief heterogeneity in return expectations. Microfoundations for differences of opinion may include overconfidence (Odean 1998; Daniel and Hirshleifer 2015) or motivated reasoning (Banerjee, Davis, and Gondhi 2024).

[17] The treatment of s_t^i as having variance σ_η^2, rather than $\sigma_\eta^2 + \sigma_\phi^2$, means that the average fundamental belief is an unbiased signal of $\mathbf{d}_t$. In the alternative case, the average fundamental belief underreacts to news. This would not meaningfully affect our results, but we shut down this channel to clearly isolate the role of higher-order beliefs.

where

$$\begin{bmatrix}\kappa_1\\ \kappa_2\end{bmatrix} = \Sigma H^T(H\Sigma H^T + R)^{-1}, H = \begin{bmatrix}1\\ 1\end{bmatrix}, R = \begin{bmatrix}\sigma_v^2 & 0\\ 0 & \sigma_\eta^2\end{bmatrix}, \text{ and}$$

$$\Sigma = \rho^2\Sigma - \rho^2\Sigma H^T(H\Sigma H^T + R)^{-1}H\Sigma + \sigma_\epsilon^2.$$

Proof. All proofs are contained in Internet Appendix A. □

Investors update their beliefs in response to new information based on the signal-to-noise ratio of D_t and s_t. When these signals are informative about dividends, investors give them additional weight (higher κ_1 and κ_2), whereas they rely more on their priors when these signals are less informative. For notational convenience, we denote investor i's beliefs about fundamentals as $\mathbf{d}_t^i \equiv \mathbb{E}_t^i(\mathbf{d}_t)$.

2.1.3 Higher-order beliefs and equilibrium.

Next, we define equilibrium, describe investors' higher-order beliefs in the context of equilibrium, and derive expressions for the risky asset price.

Definition 2.1 (Equilibrium) *An equilibrium in period t is a combination of a price, P_t, and beliefs, such that*

(i) Investor i's demand, Q_t^i, maximizes their subjective expected utility;

(ii) Markets clear $\left(\int_0^1 Q_t^i di = 0\right)$; and

(iii) Investors' (potentially incorrect) higher-order beliefs and beliefs about fundamentals are consistent with the price they observe.

Next, we define a level k equilibrium, which imposes a particular structure on speculators' higher-order beliefs.

Definition 2.2 (Level k equilibrium) *A level $k = 1$ equilibrium is an equilibrium where speculators mistakenly believe that all other investors perceive the persistence of fundamentals as $\hat{\rho} \in (0, 1)$, and that all others see this perception as common knowledge.*[18] *For $k = 2, 3, \ldots$, a level k equilibrium is an equilibrium where speculators mistakenly believe that the equilibrium is a level $k-1$ equilibrium. In all level k equilibria, arbitrageurs have correct higher-order beliefs.*

In a level 1 equilibrium, speculators have a (mistaken) second-order belief that all other investors hold an identical, incorrect belief about the persistence of the fundamentals process. This belief means that a level 1 speculator i perceives that the average fundamental belief

[18] Here, the perception of common knowledge is that speculators perceive that all other investors agree on $\hat{\rho}$ as the persistence, and they know (that they know that they know) that they all agree (see, e.g., Aumann 1976).

evolves according to

$$\mathbf{d}_{t,2}^{i} \equiv \mathbb{E}_t^i\left(\int\limits_0^1 \mathbf{d}_t^j dj\right) = (1-(\hat{\kappa}_1+\hat{\kappa}_2))\hat{\rho}\mathbf{d}_{t-1,2}^{i} + \hat{\kappa}_1 D_t + \hat{\kappa}_2 s_t^{-i}, \tag{5}$$

where $\begin{bmatrix}\hat{\kappa}_1 & \hat{\kappa}_2\end{bmatrix}^T$ are speculators' second-order beliefs about Kalman gains, following the expression in Lemma 1. These differ from speculators' own Kalman gains, because speculators' belief that others misperceive the persistence of fundamentals means that they also believe that others differ in their speed of learning.

For $k > 1$, in a level k equilibrium, each speculator has a (mistaken) $k+1^{st}$-order belief that others misperceive the persistence of fundamentals.[19] For example, in a level 2 equilibrium, speculators hold a (mistaken) third-order belief that other speculators believe that all others misperceive the persistence of fundamentals. Speculators also believe that arbitrageurs share their higher-order beliefs, when in fact, arbitrageurs (correctly) hold fourth-order beliefs about the misperception of the persistence of fundamentals. A higher k reflects more sophisticated thinking, in the sense that speculators engage in more rounds of strategic reasoning.[20] Later, we consider the impact of increasing strategic reasoning (higher k).

With the definition of equilibrium, we next derive the equilibrium pricing function.

Lemma 2 (Level k Equilibrium Pricing Function) *For $k = 1, 2, \ldots$, the linear pricing rule for the risky asset in the stationary-level k equilibrium is*

$$P_t = B_k \mathbf{d}_t^s,$$

where $B_0 \equiv \frac{\hat{\rho}}{1-\hat{\rho}}$, B_k is defined recursively as the positive root of a cubic equation that is a function of B_{k-1} and deep parameters of the model, and $\mathbf{d}_t^s \equiv \int_0^1 \mathbf{d}_t^i di$ is the average of investors' first-order beliefs.

We briefly outline the proof of Lemma 2. The price of the risky asset is determined as the equilibrium outcome of speculators' and arbitrageurs' demand. The total demand from each investor type is proportional to the average of the ratio of expected returns to perceived variance of that investor type.

Given their higher-order beliefs, in a level k equilibrium, speculators (incorrectly) perceive that the pricing coefficient is B_{k-1} when it is actually B_k. In equilibrium, each speculator i's second-order belief about fundamentals ($\mathbf{d}_{t,2} \equiv \mathbb{E}_t^i(\mathbf{d}_t^s)$) must rationalize the true price of the risky asset with their perceived pricing function, that is, $\mathbb{E}_t^i(B_{k-1}\mathbf{d}_t^s) = B_{k-1}\mathbf{d}_{t,2} = B_k \mathbf{d}_t^s$. From

[19] The recursive structure of the level k equilibrium also implicitly pins down higher-order beliefs about the average belief about the level of fundamentals.

[20] Speculators in the level k equilibrium can be thought of as level k thinkers; that is, they treat other speculators as less sophisticated and as not playing the best response to their own behavior (see, e.g., Crawford, Costa-Gomes, and Iriberri 2013).

the asset price, speculators infer the signal that the average investor received, and hence the average belief of other investors, such that $\mathbf{d}_{t,2} = \frac{B_k}{B_{k-1}} \mathbf{d}_t^s$. Given their misperception of the pricing function, the average speculator misforecasts the risk and return of the risky asset.

The average arbitrageur has correct return expectations and risk perceptions regarding the risky asset. As is a common feature of models featuring short-lived investors trading a long-lived asset, arbitrageurs' beliefs about volatility in the model can become self-fulfilling, with the potential for multiple equilibria.[21] We discuss this multiplicity in more detail when discussing how the model matches the survey evidence.

2.2 Survey evidence and asset pricing implications

We describe how the model matches the evidence on investors' expectations and then explore its equilibrium asset pricing implications.

2.2.1 Matching the evidence.

In the context of the model, we can summarize the conditions required to match the evidence:

(i) **(Perceived overreaction and overvaluation)**: on average, when fundamentals are positive, the risky asset price exceeds the average speculator's valuation (their expected sum of future dividends). That is, $\mathbf{d}_t > 0 \implies P_t > \sum_{h=1}^{\infty} \rho^h \mathbf{d}_t^s = \frac{\rho}{1-\rho} \mathbf{d}_t^s$.

(ii) **(Perceived time-series momentum and reversal)**: on average, following a positive fundamental innovation, speculators perceive that the risky asset will exhibit positive short-term returns and negative long-term returns. That is, given an innovation $\epsilon_t > 0$, $\mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) > 0$ (momentum); and $\lim_{h\to\infty} \mathbb{E}_t^s P_{t+h} + \sum_{j=1}^{h} \rho^j \mathbf{d}_t^s - P_t < 0$ (reversal), where $\mathbb{E}_t^s$ is the expectation of the average speculator.

(iii) **(Nonfundamental speculation)**: the average speculator buys into the risky asset when they perceive other investors as overly optimistic and the risky asset as overvalued.

Based on these conditions, we derive the parameters for which the model matches the survey evidence:

Proposition 1 (Matching the Survey Evidence) *In a level k equilibrium, if, and only if, $\hat{\rho} > \rho$ (level 1 speculators perceive that other investors overestimate the persistence of fundamentals), the average speculator*

(i) *perceives that the risky asset is overvalued when fundamentals are positive;*

(ii) *perceives that the risky asset exhibits time-series momentum and reversal;*

[21] See Spiegel (1998), Bacchetta and Van Wincoop (2006), Banerjee (2011), Greenwood and Vayanos (2014), and Albagli (2015) for examples and discussions. We note that while other models commonly select a particular equilibrium to study, equilibrium selection does not affect the qualitative conclusions that we draw.

(iii) *and engages in nonfundamental speculation*.

In a level k equilibrium, when $\hat{\rho} > \rho$, speculators expect other investors to bid up the price of the risky asset when fundamentals are positive. But speculators' mistaken higher-order beliefs lead them to overestimate the degree to which the risky asset overreacts and to overestimate the equilibrium pricing coefficient as $B_{k-1} > B_k$. In the level 1 equilibrium, this overestimation is driven by a mistaken belief that other investors' valuations overreact due to their overestimation of the persistence of fundamentals. For $k > 1$, this overestimation is driven by a misunderstanding of the driver of other speculators' demand; in particular, a misperception that other speculators' higher-order beliefs about persistence are kth-order beliefs, when they are in fact $(k+1)^{st}$-order beliefs. Given their overestimation of the pricing coefficient, when fundamentals are positive, speculators infer a second-order belief about fundamentals that is lower than the true average belief about fundamentals, that is, $\mathbf{d}_{t,2} = \frac{B_k}{B_{k-1}}\mathbf{d}_t^s < \mathbf{d}_t^s$. The average speculator, whose belief about fundamentals is $\mathbf{d}_t^s$, knows the risky asset to be overvalued. But their mistakenly inferred second-order belief leads them to forecast that other investors will revise their fundamental beliefs upward and further demand the risky asset in the future. Speculators accordingly engage in nonfundamental speculation.

Note that speculators' recognition of the risky asset's overvaluation and positive expected returns coexist in equilibrium because speculators' positive expected returns cause overvaluation. When fundamentals are positive, the average speculator takes a long position in the risky asset, and the average arbitrageur takes an opposing short position.

Remark 1 (Mapping the model to the data) *The HO belief variable maps to the proportion of speculators that perceive the average investor to be overly optimistic and the risky asset to be overvalued. As we show in the Internet Appendix, with stronger fundamentals (higher $\mathbf{d}_t$), the average speculators' short-horizon return expectations are higher; and more speculators find the risky asset to be overvalued. This relationship between fundamentals, return expectations, and HO belief means that the model matches the evidence in Sections 1.4 and 1.6.*

Remark 2 (Equilibrium multiplicity) *Though there may be multiple equilibria, Proposition 1 applies generically to all potential equilibria. With multiple equilibria, the difference is in the quantitative degree of overvaluation, momentum, and reversal that speculators perceive. Moreover, multiplicity is only present under specific parameterizations, as we discuss in the Internet Appendix.*

2.2.2 Equilibrium asset pricing implications

Having matched the survey evidence, we explore the model's asset pricing implications.

Result 1 (Overreaction and reversal in equilibrium.) *Whenever speculators engage in nonfundamental speculation, given positive fundamentals in period t, the risky asset is overvalued, and has negative objective expected returns in subsequent periods.*

When fundamentals are positive, the risky asset is overvalued in period t, in spite of the average investor's belief about fundamentals matching rational expectations, due to speculators' higher-order belief-induced speculation. This also corresponds with asset prices overreacting to news since, on average, fundamentals are positive following good news and negative following bad news.

Following overvaluation, the risky asset price experiences a gradual reversal, corresponding with speculators revising their second-order beliefs about the average investor's belief about fundamentals downward. Speculators' initial excitement—that other investors would overreact even more, leading to short-term profits—turns out to be incorrect, resulting in negative forecast errors of returns.

Next, we explore how these asset pricing implications vary as speculators engage in more strategic reasoning (higher k).

Result 2 (Equilibrium and strategic reasoning)

(i) For a given $\hat{\rho} > \rho$, asset price overreaction is lower as investors engage in more rounds of strategic reasoning (higher k).

(ii) In the limit, as speculators have infinite depth of reasoning ($k \to \infty$),

- *(a) The asset price converges to its rational expectations fundamental value, that is, $\lim_{k\to\infty} B_k = \frac{\rho}{1-\rho}$;*
- *(b) Nonfundamental speculation disappears; speculators become arbitrageurs.*

Result 2 indicates that as we increase k, the risky asset price price overreacts less, and converges to the rational expectations fundamental value in the limit. For each level k of reasoning, equilibrium overreaction is attenuated relative to the level $k-1$ equilibrium that speculators believe holds. This attenuation is because in a level k equilibrium, speculators hold a $k+1^{st}$-order belief that other investors overestimate the persistence of fundamentals, when the rational belief is a $k+2^{nd}$-order belief that investors overestimate the persistence of fundamentals. Higher-order belief mistakes are less meaningful as we ascend the belief hierarchy; for example, speculators' inference about other investors' signals and behavior is more distorted when they have a mistaken second-order belief about persistence than a mistaken third-order belief about persistence. As $k \to \infty$, the impact of speculators' higher-order belief mistake vanishes, and speculators correctly understand other investors' behavior.

Figure 3 summarizes Proposition 1 and Results 1 and 2. For a set of chosen parameters, the figure plots the unique equilibrium price in period t for $\mathbf{d}_t^s = \mathbf{d}_t = 1$, for different levels of k. The figure displays the period t cumulative return expectations of the average speculator in blue and the average realized returns in red. The asset price exceeds the average speculator's valuation in period t. Despite this perceived overvaluation, the average speculator expects to earn even more positive returns in period $t+1$, though they expect the cumulative returns to eventually revert to their buy-and-hold valuations. This pattern matches the survey evidence.

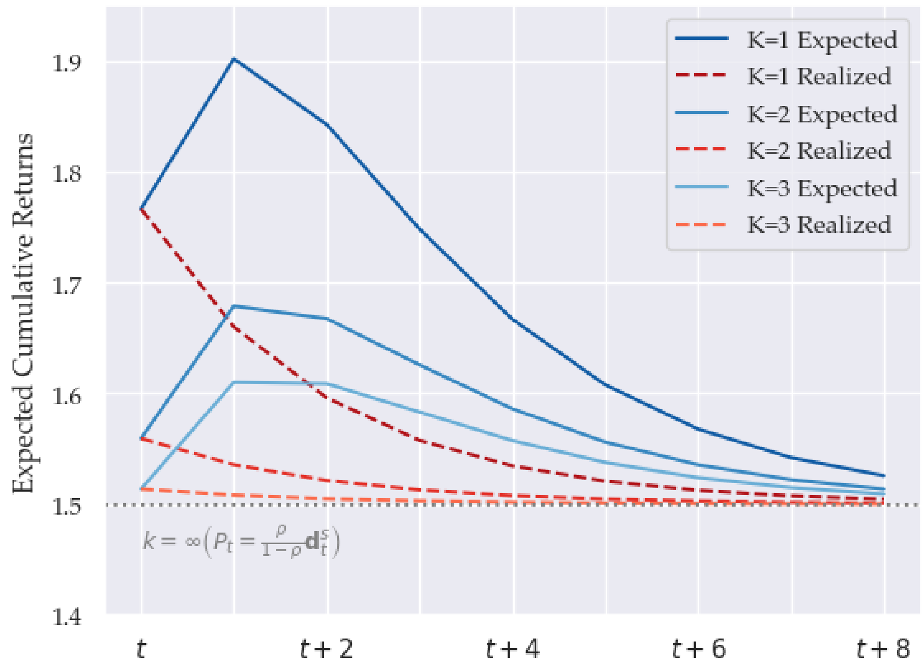

Figure 3: Nonfundamental speculation, overreaction, and reversal

The figure plots the asset price in period t given $\mathbf{d}_t^s = \mathbf{d}_t = 1$, for different levels of strategic sophistication, k. The blue line represents speculators' cumulative return expectations from period t to $t+h$. The red line represents the average realized cumulative returns from period t to $t+h$. The illustrative parameter values used are $(\theta, \rho, \hat{\rho}, \sigma_\epsilon^2, \sigma_v^2, \sigma_\eta^2, \sigma_\phi^2) = (0.5, 0.6, 0.7, 1, 1.13, 1.13, 1.13)$. The choice of σ_v^2 and σ_η^2 is made to set the Kalman gains to be $\kappa_1 = \kappa_2 = 0.2$.

Alt text: Line chart plotting the average speculator's cumulative return expectations and the corresponding realized returns for different levels of strategic sophistication k.

Realized cumulative returns are negative as the risky asset price reverts in periods subsequent to t, as speculators' forecasts of increasing optimism do not manifest. As we increase strategic reasoning, k, overvaluation decreases, as seen by lower risky asset prices in period t, and reversals that are less sharp.

We conclude our theoretical analysis with two additional remarks.

Remark 3 (Interaction with fundamental beliefs) *Result 1 isolates the impact of higher-order beliefs on the asset price by assuming that the average belief about fundamentals* $\mathbf{d}_t^s$ *matches rational expectations. Relaxing this assumption, asset prices rely on the interaction of fundamental and higher-order beliefs. If expectations of fundamentals respond sluggishly to news, then asset prices may display momentum in addition to overreaction and reversal. If fundamental beliefs overreact to news, then asset price overreaction may be amplified.*

Remark 4 (Models of rational nonfundamental speculation) *Our conclusion that nonfundamental speculation does not survive as* $k \to \infty$ *differs from those of models where sophisticated investors profitably engage in nonfundamental speculation, such as De Long et al. (1990) and Abreu and Brun-*

nermeier (2002, 2003). The difference is that we endogenize mispricing as coming from speculators that engage in nonfundamental speculation, whereas the mispricing in those models arises from exogenous sources or mechanical feedback traders.

3 Conclusion

We study investors' higher-order beliefs using survey data from the Robert Shiller Investor Confidence surveys. We find that nonfundamental speculation—investors taking positions in a risky asset in a direction that conflicts with their fundamental views—is pervasive in the U.S. stock market. The majority of Shiller survey respondents, an important class of investors, report that other investors have mistaken beliefs, but nevertheless report positive return expectations from speculating in the direction of these mistaken beliefs. Investors' nonfundamental speculation is unprofitable, however; investors' short-term return expectation perform poorly in predicting market returns. We rationalize the evidence in a model of higher-order beliefs, which reveals that investors' nonfundamental speculation may amplify stock market overreaction and excess volatility.

Code Availability: The replication code and data are available in the Harvard Dataverse at https://doi.org/10.7910/DVN/PXXSXP.

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Internet Appendix for

Speculating on Higher-Order Beliefs

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There are three sections. Appendix A contains proofs for the theoretical results in the paper. Appendix B presents a version of the model in the paper with the risky asset in fixed supply rather than zero net supply, which suggests that higher-order beliefs may induce a bias in speculators' return expectations in addition to affecting their cyclicality. Appendix C provides additional empirical analyses, including discussions of the Shiller survey, return extrapolation, and replications of the main results for different subsets of the data.

A Proofs

Proof of Lemma 1

Proof. Given the linear-Gaussian environment, belief updating follows the standard steady-state Kalman filter recursion method derived in Hamilton (2020). □

Proof of Lemma 2

Proof. We split our proof into parts. First, we derive B_0, the pricing function that speculators believe holds in a level 1 equilibrium. We then derive B_1, which is a special case, because speculators believe that other investors misperceive the persistence of fundamentals in a level 1 equilibrium, whereas they believe that others correctly perceive persistence when $k > 1$. Then, we derive B_k for $k > 1$. For notational simplicity, we define $\kappa = \kappa_1 + \kappa_2$, $\hat{\kappa} = \hat{\kappa}_1 + \hat{\kappa}_2$. Throughout the proofs, we freely make use of the substitution $\frac{1}{\theta}\int_0^\theta \mathbf{d}_t^i di = \frac{1}{1-\theta}\int_\theta^1 \mathbf{d}_t^i di = \mathbf{d}_t^s = \mathbf{d}_t$, i.e., the average speculator and the average arbitrageur have the same beliefs about fundamentals, which are equal to the true level of the unobserved fundamental.

Derivation of B_0: Consider an economy where all investors believe that the persistence of fundamentals is $\hat{\rho}$ and this perception is common knowledge.

Each investor i conjectures a pricing formula of the form $B_0\mathbf{d}_t^s$, where $\mathbf{d}_t^s \equiv \int_0^1 \mathbf{d}_t^i di$ is the average investor's belief about fundamentals. The market clearing condition is

$$
\begin{aligned}
0 = \int_0^1 Q_t^i di = \int_0^1 \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t)}{\mathbb{V}_t^i(P_{t+1} + D_{t+1})} di = \int_0^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t) di \\
= \hat{\rho}\mathbf{d}_t^s + B_0\hat{\rho}\mathbf{d}_t^s - P_t.
\end{aligned} \tag{A.1}
$$

Re-writing Equation (A.1) yields that $P_t = \hat{\rho}(1 + B_0)\mathbf{d}_t^s$. Matching coefficients, we get that $B_0 = (1 + B_0)\hat{\rho}$, yielding that $B_0 = \frac{\hat{\rho}}{1-\hat{\rho}}$.

Derivation of B_1: We conjecture that the pricing formula is of the form $P_t = B_1\mathbf{d}_t^s$. Then note that, by the definition of equilibrium, in particular that speculators' perceived price must coincide with the true price, we have that $\mathbf{d}_{t,2} = \frac{B_1}{B_0}\mathbf{d}_t^s$.

To forecast the price in period $t+1$, speculator i forecasts the average belief in period $t+1$, based on their forecast of $\mathbf{d}_{t+1}$ and s_{t+1}:

$$
\begin{aligned}
\mathbb{E}_t^i(\mathbf{d}_{t+1,2}) = \mathbb{E}_t^i \int_0^1 \mathbf{d}_{t+1}^j dj &= (1 - (\hat{\kappa}_1 + \hat{\kappa}_2))\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1 \underbrace{\mathbb{E}_t(D_{t+1})}_{=\rho\mathbf{d}_t^i} + \hat{\kappa}_2 \underbrace{\mathbb{E}_t(s_{t+1}^{-i})}_{=\rho\mathbf{d}_t^i} \\
&= (1 - \hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}\rho\mathbf{d}_t^i.
\end{aligned}
$$

Speculator i's expected period $t+1$ payoff is

$$
\begin{aligned}
\mathbb{E}_t^i(D_{t+1} + P_{t+1}) = & \underbrace{\rho\mathbf{d}_t^i}_{=\mathbb{E}_t^i(D_{t+1})} + \underbrace{\mathbb{E}_t^i(B_0\mathbf{d}_{t+1,2})}_{=\mathbb{E}_t^i(P_{t+1})} \\
= & (1 + \hat{\kappa}B_0)\rho\mathbf{d}_t^i + (1 - \hat{\kappa})\hat{\rho}B_0\mathbf{d}_{t,2}.
\end{aligned}
$$

Speculator i's subjective perceived variance of the period $t+1$ dividend is

$$
\mathbb{V}_t^j(D_{t+1}) = \sigma_\epsilon^2 + \sigma_v^2,
$$

and his subjective perceived variance of the period $t+1$ price is

$$
\begin{aligned}
\mathbb{V}_t^j(P_{t+1}) &= \mathbb{V}_t^j\left(\frac{\hat{\rho}}{1-\hat{\rho}}\mathbf{d}_{t+1}^j\right) \\
&= B_0^2\mathbb{V}_t^j((1 - \hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1 D_{t+1} + \hat{\kappa}_2 s_{t+1}^{-i}) \\
&= B_0^2\left(\hat{\kappa}_1^2(\sigma_\epsilon^2 + \sigma_v^2) + \hat{\kappa}_2^2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\hat{\kappa}_1\hat{\kappa}_2\sigma_\epsilon^2\right).
\end{aligned}
$$

Hence, his perceived variance of the period $t+1$ payoff is given by

$$
\begin{aligned}
\mathbb{V}_t^i(P_{t+1}+D_{t+1}) =& \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbb{C}(P_{t+1}, D_{t+1})}_{=B_0(\hat{\kappa}\sigma_\epsilon^2+\hat{\kappa}_1\sigma_v^2)}) \\
=&(1+\hat{\kappa}B_0)^2\sigma_\epsilon^2 + (1+\hat{\kappa}_1 B_0)^2\sigma_v^2 + B_0^2\hat{\kappa}_2^2(\sigma_\eta^2+\sigma_\phi^2).
\end{aligned}
$$

This perceived variance does not depend on the coefficient of interest, B_1, so, we denote it as a constant $A_S \equiv \mathbb{V}_t^i(P_{t+1}+D_{t+1})$. Aggregate speculator demand is then given by

$$
\begin{aligned}
\int_0^\theta Q_t^i di =& \int_0^\theta \frac{\mathbb{E}_t^i(P_{t+1}+D_{t+1}) - P_t}{\gamma \mathbb{V}_t^i(P_{t+1}+D_{t+1})} di \\
=& \frac{\theta}{\gamma} \frac{(1+\hat{\kappa}B_0)\rho \mathbf{d}_t^s + (1-\hat{\kappa})\hat{\rho}B_0 \mathbf{d}_{t,2} - P_t}{A_S},
\end{aligned}
$$

where the second line comes from substituting $\int_0^\theta \mathbf{d}_t^i di = \theta \mathbf{d}_t^s$. Substituting $\mathbf{d}_{t,2} = \frac{B_1}{B_0}\mathbf{d}_t^s$, this becomes

$$
\int_0^\theta Q_t^i di = \frac{\theta}{\gamma} \frac{((1+\hat{\kappa}B_0)\rho + (1-\hat{\kappa})\hat{\rho}B_1)\mathbf{d}_t^s - P_t}{A_S}.
$$

Turning to the arbitrageurs, they know that the form of the pricing rule is $B_1\mathbf{d}_t^s$, and that other investors correctly perceive the parameters governing the risky asset's fundamentals. Hence, arbitrageur i's expected period $t+1$ payoff is

$$
\mathbb{E}_t^i(P_{t+1}+D_{t+1}) = \rho \mathbf{d}_t^i + B_1((1-\kappa)\rho \mathbf{d}_t^s + \kappa\rho \mathbf{d}_t^i).
$$

Arbitrageur i's perceived variance of dividends and the next period's price are

$$
\begin{aligned}
\mathbb{V}_t^i(D_{t+1}) =& \sigma_\epsilon^2 + \sigma_v^2, \text{ and} \\
\mathbb{V}_t^i(P_{t+1}) =& B_1^2(\kappa_1(\sigma_\epsilon^2+\sigma_v^2) + \kappa_2(\sigma_\epsilon^2+\sigma_\eta^2+\sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2).
\end{aligned}
$$

Hence, arbitrageur i's perceived variance of the period $t+1$ payoff is

$$
\begin{aligned}
\mathbb{V}_t^i(P_{t+1}+D_{t+1}) =& \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbb{C}(P_{t+1}, D_{t+1})}_{=B_1(\kappa\sigma_\epsilon^2+\kappa_1\sigma_v^2)} \\
=&(1+\kappa B_1)^2\sigma_\epsilon^2 + (1+\kappa_1 B_1)^2\sigma_v^2 + B_1^2\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2).
\end{aligned}
$$

This perceived variance is quadratic in B_1. For notational simplicity, we define A_0, A_1, and

A_2 as the quadratic equation coefficients, i.e.,

$$\mathbb{V}_t^i(P_{t+1}+D_{t+1}) = \underbrace{(\kappa_1^2\sigma_v^2+\kappa^2\sigma_\epsilon^2+\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2))}_{A_2\equiv} B_1^2 + \underbrace{2(\kappa_1\sigma_v^2+\kappa\sigma_\epsilon^2)}_{A_1\equiv} B_1 + \underbrace{\sigma_v^2+\sigma_\epsilon^2}_{A_0\equiv},$$

and note that $A_0, A_1, A_2 > 0$.

Arbitrageur demand is then given by

$$\begin{aligned}\int_\theta^1 Q_t^i di &= \int_\theta^1 \frac{\mathbb{E}_t^i(P_{t+1}+D_{t+1})-P_t}{\gamma\mathbb{V}_t^i(P_{t+1}+D_{t+1})}di \\ &= \frac{(1-\theta)}{\gamma}\frac{\rho(1+B_1)\mathbf{d}_t^s-P_t}{A_2B_1^2+A_1B_1+A_0},\end{aligned}$$

where the second line comes from the fact that $\int_\theta^1 \mathbf{d}_t^i di = (1-\theta)\mathbf{d}_t^s$. Imposing market clearing ($\int_0^1 Q_t^i = \int_0^\theta Q_t^i di + \int_\theta^1 Q_t^i di = 0$), and solving for P_t, we get that

$$P_t = \frac{\theta(A_0+A_1B_1+A_2B_1^2)(\rho(1+\hat{\kappa}B_0)+\hat{\rho}(1-\hat{\kappa})B_1)+(1-\theta)A_S\rho(1+B_1)}{(1-\theta)A_S+\theta(A_0+A_1B_1+A_2B_1^2)}\mathbf{d}_t^s.$$

Matching coefficients, we have that

$$B_1 = \frac{\theta(A_0+A_1B_1+A_2B_1^2)(\rho(1+\hat{\kappa}B_0)+\hat{\rho}(1-\hat{\kappa})B_1)+(1-\theta)A_S\rho(1+B_1)}{(1-\theta)A_S+\theta(A_0+A_1B_1+A_2B_1^2)}.$$

Multiplying both sides by $(1-\theta)A_S+\theta(A_0+A_1B_1+A_2B_1^2)$, subtracting the resulting left hand side from both sides, and simplifying, we get that B_1 is the solution to the cubic equation

$$\begin{aligned}0 = {} & \theta(1-(1-\hat{\kappa})\hat{\rho})A_2B_1^3 - \frac{\theta(1-(1-\hat{\kappa})\hat{\rho})(\rho A_2-(1-\hat{\rho})A_1)}{1-\hat{\rho}}B_1^2 \\ & + \left(A_S(1-\theta)(1-\rho)+\theta(1-(1-\hat{\kappa})\hat{\rho})A_0-\theta\rho\left(\frac{1-(1-\hat{\kappa})\hat{\rho}}{1-\hat{\rho}}\right)A_1\right)B_1 \\ & - \rho\left(A_S(1-\theta)+\theta\left(\frac{1-(1-\hat{\kappa})\hat{\rho}}{1-\hat{\rho}}\right)A_0\right).\end{aligned} \tag{A.2}$$

Derivation of B_k, $k>1$: Speculators perceive the period t price as governed by the level $k-1$ pricing function, i.e., speculator i perceives that the equilibrium price is given by

$$P_t = \mathbb{E}_t^i\left(B_{k-1}\int_0^1 \mathbf{d}_t^j dj\right) = B_{k-1}\mathbf{d}_{t,2},$$

where the second-order belief about fundamentals, $\mathbf{d}_{t,2}$, is equal across all speculators because B_{k-1} is a constant and P_t is the same across all investors. We conjecture that the true pricing formula is of the form $B_k \mathbf{d}_t^s$. Note that in equilibrium, for speculators' beliefs to be consistent with the price they observe, we must have that $\mathbf{d}_{t,2} = \frac{B_k}{B_{k-1}} \mathbf{d}_t^s$.

The average speculator's forecasted expected period $t+1$ payoff is then given by

$$\begin{aligned} \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1}) di =& \rho \mathbf{d}_t^s + B_{k-1}((1-\kappa)\rho \mathbf{d}_{t,2} + \kappa \rho \mathbf{d}_t^s) \\ =& (1 + \kappa B_{k-1} + (1-\kappa) B_k) \rho \mathbf{d}_t^s. \end{aligned} \tag{A.3}$$

Each speculator i's forecast variance of the period $t+1$ dividend and price are given by

$$\begin{aligned} \mathbb{V}_t^i(D_{t+1}) =& \sigma_\epsilon^2 + \sigma_v^2, \text{ and} \\ \mathbb{V}_t^i(P_{t+1}) =& B_{k-1}^2 (\kappa_1^2(\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2^2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2), \end{aligned}$$

and speculator i's forecast variance of the period $t+1$ payoff is

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = (1 + \kappa B_{k-1})^2 \sigma_\epsilon^2 + (1 + \kappa_1 B_{k-1})^2 \sigma_v^2 + B_{k-1}^2 \kappa_2^2 (\sigma_\eta^2 + \sigma_\phi^2).$$

Defining, A_2, A_1, and A_0 as in the $k = 1$ case, note that this forecast variance can be written as

$$\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = \underbrace{(\kappa_1^2 \sigma_v^2 + \kappa^2 \sigma_\epsilon^2 + \kappa_2^2(\sigma_\eta^2 + \sigma_\phi^2))}_{A_2 =} B_{k-1}^2 + \underbrace{2(\kappa_1 \sigma_v^2 \kappa + \kappa \sigma_\epsilon^2)}_{A_1 =} B_{k-1} + \underbrace{\sigma_v^2 + \sigma_\epsilon^2}_{A_0 =}.$$

Hence, the total speculator demand can be written as

$$\int_0^\theta Q_t^i di = \int_0^\theta \frac{\theta}{\gamma} \frac{(1 + \kappa B_{k-1} + (1-\kappa) B_k) \rho \mathbf{d}_t^s - P_t}{A_2 B_{k-1}^2 + A_1 B_{k-1} + A_0} di. \tag{A.4}$$

The average arbitrageur, on the other hand, knows that the form of the period t price is $B_k \mathbf{d}_t^s$. Accordingly, their forecasted expected period $t+1$ payoff is

$$\begin{aligned} \frac{1}{1-\theta} \int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1}) di =& \rho \mathbf{d}_t^s + B_k \rho \mathbf{d}_t^s \\ =& (1 + B_k) \rho \mathbf{d}_t^s. \end{aligned} \tag{A.5}$$

Each arbitrageur i's forecast variance of the period $t+1$ price and dividend is

$$\begin{aligned} \mathbb{V}_t^i(D_{t+1}) =& \sigma_\epsilon^2 + \sigma_v^2, \text{ and} \\ \mathbb{V}_t^i(P_{t+1}) =& B_k^2 (\kappa_1^2(\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2^2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2), \end{aligned}$$

and arbitrageur i's forecast variance of the period $t+1$ payoff is

$$\mathbb{V}_t^i(P_{t+1}+D_{t+1}) = (1+\kappa B_k)^2\sigma_\epsilon^2 + (1+\kappa_1 B_k)^2\sigma_v^2 + B_k^2\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2),$$

which can be re-written as

$$\mathbb{V}_t^i(P_{t+1}+D_{t+1}) = \underbrace{(\kappa_1^2\sigma_v^2+\kappa^2\sigma_\epsilon^2+\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2))}_{A_2=} B_k^2 + \underbrace{2(\kappa_1\sigma_v^2\kappa+\kappa\sigma_\epsilon^2)}_{A_1=} B_k + \underbrace{\sigma_v^2+\sigma_\epsilon^2}_{A_0=}.$$

Then, the total arbitrageur demand can be written as

$$\int_\theta^1 Q_t^i di = \frac{1-\theta}{\gamma}\frac{(1+B_k)\rho\mathbf{d}_t^s - P_t}{A_2B_k^2+A_1B_k+A_0}.$$

Imposing the market clearing condition, $\int Q_t^i di = 0$, re-writing in terms of P_t, and matching coefficients, we find that B_k is the solution to the cubic equation

$$0 = y_3B_k^3 + y_2B_k^2 + y_1B_k + y_0,$$

where

$$\begin{aligned}
y_3 \equiv& \theta(1-(1-\kappa)\rho)A_2,\\
y_2 \equiv& \theta((1-(1-\kappa)\rho)A_1 - \rho A_2(1+\kappa B_{k-1})),\\
y_1 \equiv& (1-(1-\theta\kappa)\rho)A_0 + (1-\theta)(1-\rho)A_2B_{k-1}^2 + A_1((1-\rho-\theta(1-(1-\kappa)\rho))B_{k-1}-\theta\rho), \text{ and}\\
y_0 \equiv& -A_0\rho(1+\theta\kappa B_{k-1}) - (1-\theta)\rho(A_1B_{k-1}+A_2B_{k-1}^2).
\end{aligned}$$

Positivity of B_k: We prove by induction. Assume, by contradiction, that $B_1 \le 0$. Assume without loss of generality that $\mathbf{d}_t > 0$. We know that $B_0 = \frac{\hat{\rho}}{1-\hat{\rho}} > 0$.

We denote the average speculator's expected return and perceived variance as

$$\begin{aligned}
ER_t^S =& \frac{1}{\theta}\int_0^\theta \mathbb{E}_t^i(P_{t+1}+D_{t+1}-P_t)di, \text{ and}\\
V_t^S =& \frac{1}{\theta}\int_0^\theta \mathbb{V}_t^i(P_{t+1}+D_{t+1}-P_t)di,
\end{aligned}$$

and the average arbitrageur's expected return and perceived variance as

$$ER_t^A = \frac{1}{1-\theta}\int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t)di, \text{ and}$$
$$V_t^A = \frac{1}{1-\theta}\int_\theta^1 \mathbb{V}_t^i(P_{t+1} + D_{t+1} - P_t)di.$$

The market clearing condition implies that

$$0 = \frac{\theta ER_t^S V_t^A + (1-\theta)ER_t^A V_t^S}{V_t^A V_t^S}.$$

Since θ, $(1-\theta)$, V_t^S, and V_t^A are all positive, for the market to clear, we must have that $\text{Sign}(ER_t^S) = -\text{Sign}(ER_t^A)$, i.e., the average speculator and arbitrageur must have opposite sign expected returns.

The objective expected return of the risky asset is positive, i.e.,

$$\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = \underbrace{\rho \mathbf{d}_t}_{>0} + \underbrace{B_1}_{\leq 0}(\underbrace{\rho \mathbf{d}_t - \mathbf{d}_t}_{\leq 0}) > 0.$$

Because arbitrageurs have correct fundamental beliefs on average ($\frac{1}{1-\theta}\int \mathbf{d}_t^i di = \mathbf{d}_t$), and because they know the form of the true pricing formula, the average arbitrageur's expected return is correct. It follows that the average speculator must have negative expected returns.

Next note that $\mathbf{d}_{t,2} = \frac{B_1}{B_0}\mathbf{d}_t < 0$, since $B_0 > 0$, $B_1 < 0$, and $\mathbf{d}_t > 0$. The average speculator's expected return is

$$ER_t^S = \underbrace{\rho \mathbf{d}_t^s}_{>0} + B_0(\underbrace{(1-\hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}\rho \mathbf{d}_t - \mathbf{d}_{t,2}}_{>0}) > 0.$$

But this contradicts that the average speculator's expected return is negative. Hence $B_1 > 0$.

For $k > 1$, assume that $B_{k-1} > 0$. Positivity follows by an identical argument as for $B_1 > 0$, replacing B_0 and B_1 with B_{k-1} and B_k.

□

Proof of Proposition 1

Proof. Without loss of generality, we assume that $\mathbf{d}_t > 0$. We first prove a set of lemmas.

Lemma A.1 *Objective expected returns are negative if and only if* $B_k > \frac{\rho}{1-\rho}$.

Proof.

$$
\begin{aligned}
0 > & \mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = \rho \mathbf{d}_t + B_k \underbrace{\mathbb{E}_t(\mathbf{d}_{t+1} - \mathbf{d}_t)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\
\iff & B_k > \frac{\rho}{1-\rho}.
\end{aligned}
$$

□

Lemma A.2 *$B_1 > \frac{\rho}{1-\rho}$ if and only if $B_0 > \frac{\rho}{1-\rho}$.*

Proof. First we note that $\hat{\rho} > \rho \iff B_0 = \frac{\hat{\rho}}{1-\hat{\rho}} > \frac{\rho}{1-\rho}$. For the if direction, assume that $\hat{\rho} > \rho$, and assume by contradiction that $B_1 \leq \frac{\rho}{1-\rho}$. Then the objective one-period ahead expected return of the risky asset is positive:

$$
\begin{aligned}
\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) = & \rho \mathbf{d}_t + B_1 \underbrace{\mathbb{E}_t(\mathbf{d}^s_{t+1} - \mathbf{d}^s_t)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\
\geq & \rho \mathbf{d}_t - \frac{\rho}{1-\rho}(1-\rho)\mathbf{d}_t \\
= & 0.
\end{aligned}
$$

Because the average arbitrageur has correct beliefs and knows the form of the equilibrium pricing rule, they also have must have a positive one-period ahead expected return. Hence, by the same argument in the proof of Lemma 2, the average speculator must have a negative one-period ahead expected return.

Next, considering speculators' perception of the economy, their second-order belief must satisfy $\mathbf{d}_{t,2} = \frac{B_1}{B_0}\mathbf{d}_t$. Making use of the substitution that $\mathbf{d}^s_t = \frac{1}{\theta}\int_0^\theta \mathbf{d}^i_t di = \mathbf{d}_t$, we can write the average speculator's expected returns as

$$
\begin{aligned}
\frac{1}{\theta}\int_0^\theta \mathbb{E}^i_t(P_{t+1} + D_{t+1})di - P_t = & \rho \mathbf{d}_t + B_0((1-\hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}\rho \mathbf{d}_t - \mathbf{d}_{t,2}) \\
= & (\rho - (1-\hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho}B_1))\mathbf{d}_t \left(\text{substituting } \mathbf{d}_{t,2} = \frac{B_1}{B_0}\mathbf{d}_t\right) \qquad \text{(A.6)} \\
\geq & \frac{\rho((1-\hat{\kappa})\hat{\rho} + (1-\rho)\hat{\kappa}B_0 - \rho)}{1-\rho}\mathbf{d}_t \left(\text{substituting } B_1 \leq \frac{\rho}{1-\rho}\right) \\
> & \frac{\rho(1-\hat{\kappa})(\hat{\rho}-\rho)}{1-\rho}\mathbf{d}_t \text{ since } B_0 > \frac{\rho}{1-\rho} \\
> & 0 \text{ since } \hat{\rho} > \rho.
\end{aligned}
$$

But this is a contradiction since the average speculator's expected return must be negative. Hence we must have that $B_1 > \frac{\rho}{1-\rho}$.

For the only if direction, assume that $B_1 > \frac{\rho}{1-\rho}$. Then the objective one-period ahead expected return is negative:

$$\begin{aligned}
\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) =& \rho \mathbf{d}_t + B_1 \underbrace{\mathbb{E}_t(\mathbf{d}_{t+1} - \mathbf{d}_t)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\
<& \rho \mathbf{d}_t - \frac{\rho}{1-\rho}(1-\rho)\mathbf{d}_t \\
=& 0.
\end{aligned} \tag{A.7}$$

Hence, by similar argument as before, the average speculator's one-period ahead expected return is positive. This implies that

$$\begin{aligned}
0 <& \frac{1}{\theta}\int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1})di - P_t \\
=& (\rho - (1-\hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho} B_1)) \\
<& \frac{\rho(\hat{\rho} - \rho)(1 - (1-\hat{\kappa})\hat{\rho})}{(1-\rho)(1-\hat{\rho})}\mathbf{d}_t \left(\text{substituting } B_0 = \frac{\hat{\rho}}{1-\hat{\rho}} \text{ and } B_1 > \frac{\rho}{1-\rho}\right),
\end{aligned}$$

which is true if and only if $\hat{\rho} > \rho$ (and hence if and only if $B_0 > \frac{\rho}{1-\rho}$).

□

Lemma A.3 *$B_k > \frac{\rho}{1-\rho}$ if and only if $B_{k-1} > \frac{\rho}{1-\rho}$, for $k = 2, 3, \ldots$.*

Proof. The proof of the if direction is identical to the proof of A.2, replacing $\hat{\kappa}$, $\hat{\rho}$, B_0, and B_1 with κ, ρ, B_{k-1} and B_k.

For the only if direction, assume that $B_k > \frac{\rho}{1-\rho}$. Then the objective one-period ahead expected return is negative:

$$\begin{aligned}
\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) =& \rho \mathbf{d}_t + B_k \underbrace{\mathbb{E}_t(\mathbf{d}_{t+1} - \mathbf{d}_t)}_{=\rho \mathbf{d}_t - \mathbf{d}_t} \\
<& \rho \mathbf{d}_t - \frac{\rho}{1-\rho}(1-\rho)\mathbf{d}_t \\
=& 0.
\end{aligned}$$

Again, this means that the average speculator's one-period ahead expected return is positive. Hence,

$$
\begin{aligned}
0 <& \frac{1}{\theta} \int_0^\theta \mathbb{E}_t^i (P_{t+1} + D_{t+1}) di - P_t \\
=& (\rho - (1-\rho) B_k + \kappa\rho(B_{k-1} - B_k)) \mathbf{d}_t \qquad \text{(A.8)} \\
<& \kappa\rho(B_{k-1} - \frac{\rho}{1-\rho}) \mathbf{d}_t \left(\text{substituting } B_k > \frac{\rho}{1-\rho} \right),
\end{aligned}
$$

which is true if and only if $B_{k-1} > \frac{\rho}{1-\rho}$. □

Note that by induction, Lemmas A.2 and A.3 are equivalent to $\hat{\rho} > \rho \iff B_k > \frac{\rho}{1-\rho}, k = 1, 2, \ldots$. With Lemmas A.1, A.2, and A.3 in hand, we turn to the proof of the main claim.

For claim (i), we can see the average speculator's belief about fundamentals is correct, i.e., $\mathbf{d}_t^s = \frac{1}{\theta} \int_0^\theta \mathbf{d}_t^i di = \mathbf{d}_t$. Because they also know that the true persistence of the fundamental process is ρ, the average speculator perceives that the fundamental (buy-and-hold) value of the asset is $\frac{\rho}{1-\rho} \mathbf{d}_t^s$. Using Lemmas A.2 and A.3, this valuation is less than the price, $B_k \mathbf{d}_t^s$, if and only if $\hat{\rho} > \rho$, hence proving claim (i).

For time series momentum in claim (ii), using Lemma A.1, because the arbitrageur has correct beliefs on average and knows the form of the equilibrium pricing rule, they expect negative returns for the risky asset one period ahead if and only if $\hat{\rho} > \rho$. By similar argument as before regarding the opposing signs of the average speculator's and arbitrageur's return expectations, the average speculator must have positive expected returns if and only if $\hat{\rho} > \rho$. Since this is true for $\mathbf{d}_t > 0$, positive expected returns are sufficient for perceived time series momentum since $\mathbf{d}_t > 0$ on average following positive news. Perceived long-term reversals follow immediately from perceived overvaluation in (i), since the average speculator perceives the long-term buy-and-hold return of the risky asset to be negative.

For non-fundamental speculation in (iii), this follows immediately from perceived overvaluation in (i) and positive return expectations in (ii).

□

Proof of Mapping the Model to the Data (Remark 1)

Proof. We can observe that the proportion of speculators seeing the risky asset as overvalued is $\frac{1}{\theta} \int_0^\theta \mathbb{1}_{P_t > \frac{\rho}{1-\rho} \mathbf{d}_t^i} di = \mathbb{P}(B_k \mathbf{d}_t > \frac{\rho}{1-\rho} \mathbf{d}_t^i)$, where $\mathbb{P}$ is the probability operator. Note that given the linear Gaussian structure of the model, and the fact that the average speculator's belief about fundamentals matches rational expectations, speculator's fundamental belief can be written as $\mathbf{d}_t^i = \mathbf{d}_t + \xi_i$, where $\xi_i \sim N(0, \sigma_\xi^2)$ for some variance σ_ξ^2 that is pinned down in the model by the variance of the signals observed by speculators. Hence, we can observe that

$\mathbb{P}(B_k \mathbf{d}_t > \frac{\rho}{1-\rho}\mathbf{d}_t^i) = \mathbb{P}((B_k - \frac{\rho}{1-\rho})\mathbf{d}_t > \xi_i)$. Since $B_k > \frac{\rho}{1-\rho}$ when $\hat{\rho} > \rho$, and ξ_i is a random variable with mean zero and fixed variance, this probability is strictly increasing in $\mathbf{d}_t$.

Additionally, observe that in the level 1 equilibrium, from Equation (A.6), the average speculator's one-period ahead return expectation is given by

$$\mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) = (\rho - (1-\hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho}B_1))\mathbf{d}_t^s,$$

which is strictly increasing in $\mathbf{d}_t^s$, since $(\rho - (1-\hat{\rho})B_1 + \hat{\kappa}(\rho B_0 - \hat{\rho}B_1)) > 0$, per Lemma A.2. Similarly, in the level k equilibrium, per Equation (A.8), the average speculator's one-period return expectation is given by

$$\mathbb{E}_t^s(P_{t+1} + D_{t+1} - P_t) = (\rho - (1-\rho)B_k + \kappa\rho(B_{k-1} - B_k))\mathbf{d}_t^s,$$

which is strictly increasing in $\mathbf{d}_t^s$, using positivity of expected returns from Lemma A.3.

□

Proof of Necessary Condition for Multiplicity (Remark 2)

Proof. We first focus on the level 1 equilibrium. From the proof of Proposition 1, we know that all equilibria have to satisfy $B_1 > \frac{\rho}{1-\rho}$. Hence, it is useful to perform a change of variables, $B_1 = \beta_1 + \frac{\rho}{1-\rho}$, and re-write the cubic equation that B_1 must satisfy (Equation (A.2)) as a function of β_1. Doing so and simplifying, any equilibrium must coincide with a real and positive root of the equation

$$0 = z_3\beta_1^3 + z_2\beta_1^2 + z_1\beta_1 + z_0,$$

where

$$\begin{aligned}
z_3 \equiv& \theta(1-(1-\hat{\kappa})\hat{\rho})A_2, \\
z_2 \equiv& \frac{\theta(1-(1-\hat{\kappa})\hat{\rho})((1-\rho)(1-\hat{\rho})A_1 + \rho(2+\rho-3\hat{\rho})A_2)}{(1-\rho)(1-\hat{\rho})}, \\
z_1 \equiv& ((1-\rho)^2(1-\hat{\rho}))^{-1}\left(\theta(1-\rho)^2(1-\hat{\rho})(1-(1-\hat{\kappa})\hat{\rho})A_0\right. \\
&+ \theta(1-\rho)\rho(1+\rho-2\hat{\rho})(1-(1-\hat{\kappa})\hat{\rho})A_1 \\
&\left.- \theta\rho^2(3\hat{\rho}-1-2\rho)(1-(1-\hat{\kappa})\hat{\rho})A_2 + (1-\theta)(1-\rho)^3(1-\hat{\rho})A_S\right), \text{ and} \\
z_0 \equiv& \frac{\theta\rho(\rho-\hat{\rho})(1-(1-\hat{\kappa})\hat{\rho})((1-\rho)^2A_0 + \rho((1-\rho)A_1 + \rho A_2))}{(1-\rho)^3(1-\hat{\rho})}.
\end{aligned}$$

Descartes' Rule of Signs states that the upper bound for the number of real and positive roots of a polynomial is equal to the number of sign changes in the coefficients (e.g., a sign change is if z_3 and z_2 have different signs or if z_2 and z_1 have different signs). Assuming that $\hat{\rho} > \rho$, as in Proposition 1, we observe that $z_3 > 0$ and $z_0 < 0$. Hence, for there to be more than one sign change in the coefficients (and more than one equilibrium), a necessary condition is that $z_2 < 0$, which in turn requires that $\hat{\rho} > 2/3 + 1/3\rho$.

An additional necessary condition is that $z_1 > 0$. This can be seen as placing additional joint restrictions on the noise in signals observed by investors, the proportion of speculators in the economy, and higher-order beliefs about persistence.

We note that these conditions are not sufficient. In particular, it may be the case that there is only one real root, even if there are three sign changes in the coefficients. Additionally, inspecting z_2, for it to be negative, we also need that $\rho(3\hat{\rho} - 2 - \rho)A_2 > (1-\rho)(1-\hat{\rho})A_1$, i.e., there are additional restrictions on A_1 and A_2, which depend upon the noise in public and private signals relative to fundamentals, in order for z_2 to be negative.

We can similarly consider the level $k > 1$ equilibrium. Performing a change of variables, $\beta_k \equiv B_k + \frac{\rho}{1-\rho}$, we have that any equilibrium has to satisfy that β_k is a real and positive root of the equation

$$0 = z_{k,3}\beta_k^3 + z_{k,2}\beta_k^2 + z_{k,1}\beta_k + z_{k,0},$$

where

$$\begin{aligned}
z_{k,3} =& \theta(1-(1-\kappa)\rho)A_2,\\
z_{k,2} =& \theta((1-(1-\kappa)\rho)A_1 + \frac{\theta\rho A_2(2-2(1-\kappa)\rho - \kappa(1-\rho)\beta_{k-1})}{1-\rho},\\
z_{k,1} =& (1-(1-\theta\kappa)\rho)A_0 + \frac{A_1(\rho - (1-\theta\kappa)\rho^2 + (1-\rho)(1-\theta-\rho+\theta(1-\kappa)\rho)\beta_{k-1})}{1-\rho}\\
&+ (1-\rho)^{-2}A_2(\rho^2 - (1-\theta\kappa)\rho^3 + 2(1-\rho)\rho(1-\theta-\rho+\theta(1-\kappa)\rho)\beta_{k-1} + (1-\theta)(1-\rho)^3\beta_{k-1}^2),
\end{aligned}$$

and

$$z_{k,0} = -\frac{\theta\kappa\rho((1-\rho)^2A_0 + \rho((1-\rho)A_1 + \rho A_2))\beta_{k-1}}{(1-\rho)^2}.$$

Descartes' Rule of Signs similarly applies here. Observing that $z_{k,3} > 0$ and $z_{k,0} < 0$, a necessary condition for multiplicity is that $z_{k,2} < 0$ and $z_{k,1} > 0$. Note that as in the level 1 equilibrium case, these conditions depend on the proportion of speculators, the noise in public and private signals relative to fundamentals, etc. Additionally, however, the choice of β_{k-1} also is a relevant (which is determined by the pricing coefficient that speculators believe holds in the $k-1$ equilibrium).

□

Proof of Result 1

Proof. All claims follow immediately from the proof of Proposition 1. Perceived overvaluation and long term reversal follow from the fact that the average speculator has correct fundamental beliefs on average, and correctly recognizes that the risky asset is overvalued. Short term reversal is also proven in the proof of Proposition 1, since arbitrageurs have correct one-period ahead return expectations and expected negative returns whenever $\mathbf{d}_t > 0$, which is true on average following positive news. □

Proof of Result 2

Proof. First, we show that $B_k < B_{k-1}$. Assume that $\hat{\rho} > \rho$, and assume without loss of generality that $\mathbf{d}_t^s > 0$.

From Proposition 1, we know that $B_k > \frac{\rho}{1-\rho}$, so the objective one-period ahead expected return of the risky asset is negative:

$$\begin{aligned}\mathbb{E}_t(P_{t+1} + D_{t+1} - P_t) =& \rho\mathbf{d}_t + B_k(\rho\mathbf{d}_t - \mathbf{d}_t)\\ <& \rho\mathbf{d}_t - \frac{\rho}{1-\rho}(1-\rho)\mathbf{d}_t\\ =& 0.\end{aligned}$$

Because the average arbitrageur has correct expectations on average, the average arbitrageur has negative expected returns, and accordingly, the average speculator has positive expected returns. This, in turn, holds if and only if $\frac{1}{\theta}\int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1})di > \frac{1}{1-\theta}\int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1})di$. Using Equations (A.3) and (A.5), we can observe that

$$\begin{aligned}\frac{1}{\theta}\int_0^\theta \mathbb{E}_t^i(P_{t+1} + D_{t+1})di > \frac{1}{1-\theta}\int_\theta^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1})di&\\ \iff (1 + \kappa B_{k-1} + (1-\kappa)B_k)\rho\mathbf{d}_t^s > (1 + B_k)\rho\mathbf{d}_t^s&\\ \iff B_{k-1} > B_k.&\end{aligned}$$

It immediately follows that when fundamentals are positive, the price in the level k equilibrium is lower than the price in the level $k-1$ equilibrium, i.e., there is less overvaluation when fundamentals are positive.

Moreover, this holds for each k, so we have a sequence, $B_1, B_2, \ldots, B_k, \ldots$ such that $B_k < B_{k-1}$, where $B_k > \frac{\rho}{1-\rho}, \forall k$, i.e., we have a monotonically decreasing sequence that is bounded below. By the monotone convergence theorem, $\lim_{k\to\infty} B_k \to \bar{B}$ for some value $\bar{B}$.

To solve for $\bar{B}$, consider the limit as $k \to \infty$, where $\bar{B} = B_{k-1} = B_k$, i.e., arbitrageurs and speculators conjecture the same pricing rule. Then, we can summarize the market clearing condition as

$$
\begin{aligned}
0 = & \int_0^1 \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t)}{\mathbb{V}_t^i(P_{t+1} + D_{t+1})} di \\
= & \int_0^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1} - P_t) di \\
= & \rho \mathbf{d}_t^s + \bar{B}(\rho \mathbf{d}_t^s - \mathbf{d}_t^s).
\end{aligned}
$$

Solving for $\bar{B}$, we get that $\bar{B} = \frac{\rho}{1-\rho}$. Hence, $\lim_{k\to\infty} B_k = \frac{\rho}{1-\rho}$.

$\square$

B Model with Positive Supply

Here, we briefly outline a version of the model with the risky asset in positive supply. The solution to the model is similar to that of the main model, but as to be expected with positive supply, the risky asset price embeds a risk premium. The analysis here reveals that due to their higher-order beliefs, speculators also have biased return expectations.

The model is identical to the main specification, with two modifications. First, the risky asset is in fixed supply $Q > 0$. Second, rather than normalizing the payoff of the riskless asset to zero, we assume that it pays a gross return of $(1+r)$.

With these modifications, we derive the equilibrium pricing function.

Lemma B.1 (Level k Equilibrium Pricing Function with Positive Supply) *For $k = 1, 2, \ldots,$ the linear equilibrium pricing rule for the risky asset in the stationary level k equilibrium is*

$$P_t = C_k Q + B_k \mathbf{d}_t^s,$$

where $B_1 > 0$ is the solution to a cubic equation that is a function of deep parameters of the model, B_k is defined recursively as the solution to a cubic equation that is a function of B_{k-1} and deep parameters of the model,

$$C_0 = -\gamma \frac{(1+r-(1-\hat{\kappa})\hat{\rho})^2\sigma_\epsilon^2 + (1+r-\hat{\rho}(1-\hat{\kappa}_1))^2\sigma_v^2 + \hat{\rho}^2(\sigma_\eta^2+\sigma_\phi^2)\hat{\kappa}_2^2}{r(1+r-\hat{\rho})^2},$$

$$C_1 = -\frac{(A_0 + B_1(A_1+A_2B_1))(\gamma A_S - \theta(1-(1-\hat{\kappa})\hat{\rho})C_0)}{\theta(1+r-(1-\hat{\kappa})\hat{\rho})A_0 + (1-\theta)rA_S + \theta(1+r-(1-\hat{\kappa})\hat{\rho})B_1(A_1+A_2B_1)},$$

$$\begin{aligned} C_k = &- \left(A_0 + A_1B_k + A_2B_k^2\right)\left(\gamma(A_0 + A_1B_{k-1} + A_2B_{k-1}^2) - \theta C_{k-1}(1-\theta\rho(1-\kappa))\right) \\ &\times \big((r+\theta(1-(1-\kappa)\rho))A_0 + (r(1-\theta)B_{k-1} + \theta(1+r-(1-\kappa)\rho)B_k)A_1 \\ &+ (r(1-\theta)B_{k-1}^2 + \theta(1+r-(1-\kappa)\rho)B_k^2)A_2\big)^{-1}, \end{aligned}$$

and A_0, A_1, A_2, and $A_S > 0$ are functions of deep parameters of the model.

Proof. Proofs presented at the end of the section. □

For simplicity, we discuss the level 1 equilibrium, but can extend the logic to apply to the level k equilibrium. Similar to the main analysis, the cyclicality of speculators' beliefs that the market is overvalued and the cyclicality of their return expectations with respect to the level of dividends, $\mathbf{d}_t$, are determined by $\hat{\rho}$. By similar arguments as the main proofs, both return expectations and the proportion of investors seeing the market as overvalued are procyclical when $\hat{\rho} > \rho$. Namely, objective expected returns (those held by the average arbitrageur) are countercyclical when $B_1 > \frac{\rho}{1+r-\rho}$, and accordingly, the average speculator's expected returns are procyclical in this case. And $B_1 > \frac{\rho}{1+r-\rho}$ when $\hat{\rho} > \rho$.

Speculators also have biased expected returns. Assuming $\hat{\rho} > \rho$, there are two forces that operate on investors' return expectations. First, the coefficient on risky asset supply, C_1, which captures the risk premium, is different than the risk premium coefficient perceived by speculators, which is C_0. When $\hat{\rho} > \rho$, speculators overestimate the risk premium ($0 > C_1 > C_0$). This is because a belief that $B_0 > B_1$, which arises from the belief that other investors overestimate the persistence of fundamentals, induces speculators to overestimate the volatility of the risky asset. In turn, for a given level of risk aversion, speculators perceive a higher risk premium. Second, speculators' higher-order beliefs about persistence also influence the second-order belief about fundamentals that they extract from prices, $\mathbf{d}_{t,2} = \frac{C_1 - C_0}{B_1} Q + \frac{B_1}{B_0} \mathbf{d}_t^s$. On average, speculators perceive the price of the risky asset as higher than they expect, given their overestimation of the risk premium. This induces them to extract an upward biased signal about other investors' beliefs, and serves to lower their average return expectations. The second effect dominates, leading speculators to hold downward biased return expectations on average. The bias component of return expectations is not the main focus of our empirical analysis, though we find some evidence that speculators' return expectations may be biased.

B.1 Proofs

Proof of Lemma B.1

Proof. As with the proof of the analogous result in the main text, the proof follows in parts. First, we derive B_0 and C_0, which are the pricing coefficients that speculators perceive in the level 1 equilibrium. Then we derive B_1 and C_1, and finally we derive B_k and C_k for $k > 1$.

Derivation of B_0 and C_0: Consider an economy where all investors perceive the persistence of fundamentals as $\hat{\rho}$, and see this as common knowledge. This gives rise to the pricing rule that speculators perceive.

All investors conjecture that the pricing rule is $C_0 Q + B_0 \mathbf{d}_{t,s}$, where $\mathbf{d}_t^s$ is the average investor's belief about fundamentals. The market clearing condition in this economy is

$$Q = \int_0^1 \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - (1+r)P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})} di.$$

The average investor's forecasted period $t+1$ payoff is given by

$$\begin{aligned}\int_0^1 \mathbb{E}_t^i(P_{t+1} + D_{t+1}) di = & \int_0^1 C_0 Q + (1 + \hat{\kappa} B_0)\hat{\rho} \mathbf{d}_t^i + \left(B_0 (1 - \hat{\kappa}) \hat{\rho} \int_0^1 \mathbf{d}_t^j dj \right) di \\ = & C_0 Q + (1 + B_0) \hat{\rho} \mathbf{d}_t^s.\end{aligned}$$

Each investor i's subjective perceived variance of the period $t+1$ dividend is

$$\mathbb{V}_t^j(D_{t+1}) = \sigma_\epsilon^2 + \sigma_v^2,$$

and his subjective perceived variance of the period $t+1$ price is

$$\begin{aligned}
\mathbb{V}_t^j(P_{t+1}) =& \mathbb{V}_t^j\left(B_0 \mathbf{d}_{t+1}^j\right) \\
=& B_0^2 \mathbb{V}_t^j((1-\hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1 D_{t+1} + \hat{\kappa}_2 s_{t+1}^{-i}) \\
=& B_0^2\left(\hat{\kappa}_1^2(\sigma_\epsilon^2+\sigma_v^2) + \hat{\kappa}_2^2(\sigma_\epsilon^2+\sigma_\eta^2+\sigma_\phi^2) + 2\hat{\kappa}_1\hat{\kappa}_2\sigma_\epsilon^2\right).
\end{aligned}$$

Hence, his perceived variance of the period $t+1$ payoff is given by

$$\begin{aligned}
\mathbb{V}_t^i(P_{t+1}+D_{t+1}) =& \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbb{C}(P_{t+1}, D_{t+1}))}_{=B_0(\hat{\kappa}\sigma_\epsilon^2+\hat{\kappa}_1\sigma_v^2)} \\
=& (1+\hat{\kappa}B_0)^2\sigma_\epsilon^2 + (1+\hat{\kappa}_1 B_0)^2\sigma_v^2 + B_0^2\hat{\kappa}_2^2(\sigma_\eta^2+\sigma_\phi^2).
\end{aligned}$$

Hence, the market clearing condition can be written as

$$\gamma Q\left((1+\hat{\kappa}B_0)^2\sigma_\epsilon^2 + (1+\hat{\kappa}_1 B_0)^2\sigma_v^2 + B_0^2\hat{\kappa}_2^2(\sigma_\eta^2+\sigma_\phi^2)\right) = C_0 Q + (1+B_0)\hat{\rho}\mathbf{d}_{t,s} - (1+r)P_t.$$

Re-arranging in terms of P_t, simplifying, and matching coefficients, we get the following system of equations:

$$\begin{aligned}
B_0 =& \frac{\hat{\rho}(1+B_0)}{1+r}, \text{ and} \\
C_0 =& \frac{C_0 - \gamma((1+\hat{\kappa}B_0)^2\sigma_\epsilon^2 + \sigma_v^2(1+B_0\hat{\kappa}_1)^2 + B_0^2(\sigma_\eta^2+\sigma_\phi^2)\hat{\kappa}_2^2)}{1+r}.
\end{aligned}$$

Solving the system of equations yields that $B_0 = \frac{\hat{\rho}}{1+r-\hat{\rho}}$ and the expression for C_0 provided in the lemma.

Derivation of B_1 and C_1: For notational simplicity, we define $\kappa = \kappa_1 + \kappa_2$ and $\hat{\kappa} = \hat{\kappa}_1 + \hat{\kappa}_2$.

We conjecture that the pricing formula is of the form $P_t = C_1 Q + B_1 \mathbf{d}_t^s$. Note that in equilibrium, we must have that $\mathbf{d}_{t,2} = \frac{C_1 - C_0}{B_0} Q + \frac{B_1}{B_0}\mathbf{d}_t^s$, since speculators' second-order beliefs at the equilibrium must equal their perceived price to the prevailing equilibrium price.

To forecast the price in period $t+1$, speculator i forecasts the average belief in period $t+1$, based on their forecast of $\mathbf{d}_{t+1}$ and s_{t+1}:

$$\begin{aligned}
\mathbb{E}_t^i(\mathbf{d}_{t+1,2}) = \mathbb{E}_t^i \int \mathbf{d}_{t+1}^j dj =& (1-(\hat{\kappa}_1+\hat{\kappa}_2))\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1 \underbrace{\mathbb{E}_t(D_{t+1})}_{=\rho\mathbf{d}_t^i} + \hat{\kappa}_2 \underbrace{\mathbb{E}_t(s_{t+1}^{-i})}_{=\rho\mathbf{d}_t^i} \\
=& (1-\hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}\rho\mathbf{d}_t^i.
\end{aligned}$$

Speculator i's expected period $t+1$ payoff is

$$\mathbb{E}_t^i(D_{t+1}+P_{t+1}) = \underbrace{\rho \mathbf{d}_t^i}_{=\mathbb{E}_t^i(D_{t+1})} + \underbrace{C_0 Q + \mathbb{E}_t^i(B_0 \mathbf{d}_{t+1,2})}_{=\mathbb{E}_t^i(P_{t+1})}$$
$$= C_0 Q + (1+\hat{\kappa} B_0)\rho \mathbf{d}_t^i + (1-\hat{\kappa})\hat{\rho} B_0 \mathbf{d}_{t,2}. \tag{B.1}$$

Speculator i's subjective perceived variance of the period $t+1$ dividend is

$$\mathbb{V}_t^j(D_{t+1}) = \sigma_\epsilon^2 + \sigma_v^2,$$

and his subjective perceived variance of the period $t+1$ price is

$$\begin{aligned}\mathbb{V}_t^j(P_{t+1}) =& \mathbb{V}_t^j\left(B_0 \mathbf{d}_{t+1}^j\right)\\ =& B_0^2 \mathbb{V}_t^j((1-\hat{\kappa})\hat{\rho}\mathbf{d}_{t,2} + \hat{\kappa}_1 D_{t+1} + \hat{\kappa}_2 s_{t+1}^{-i})\\ =& B_0^2\left(\hat{\kappa}_1^2(\sigma_\epsilon^2+\sigma_v^2) + \hat{\kappa}_2^2(\sigma_\epsilon^2+\sigma_\eta^2+\sigma_\phi^2) + 2\hat{\kappa}_1\hat{\kappa}_2\sigma_\epsilon^2\right).\end{aligned}$$

Hence, his perceived variance of the period $t+1$ payoff is given by

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1}+D_{t+1}) =& \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbb{C}(P_{t+1}, D_{t+1})}_{=B_0(\hat{\kappa}\sigma_\epsilon^2+\hat{\kappa}_1\sigma_v^2)})\\ =& (1+\hat{\kappa}B_0)^2\sigma_\epsilon^2 + (1+\hat{\kappa}_1 B_0)^2\sigma_v^2 + B_0^2\hat{\kappa}_2^2(\sigma_\eta^2+\sigma_\phi^2).\end{aligned}$$

As in the main specification, we note that this variance doesn't depend on the coefficient of interest, B_1, and accordingly denote it as A_S. Speculator demand is then given by

$$\begin{aligned}\int_0^\theta Q_t^i di =& \int_0^\theta \frac{\mathbb{E}_t^i(P_{t+1}+D_{t+1}) - (1+r)P_t}{\gamma \mathbb{V}_t^i(P_{t+1}+D_{t+1})} di\\ =& \frac{\theta}{\gamma}\frac{C_0 Q + (1+\hat{\kappa}B_0)\rho\mathbf{d}_t^i + (1-\hat{\kappa})\hat{\rho}B_0\mathbf{d}_{t,2} - (1+r)P_t}{A_S}.\end{aligned}$$

Substituting $\mathbf{d}_{t,2} = \frac{C_1 - C_0}{B_0} Q + \frac{B_1}{B_0}\mathbf{d}_t^s$, this becomes

$$\int_0^\theta Q_t^i di = \frac{\theta}{\gamma}\frac{QC_0 + (1+\hat{\kappa}B_0)\rho\mathbf{d}_t^s + (1-\hat{\kappa})\hat{\rho}(Q(C_1-C_0) + B_1\mathbf{d}_t^s) - (1+r)P_t}{A_S}.$$

Turning to the arbitrageurs, they know that the form of the pricing rule is $QC_1 + B_1\mathbf{d}_t^s$. Moreover, they know that other investors correctly perceive the parameters governing the risky asset's fundamentals. Hence, arbitrageur i's expected period $t+1$ payoff is

$$\mathbb{E}_t^i(P_{t+1}+D_{t+1}) = \rho \mathbf{d}_t^i + B_1((1-\kappa)\rho \mathbf{d}_t^s + \kappa\rho \mathbf{d}_t^i) + C_1 Q. \tag{B.2}$$

Arbitrageur i's perceived variance of dividends and the next period's price are

$$\begin{aligned}\mathbb{V}_t^i(D_{t+1}) =& \sigma_\epsilon^2 + \sigma_v^2, \text{ and}\\ \mathbb{V}_t^i(P_{t+1}) =& B_1^2(\kappa_1(\sigma_\epsilon^2+\sigma_v^2) + \kappa_2(\sigma_\epsilon^2+\sigma_\eta^2+\sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2).\end{aligned}$$

Hence, arbitrageur i's perceived variance of the period $t+1$ payoff is

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1}+D_{t+1}) =& \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbb{C}(P_{t+1}, D_{t+1})}_{=B_1(\kappa\sigma_\epsilon^2+\kappa_1\sigma_v^2)}\\ =& (1+\kappa B_1)^2\sigma_\epsilon^2 + (1+\kappa_1 B_1)^2\sigma_v^2 + B_1^2\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2).\end{aligned}$$

This perceived variance is quadratic in B_1. For notational simplicity, we define A_0, A_1, and A_2 as the quadratic equation coefficients, i.e.,

$$\mathbb{V}_t^i(P_{t+1}+D_{t+1}) = \underbrace{(\kappa_1^2\sigma_v^2 + \kappa^2\sigma_\epsilon^2 + \kappa_2^2(\sigma_\eta^2+\sigma_\phi^2))}_{A_2\equiv} B_1^2 + \underbrace{2(\kappa_1\sigma_v^2\kappa + \kappa\sigma_\epsilon^2)}_{A_1\equiv} B_1 + \underbrace{\sigma_v^2+\sigma_\epsilon^2}_{A_0\equiv},$$

and note that $A_0, A_1, A_2 > 0$.

Arbitrageur demand is then given by

$$\begin{aligned}\int_\theta^1 Q_t^i di =& \int_\theta^1 \frac{\mathbb{E}_t^i(P_{t+1}+D_{t+1}) - (1+r)P_t}{\gamma\mathbb{V}_t^i(P_{t+1}+D_{t+1})} di\\ =& \frac{(1-\theta)}{\gamma}\frac{C_1Q + \rho(1+B_1)\mathbf{d}_t^s - (1+r)P_t}{A_2B_1^2 + A_1B_1 + A_0}.\end{aligned}$$

Imposing market clearing ($\int Q_t^i di = \int_0^\theta Q_t^i di + \int_\theta^1 Q_t^i di = Q$), and solving for P_t, we get that

$$\begin{aligned}P_t =& \frac{(1-\theta)\rho A_S(1+B_1) + \theta A_0(\rho+\hat{\rho}B_1+\hat{\kappa}(\rho B_0 - \hat{\rho}B_1)) + \theta B_1(A_1+A_2B_1)(\rho+\hat{\rho}B_1+\hat{\kappa}(\rho B_0-\hat{\rho}B_1))}{(1+r)(\theta A_0 + (1-\theta)A_S + \theta B_1(A_1+A_2B_1))}\mathbf{d}_t^s\\ &+ \frac{\frac{(1-\theta)C_1 - \gamma(A_0+A_1B_1+A_2B_1^2)}{A_2B_1^2+A_1B_1+A_0} + \frac{\theta(1-(1-\hat{\kappa})\hat{\rho})C_0 + \theta(1-\hat{\kappa})\hat{\rho}C_1}{A_S}}{(1+r)\left(\frac{\theta}{A_S} + \frac{1-\theta}{A_0+B_1(A_1+A_2B_1)}\right)}Q.\end{aligned}$$

Matching coefficients, and through algebraic manipulation, we get that B_1 is the solution to a cubic equation:

$$0 = \theta(1+r-(1-\hat{\kappa})\hat{\rho})A_2B_1^3$$

$$
\begin{aligned}
&+\left(\theta(1+r-(1-\hat{\kappa})\hat{\rho})A_1 - \frac{\theta\rho(1-(1-\hat{\kappa})\hat{\rho})A_2}{1-\hat{\rho}}\right)B_1^2 \\
&+\left(\theta(1+r-(1-\hat{\kappa})\hat{\rho})A_0 - \frac{\theta\rho(1-(1-\hat{\kappa})\hat{\rho})A_1}{1-\hat{\rho}} + (1-\theta)(1+r-\rho)A_S\right)B_1, \\
&-\frac{\theta\rho(1-(1-\hat{\kappa})\hat{\rho})A_0}{1-\hat{\rho}} - (1-\theta)\rho A_S,
\end{aligned}
$$

and C_1 is the expression provided in the lemma.

Bias in Speculators' Return Expectations: Using Equations (B.1) and (B.2), averaging over investors, and substituting $\mathbf{d}_{t,2} = \frac{C_1-C_0}{B_0}Q + \frac{B_1}{B_0}\mathbf{d}_t^s$ in equilibrium, we can write the bias in the average speculators' return expectations as

$$
\begin{aligned}
\mathbb{E}(\mathbb{E}_t^s(D_{t+1}+P_{t+1}) - \mathbb{E}_t(D_{t+1}+P_{t+1})) =& \mathbb{E}(C_0Q + \rho(1+\hat{\kappa}B_0)\mathbf{d}_t^s + (1-\hat{\kappa})\hat{\rho}B_0\mathbf{d}_{t,2} - (QC_1 + \rho(1+B_1)\mathbf{d}_t^s)) \\
=& Q(1-(1-\hat{\kappa})\hat{\rho})(C_0 - C_1)).
\end{aligned}
$$

Hence, the average speculator's return expectations are downward biased if and only if $C_0 < C_1$. When $\hat{\rho} > \rho$, this is true as a consequence of $B_0 > B_1$.

Derivation of B_k and C_k, $k > 1$: We conjecture that the pricing formula is of the form $P_t = C_kQ + B_k\mathbf{d}_t^s$. Note that in equilibrium, we must have that $\mathbf{d}_{t,2} = \frac{C_k - C_{k-1}}{B_{k-1}}Q + \frac{B_k}{B_{k-1}}\mathbf{d}_t^s$, since speculators' second-order beliefs at the equilibrium must equal their perceived price to the prevailing equilibrium price.

To forecast the price in period $t+1$, speculator i forecasts the average belief in period $t+1$, based on their forecast of $\mathbf{d}_{t+1}$ and s_{t+1}:

$$
\mathbb{E}_t^i(\mathbf{d}_{t+1,2}) = (1-\kappa)\rho\mathbf{d}_{t,2} + \kappa\rho\mathbf{d}_t^i.
$$

Speculator i's expected period $t+1$ payoff is

$$
\begin{aligned}
\mathbb{E}_t^i(D_{t+1}+P_{t+1}) =& \underbrace{\rho\mathbf{d}_t^i}_{=\mathbb{E}_t^i(D_{t+1})} + \underbrace{C_{k-1}Q + \mathbb{E}_t^i(B_{k-1}\mathbf{d}_{t+1,2})}_{=\mathbb{E}_t^i(P_{t+1})} \\
=& C_{k-1}Q + (1+\kappa B_{k-1})\rho\mathbf{d}_t^i + (1-\kappa)\rho B_{k-1}\mathbf{d}_{t,2}.
\end{aligned}
$$

Speculator i's subjective perceived variance of the period $t+1$ dividend is

$$
\mathbb{V}_t^j(D_{t+1}) = \sigma_\epsilon^2 + \sigma_v^2,
$$

and his subjective perceived variance of the period $t+1$ price is

$$
\mathbb{V}_t^j(P_{t+1}) = \mathbb{V}_t^j\left(B_{k-1}\mathbf{d}_{t+1}^j\right)
$$

$$
\begin{aligned}
&= B_{k-1}^2 \mathbb{V}_t^j((1-\kappa)\rho \mathbf{d}_{t,2} + \kappa_1 D_{t+1} + \kappa_2 s_{t+1}^{-i}) \\
&= B_{k-1}^2 \left(\kappa_1^2(\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2^2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2 \right).
\end{aligned}
$$

Hence, his perceived variance of the period $t+1$ payoff is given by

$$
\begin{aligned}
\mathbb{V}_t^i(P_{t+1} + D_{t+1}) &= \mathbb{V}_t^i(D_{t+1}) + \mathbb{V}_t^i(P_{t+1}) + 2\underbrace{\mathbb{C}(P_{t+1}, D_{t+1})}_{=B_{k-1}(\kappa\sigma_\epsilon^2 + \kappa_1\sigma_v^2)}) \\
&= (1 + \kappa B_{k-1})^2\sigma_\epsilon^2 + (1 + \kappa_1 B_{k-1})^2\sigma_v^2 + B_{k-1}^2\kappa_2^2(\sigma_\eta^2 + \sigma_\phi^2).
\end{aligned}
$$

Defining A_0, A_1, and A_2 as before, we can write this perceived variance as,

$$
\mathbb{V}_t^i(P_{t+1} + D_{t+1}) = \underbrace{(\kappa_1^2\sigma_v^2 + \kappa^2\sigma_\epsilon^2 + \kappa_2^2(\sigma_\eta^2 + \sigma_\phi^2))}_{A_2\equiv} B_{k-1}^2 + \underbrace{2(\kappa_1\sigma_v^2\kappa + \kappa\sigma_\epsilon^2)}_{A_1\equiv} B_{k-1} + \underbrace{\sigma_v^2 + \sigma_\epsilon^2}_{A_0\equiv}.
$$

Speculator demand is then given by

$$
\begin{aligned}
\int_0^\theta Q_t^i di &= \int_0^\theta \frac{\mathbb{E}_t^i(P_{t+1} + D_{t+1}) - (1+r)P_t}{\gamma \mathbb{V}_t^i(P_{t+1} + D_{t+1})} di \\
&= \frac{\theta}{\gamma} \frac{C_{k-1}Q + (1 + \kappa B_{k-1})\rho \mathbf{d}_t^s + (1-\kappa)\rho B_{k-1}\mathbf{d}_{t,2} - (1+r)P_t}{A_2 B_{k-1}^2 + A_1 B_{k-1} + A_0}.
\end{aligned}
$$

Substituting $\mathbf{d}_{t,2} = \frac{C_k - C_{k-1}}{B_{k-1}}Q + \frac{B_k}{B_{k-1}}\mathbf{d}_t^s$, this becomes

$$
\int_0^\theta Q_t^i di = \frac{\theta}{\gamma} \frac{QC_{k-1} + (1 + \kappa B_{k-1})\rho \mathbf{d}_t^s + (1-\kappa)\hat{\rho}(Q(C_k - C_{k-1}) + B_k \mathbf{d}_t^s) - (1+r)P_t}{A_2 B_{k-1}^2 + A_1 B_{k-1} + A_0}.
$$

Turning to the arbitrageurs, they know that the form of the pricing rule is $QC_k + B_k\mathbf{d}_t^s$. Moreover, they know that other investors correctly perceive the parameters governing the risky asset's fundamentals. Hence, arbitrageur i's expected period $t+1$ payoff is

$$
\mathbb{E}_t^i(P_{t+1} + D_{t+1}) = \rho \mathbf{d}_t^i + B_k((1-\kappa)\rho \mathbf{d}_t^s + \kappa\rho \mathbf{d}_t^i) + C_k Q.
$$

Arbitrageur i's perceived variance of dividends and the next period's price are

$$
\begin{aligned}
\mathbb{V}_t^i(D_{t+1}) &= \sigma_\epsilon^2 + \sigma_v^2, \text{ and} \\
\mathbb{V}_t^i(P_{t+1}) &= B_k^2(\kappa_1(\sigma_\epsilon^2 + \sigma_v^2) + \kappa_2(\sigma_\epsilon^2 + \sigma_\eta^2 + \sigma_\phi^2) + 2\kappa_1\kappa_2\sigma_\epsilon^2).
\end{aligned}
$$

Hence, arbitrageur i's perceived variance of the period $t+1$ payoff is

$$\begin{aligned}\mathbb{V}_t^i(P_{t+1}+D_{t+1}) =& \mathbb{V}_t^i(D_{t+1})+\mathbb{V}_t^i(P_{t+1})+2\underbrace{\mathbb{C}(P_{t+1},D_{t+1})}_{=B_k(\kappa\sigma_\epsilon^2+\kappa_1\sigma_v^2)}\\ =&(1+\kappa B_k)^2\sigma_\epsilon^2+(1+\kappa_1 B_k)^2\sigma_v^2+B_k^2\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2).\end{aligned}$$

This perceived variance is quadratic in B_k, and can be written as

$$\mathbb{V}_t^i(P_{t+1}+D_{t+1}) = \underbrace{(\kappa_1^2\sigma_v^2+\kappa^2\sigma_\epsilon^2+\kappa_2^2(\sigma_\eta^2+\sigma_\phi^2))}_{A_2\equiv}B_k^2+\underbrace{2(\kappa_1\sigma_v^2\kappa+\kappa\sigma_\epsilon^2)}_{A_1\equiv}B_k+\underbrace{\sigma_v^2+\sigma_\epsilon^2}_{A_0\equiv},$$

and note that $A_0, A_1, A_2 > 0$.

Arbitrageur demand is then given by

$$\begin{aligned}\int_\theta^1 Q_t^i di =& \int_\theta^1 \frac{\mathbb{E}_t^i(P_{t+1}+D_{t+1})-(1+r)P_t}{\gamma\mathbb{V}_t^i(P_{t+1}+D_{t+1})}di\\ =&\frac{(1-\theta)}{\gamma}\frac{C_kQ+\rho(1+B_k)\mathbf{d}_t^s-(1+r)P_t}{A_2B_k^2+A_1B_k+A_0}.\end{aligned}$$

Imposing market clearing ($\int Q_t^i di = \int_0^\theta Q_t^i di+\int_\theta^1 Q_t^i di = Q$), and solving for P_t, we get that

$$\begin{aligned}P_t =& \left(\frac{A_0\left(1+\theta\kappa B_{k-1}+(1-\theta\kappa)B_k\right)+A_1\left(B_{k-1}\left(B_k(1-\theta(1-\kappa))+(1-\theta)\right)+\theta B_k\left(1+(1-\kappa)B_k\right)\right)}{(1+r)\left(\frac{\theta}{A_0+A_1B_{k-1}+A_2B_{k-1}^2}+\frac{1-\theta}{A_0+A_1B_k+A_2B_k^2}\right)}\right.\\ &+\left.\frac{A_2\left(\theta\kappa B_k^2B_{k-1}+\theta B_k^2\left(1+(1-\kappa)B_k\right)+\left((1-\theta)\left(B_k+1\right)B_{k-1}^2\right)\right)}{(1+r)\left(\frac{\theta}{A_0+A_1B_{k-1}+A_2B_{k-1}^2}+\frac{1-\theta}{A_0+A_1B_k+A_2B_k^2}\right)}\right)\rho\mathbf{d}_t^s\\ &-\left(\frac{1-\theta\frac{C_{k-1}(1-(1-\kappa)\rho))}{\gamma\left(A_0+A_1B_{k-1}+A_2B_{k-1}^2\right)}}{(1+r)\left(\frac{\theta}{A_0+A_1B_{k-1}+A_2B_{k-1}^2}+\frac{1-\theta}{A_0+A_1B_k+A_2B_k^2}\right)}\right.\\ &+\left.\frac{C_k\gamma^{-1}\left(\theta\left(\frac{1}{A_0+A_1B_k+A_2B_k^2}-\frac{(1-\kappa)\rho}{A_0+A_1B_{k-1}+A_2B_{k-1}^2}\right)-\frac{1}{A_0+A_1B_k+A_2B_k^2}\right)}{(1+r)\left(\frac{\theta}{A_0+A_1B_{k-1}+A_2B_{k-1}^2}+\frac{1-\theta}{A_0+A_1B_k+A_2B_k^2}\right)}\right)\gamma Q.\end{aligned}$$

Matching coefficients, and through algebraic manipulation, we get that B_k is the solution to the cubic equation

$$0 =\theta(1+r-(1-\kappa)\rho)A_2B_k^3$$

$$
\begin{aligned}
&+\theta((1+r-(1-\kappa)\rho)A_1-\rho A_2(1+\kappa B_{k-1}))B_k^2\\
&+((1+r-(1-\theta\kappa)\rho)A_0+(1-\theta)(1+r-\rho)A_2B_{k-1}^2-A_1(\theta\rho-(1+r(1-\theta)-\theta-\rho+\theta\rho(1-\kappa))B_{k-1})B_k\\
&-A_0\rho(1+\theta\kappa B_{k-1})-(1-\theta)\rho B_{k-1}(A_1+A_2B_{k-1}),
\end{aligned}
$$

and C_k is the solution given in the lemma. □

C Additional Empirical Analyses

In this section, we present additional empirical analyses. Section C.1 discusses the selection criteria and demographics of the Shiller survey, and potential survey response biases. Section C.2 discusses the evidence from the Shiller survey as it relates to previous evidence on return extrapolation. Section C.3 presents tables and figures that replicate the main results for different subsets of the data (e.g., individual versus institutional investors).

C.1 Survey responses in the Shiller survey

The Shiller survey data have been collected continuously since 1989 – semi-anually for a decade, and then monthly by the International Center for Finance at the Yale School of Management since July 2001. The surveys are conducted by a market survey firm, which mails 500 surveys to high net-worth individual investors, and 500 surveys to institutional investors each month, with a sampling goal of 20 to 50 responses by each of the two types - individual and institutional. For both institutional and individual investors, the investor mailing lists are purchased from Data Axle (previously known as InfoUSA).

The micro data do not provide detailed demographic information. There is likely to be selection into responding, as in other surveys. For example, Giglio et al. (2021) find in a survey of Vanguard investors that their respondents are older, wealthier, more likely to be male, and trade more often than nonrespondents. The selection criteria for investors in the Shiller survey, and the data that are available on their characteristics, indicate that individual respondents are likely to have high income and be wealthy, and that institutional respondents manage large portfolios. While likely not representative of the investor population, survey respondents are a substantial and important class of investors.

For individual investors, the mailing list for the surveys is constructed by sampling households with a household income of greater than $150,000 per year from the Infogroup Consumer Database. We have no additional demographic information on the respondents.

For institutional investors, the mailing list is constructed by sampling companies from the Infogroup Business Database with the SIC codes 628202 (Investment Management), 628203 (Financial Advisory Services), 628204 (Financing Consultants), and 628205 (Financial Planning Consultants). Survey respondents are asked to provide the 'Size of the common stock portfolio(s) you make decisions about." In the sample, the 25th, 50th, and 75th percentiles of responses are $2 million, $27 million, and $115 million. Summing across respondents by month, the 25th, 50th, and 75th percentiles of the sum of responses are $1.2 billion, $2.8 billion, and $19 billion.

We analyze if there is any business cycle frequency variation in responses to the Shiller survey. We regress 100 multiplied by the quarterly change in the log number of survey responses each quarter on Dow Jones Industrial Average returns, and quarterly innovations in the Conference Board Coincident indicators index (labeled 'Macro').[22] The independent variables are standardized to have zero mean and unit standard deviation. Table C.1 reports the results, and Newey-West standard errors (4 lags) are reported in parentheses. There is little evidence to indicate systematic business cycle variation in survey response counts.

	All		Indiv		Inst	
Returns	0.70		-0.18		1.76	
	(5.02)		(4.98)		(5.96)	
Macro		-0.73		-2.64		1.47
		(2.60)		(2.44)		(3.32)

Table C.1: Response counts and business cycle variation

[22]Unfortunately, we do not observe the number of questionnaires that were sent out each quarter, so we use changes in total responses to proxy for response rates.

C.2 Relationship with return extrapolation

Prior work finds that investors exhibit extrapolative return expectations; they expect high returns following positive market performance and low returns following poor market performance. Given our evidence relating investors' expectations with macroeconomic news, we naturally expect a similar relationship in our setting. We examine the relationship between past returns and expectations in the Shiller survey, and discuss it in the context of the evidence on extrapolation in Greenwood and Shleifer (2014).

Panel A of Table C.2 reports the correlations of investors' return expectations, *HO belief*, and *Overvaluation* with trailing 12-month returns. Short-term return expectations are positively correlated with trailing 12-month returns (correlation of 0.40 for 1-month return expectations; and 0.30 for 3-month return expectations), while the correlation of 6-month ahead returns is insignificantly positive (0.07), and that of 12-month ahead return expectations is negative (-0.16). *HO belief* is 0.61 correlated with trailing 12-month returns, and *Overvaluation* is 0.45 correlated with 12-month trailing returns. Tables C.3 and C.4 report similar evidence when we separate the results for individual and institutional investors.

The correlations are consistent with investors' 1- to 3-month return expectations being extrapolative and 12-month return expectations being somewhat contrarian. Studying investors' stated bullishness or bearishness about subsequent 12-month returns from the Gallup survey, Chief Financial Officers' (CFOs') expectations of the returns of the U.S. stock market over the next 12 months, individual investors' bullishness or bearishness about subsequent 6-month returns from the American Association of Individual Investors (AAII) Investor Sentiment Survey, and the bullishness or bearishness of various financial newsletters' forecasts of 'near term' stock market returns as surveyed by "Investors Intelligence," Greenwood and Shleifer (2014) find consistent evidence of extrapolative return expectations. The extrapolative 1- to 3-month return expectations we document are consistent with the findings of Greenwood and Shleifer (2014), though the contrarian 12-month ahead return expectations appear to differ.[23]

A potential explanation for the difference is survey design. The Shiller survey is unique from other surveys of return expectations, in that it asks investors about their expectations for multiple horizons. It is plausible that investors believe that the stock market may increase over the short horizon (the period relevant for their portfolio choice) and decrease over a longer horizon, but they do not mentally formulate precise frequency-specific forecasts. When asked about returns over multiple horizons, as in the Shiller survey, respondents may report numerical return expectations at different horizons consistent with their belief in

[23] Greenwood and Shleifer (2014) also analyze the Yale ICF 1-year confidence index for individual investors, which is the proportion of individual investor respondents to the Shiller survey that report strictly positive 12-month ahead return expectations. In a sample that runs through 2011, Greenwood and Shleifer (2014) find a relationship close to zero between the confidence index and trailing 12-month returns (t-statistic of 0.18).

high short term returns to be followed by lower longer term returns. But when asked only about returns at the 6- to 12-month ahead horizon, as in other surveys, investors may simply report the short horizon return expectations. That is, the omission of questions about different horizons may lead a respondent to report their beliefs differently.

Such a difference is consistent with psychological evidence on framing effects (e.g., see Zauberman et al. 2010; Read, Frederick, and Scholten 2013).[24] Additionally, we present two pieces of evidence consistent with this reasoning. First, Panel B of Table C.2 displays the correlations of expectations measures from the Shiller survey with the monthly proportion of investors that report being bullish minus the proportion that report being bearish about 6-month stock market returns in the AAII survey.[25] We find that the AAII responses are highly correlated with short-horizon return expectations in the Shiller survey (correlation of 0.59 with 1-month ahead return expectations), with the correlations declining with forecast horizon (correlations of 0.54, 0.43, and 0.21 with 3-, 6-, and 12-month ahead return expectations). These correlations suggest that the AAII responses are particularly well aligned with Shiller survey respondents' short-horizon return expectations, and less so with their 12-month ahead return expectations. Second, in Table C.5, we analyze multi-period forecasts of exchange rates by financial institutions from FX4casts. We find that respondents report expectations of high 1- to 3-month ahead returns followed by low 6- to 12-months ahead returns for developed market currencies versus the USD that experienced interest rate increases and positive excess returns in the previous quarter. The consistency of the patterns of more extrapolative short term return expectations and more contrarian longer term return expectations suggest that these might be general features of investors' expectations.

[24] Also see Hartzmark and Sussman (2024), who argue that differences in how questions are framed may significantly influence reported beliefs about return expectations. They focus on beliefs about distributions.

[25] The AAII data are weekly; we follow Greenwood and Shleifer (2014) and aggregate them to be monthly. Greenwood and Shleifer (2014) report a high correlation between the AAII survey and the other surveys they examine.

Panel A: Expectations and trailing returns						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	*HO belief*	*Overvaluation*
$R_{t-12,t}$	0.40	0.30	0.07	-0.16	0.61	0.45
	(0.08)	(0.08)	(0.11)	(0.12)	(0.11)	(0.15)
R^2	.16	.09	.01	.03	.37	.20
N	259	259	259	259	259	259

Panel B: Shiller and AAII survey expectations						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	*HO belief*	*Overvaluation*
AAII	0.59	0.54	0.43	0.21	0.36	0.03
	(0.08)	(0.07)	(0.10)	(0.12)	(0.09)	(0.13)
R^2	.35	.29	.19	.04	.13	.00
N	259	259	259	259	259	259

Table C.2: Expectations and trailing returns

Note: Panel A of the table displays time series correlation coefficients between measures of expectations from the Shiller survey, averaged across investors in a given month, with the trailing 12-month excess returns of the Dow Jones Industrial Average. Panel B of the table reports time series correlation coefficients of the same measures of expectations from the Shiller survey with the the proportion of investors that reporting being bullish minus the proportion of investors that report being bearish about the future 6-month returns of the U.S. stock market from the American Association of Individual Investors (AAII) Investor Sentiment Survey. The AAII survey data are aggregated from the weekly to monthly frequency by averaging across observations within a month. Newey-West standard errors (12 lags) are reported in parentheses.

Panel A: Expectations and trailing returns						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	*HO belief*	*Overvaluation*
$R_{t-12,t}$	0.46	0.36	0.18	-0.07	0.56	0.43
	(0.08)	(0.08)	(0.09)	(0.10)	(0.09)	(0.13)
R^2	.21	.13	.03	.01	.31	.18
N	259	259	259	258	259	259
Panel B: Shiller and AAII survey expectations						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	*HO belief*	*Overvaluation*
AAII	0.57	0.55	0.46	0.31	0.27	-0.02
	(0.09)	(0.07)	(0.08)	(0.10)	(0.09)	(0.12)
R^2	.33	.30	.21	.09	.07	.00
N	259	259	259	258	259	259

Table C.3: Expectations and trailing returns (individual investors)

Note: This table replicates Table C.2 for the subset of individual investors in our sample.

Panel A: Expectations and trailing returns						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	*HO belief*	*Overvaluation*
$R_{t-12,t}$	0.20	0.14	-0.02	-0.21	0.49	0.37
	(0.08)	(0.09)	(0.09)	(0.10)	(0.10)	(0.13)
R^2	.04	.02	.00	.04	.24	.14
N	258	258	258	259	259	259
Panel B: Shiller and AAII survey expectations						
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	*HO belief*	*Overvaluation*
AAII	0.40	0.32	0.23	0.04	0.33	0.04
	(0.08)	(0.06)	(0.09)	(0.10)	(0.08)	(0.12)
R^2	.16	.11	.05	.00	.11	.00
N	258	258	258	259	259	259

Table C.4: Expectations and trailing returns (institutional investors)

Note: This table replicates Table C.2 for the subset of institutional investors in our sample.

	$\mathbb{E}_t^s\left(r_{t,t+3}\right)$	$\mathbb{E}_t^s\left(r_{t+3,t+6}\right)$	$\mathbb{E}_t^s\left(r_{t+6,t+12}\right)$
Interest rate innovations	0.63	-0.02	-0.51
	(0.18)	(0.14)	(0.21)
Trailing 3-month return	0.39	0.14	-0.13
	(0.17)	(0.13)	(0.19)

Table C.5: Currency market expected returns in response to news

Note: The table reports regression results from regressions of consensus return expectations over different horizons on past news using data on currency market expectations. $\mathbb{E}_t^s(r_{t+h,t+h+k})$ represents the consensus k-month return expectation for h months in the future. The independent variable is standardized to have zero mean and unit standard deviation, and return expectations are multiplied by 100, so that coefficients can be interpreted as expected returns in percentage points corresponding with a one standard deviation change in the independent variable. The first row corresponds with regressions where the news measure is AR(1) innovations to interest rate differentials, and the second row corresponds with regressions where the independent variable is trailing 3-month returns. The table reports the average coefficient across countries. Standard errors are HAC-panel standard errors and are reported in parentheses. The return expectations data are from FX4casts, which provides the average forecast of 3-, 6-, and 12-month ahed exchange rate forecasts from a number of large financial institutions that actively participate in foreign exchange markets across the world. The sample begins in August 1986 and ends in December 2019, and contains monthly observations of forecasts for developed market G11 currencies versus the USD.

C.3 Additional tables and figures

Panel A: Term structure of expected cumulative returns								
	Time-series				Cross-sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
HO belief	2.02	1.72	0.74	-1.04	0.21	-0.04	-0.59	-1.16
	(0.45)	(0.48)	(0.65)	(0.93)	(0.06)	(0.08)	(0.09)	(0.13)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
N	259	259	259	258	6,116	6,116	6,116	6,116
R^2	.19	.09	.01	.01	.00	.00	.01	.02

Panel B: Short-term peaks and troughs				
	ST peak	*ST trough*	*ST peak*	*ST trough*
HO belief	0.34	-0.29		
	(0.04)	(0.05)		
Overvaluation			0.58	-0.09
			(0.08)	(0.14)
Time FE	NA	NA	NA	NA
N	259	259	259	259
R^2	.21	.15	.26	.01

Table C.6: Higher-order beliefs and return expectations (individual investors)

Note: This table replicates Table 3 for the individual investor subset of our sample.

Panel A: Term structure of expected cumulative returns								
	Time-series				Cross-sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
HO belief	0.42	-0.15	-0.72	-2.52	-0.12	-0.61	-1.48	-2.42
	(0.22)	(0.40)	(0.61)	(1.03)	(0.05)	(0.08)	(0.13)	(0.18)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
N	258	258	258	259	4,841	4,841	4,841	4,841
R^2	.01	.00	.01	.06	.00	.01	.04	.06

Panel B: Short-term peaks and troughs				
	ST peak	*ST trough*	*ST peak*	*ST trough*
HO belief	0.30	-0.14		
	(0.08)	(0.06)		
Overvaluation			0.64	0.01
			(0.15)	(0.14)
Time FE	NA	NA	NA	NA
N	259	259	259	259
R^2	.09	.04	.17	.00

Table C.7: Higher-order beliefs and return expectations (institutional investors)

Note: This table replicates Table 3 for the institutional investor subset of our sample.

Panel A: Term structure of expected cumulative returns and higher-order optimism								
	Time-series				Cross-sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
HO optimism	2.32	1.81	0.24	-2.30	0.04	-0.34	-1.00	-1.75
	(0.39)	(0.52)	(0.78)	(1.53)	(0.05)	(0.07)	(0.11)	(0.16)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
N	259	259	259	259	11,020	11,020	11,020	11,020
R^2	.17	.06	.00	.03	.00	.00	.01	.02

Panel B: Term structure of expected cumulative returns and higher-order pessimism								
	Time-series				Cross-sectional			
	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$	$\mathbb{E}_t(R_{t,t+1})$	$\mathbb{E}_t(R_{t,t+3})$	$\mathbb{E}_t(R_{t,t+6})$	$\mathbb{E}_t(R_{t,t+12})$
HO pessimism	-2.07	-0.92	-0.25	3.58	-0.06	0.18	0.76	1.36
	(0.66)	(0.88)	(1.13)	(1.71)	(0.05)	(0.06)	(0.10)	(0.15)
Time FE	NA	NA	NA	NA	Yes	Yes	Yes	Yes
N	259	259	259	259	10,987	10,987	10,987	10,987
R^2	.08	.01	.00	.04	.00	.00	.01	.01

Table C.8: Higher-order optimism, pessimism, and return expectations

Note: This table replicates Table 3, separately breaking down the results for *HO optimism* and *HO pessimism*.

Panel A: Levels regressions										
	DJIA futures					S&P 500 futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.22				0.27	0.32				0.31
	(0.08)				(0.17)	(0.14)				(0.21)
$\mathbb{E}_t(R_{t,t+3})$		0.07			-0.05		0.29			-0.20
		(0.10)			(0.21)		(0.10)			(0.26)
$\mathbb{E}_t(R_{t,t+6})$			-0.07		0.02			0.25		0.30
			(0.11)		(0.20)			(0.08)		(0.18)
$\mathbb{E}_t(R_{t,t+12})$				-0.11	-0.12				0.10	-0.01
				(0.07)	(0.09)				(0.05)	(0.07)
R^2	.05	.01	.01	.07	.13	.11	.13	.15	.06	.18
N	69	69	69	69	69	69	69	69	69	69

Panel B: Changes regressions										
	DJIA futures					S&P 500 futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.12				0.02	0.46				0.57
	(0.14)				(0.24)	(0.09)				(0.16)
$\mathbb{E}_t(R_{t,t+3})$		0.10			0.16		0.29			-0.26
		(0.10)			(0.22)		(0.09)			(0.24)
$\mathbb{E}_t(R_{t,t+6})$			0.03		-0.08			0.20		0.28
			(0.11)		(0.19)			(0.09)		(0.15)
$\mathbb{E}_t(R_{t,t+12})$				0.00	0.00				0.06	-0.05
				(0.05)	(0.08)				(0.06)	(0.07)
R^2	.02	.02	.00	.00	.02	.25	.14	.09	.01	.29
N	68	68	68	68	68	68	68	68	68	68

Table C.9: Return expectations and asset manager futures positions

Note: This table replicates Table 4, using the positioning of asset managers rather than dealers as the dependent variable.

	Panel A: Levels regressions									
	DJIA futures					S&P 500 futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.33				0.36	0.18				-0.22
	(0.11)				(0.21)	(0.12)				(0.20)
$\mathbb{E}_t(R_{t,t+3})$		0.20			-0.03		0.28			0.44
		(0.08)			(0.25)		(0.10)			(0.26)
$\mathbb{E}_t(R_{t,t+6})$			0.08		-0.01			0.24		-0.05
			(0.07)		(0.16)			(0.09)		(0.26)
$\mathbb{E}_t(R_{t,t+12})$				0.00	-0.01				0.12	0.07
				(0.07)	(0.07)				(0.07)	(0.12)
R^2	.12	.06	.01	.00	.12	.03	.13	.14	.08	.17
N	69	69	69	69	69	69	69	69	69	69

	Panel B: Changes regressions									
	DJIA futures					S&P 500 futures				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\mathbb{E}_t(R_{t,t+1})$	0.19				0.29	0.11				-0.09
	(0.14)				(0.16)	(0.12)				(0.12)
$\mathbb{E}_t(R_{t,t+3})$		0.09			-0.15		0.14			0.33
		(0.12)			(0.22)		(0.11)			(0.20)
$\mathbb{E}_t(R_{t,t+6})$			0.07		0.00			0.05		-0.22
			(0.09)		(0.14)			(0.10)		(0.16)
$\mathbb{E}_t(R_{t,t+12})$				0.09	0.11				0.05	0.09
				(0.06)	(0.08)				(0.07)	(0.08)
R^2	.04	.01	.01	.03	.08	.01	.03	.01	.01	.06
N	68	68	68	68	68	68	68	68	68	68

Table C.10: Return expectations and hedge fund futures positions

Note: This table replicates Table 4, using the positioning of leverage funds (hedge funds) rather than dealers as the dependent variable.

Panel A: Levels regressions						
	DJIA futures			S&P 500 futures		
	(1)	(2)	(3)	(4)	(5)	(6)
HO belief	0.37		0.57	0.08		0.66
	(0.14)		(0.21)	(0.20)		(0.20)
Overvaluation		0.11	-0.28		-0.34	-0.79
		(0.17)	(0.24)		(0.11)	(0.16)
R^2	.14	.01	.19	.01	.14	.37
N	69	69	69	69	69	69

Panel B: Changes regressions						
	DJIA futures			S&P 500 futures		
	(1)	(2)	(3)	(4)	(5)	(6)
HO belief	0.21		0.26	0.14		0.08
	(0.09)		(0.14)	(0.11)		(0.12)
Overvaluation		0.07	-0.08		0.14	0.10
		(0.11)	(0.15)		(0.10)	(0.10)
R^2	.04	.00	.05	.02	.02	.03
N	68	68	68	68	68	68

Table C.11: Higher-order beliefs, valuations, and investor futures positions

Note: The table reports results from regressions of *Net positioning* on *HO belief* and *Overvaluation*. Observations are quarterly levels in Panel A ("Level regressions"). In Panel B ("Changes regressions"), observations are quarterly changes in the independent variables and the change in short minus long futures contracts held by dealers, normalized by lagged open interest. All regression variables are normalized to have zero mean and unit standard deviation. The first four columns in the table report results where futures positions are those of dealers in Dow Jones Industrial Average (DJIA) futures. The last four columns report results where futures positions are those of dealers in S&P 500 futures. Newey-West standard errors (4 lags) of coefficients are reported in parentheses.

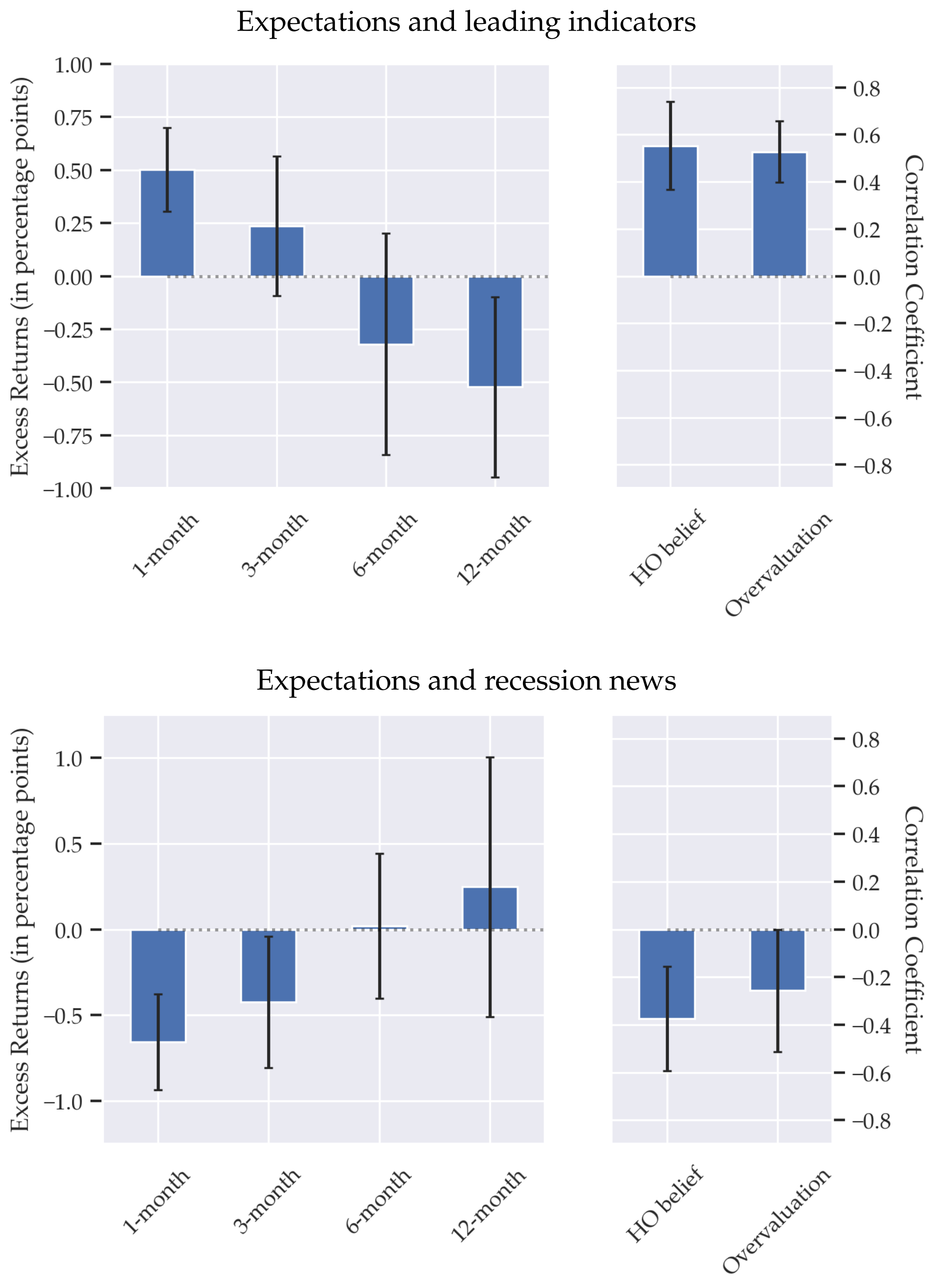

Figure C.1: Macroeconomic news and expectations (individual investors)

Note: The figure replicates Figure 2 for the individual investor subset of our sample.

Alt text: Bar graph depicting the coefficients from regressions of return expectations, *HO belief*, and *Overvaluation* on macroeconomic news measures, with 95% confidence intervals.

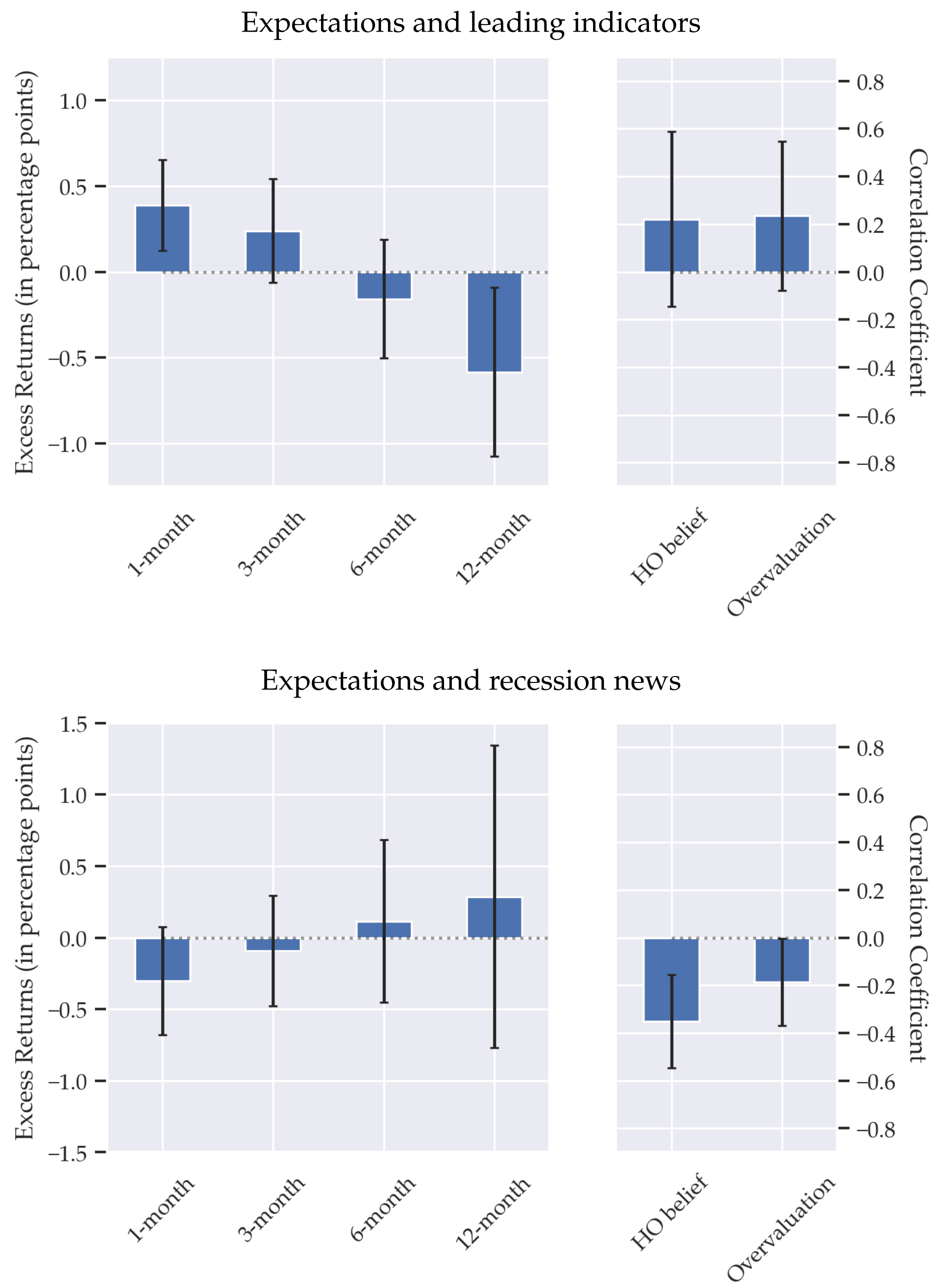

Figure C.2: Macroeconomic news and expectations (institutional investors)

Note: The figure replicates Figure 2 for the institutional investor subset of our sample.

Alt text: Bar graph depicting the coefficients from regressions of return expectations, *HO belief*, and *Overvaluation* on macroeconomic news measures, with 95% confidence intervals.

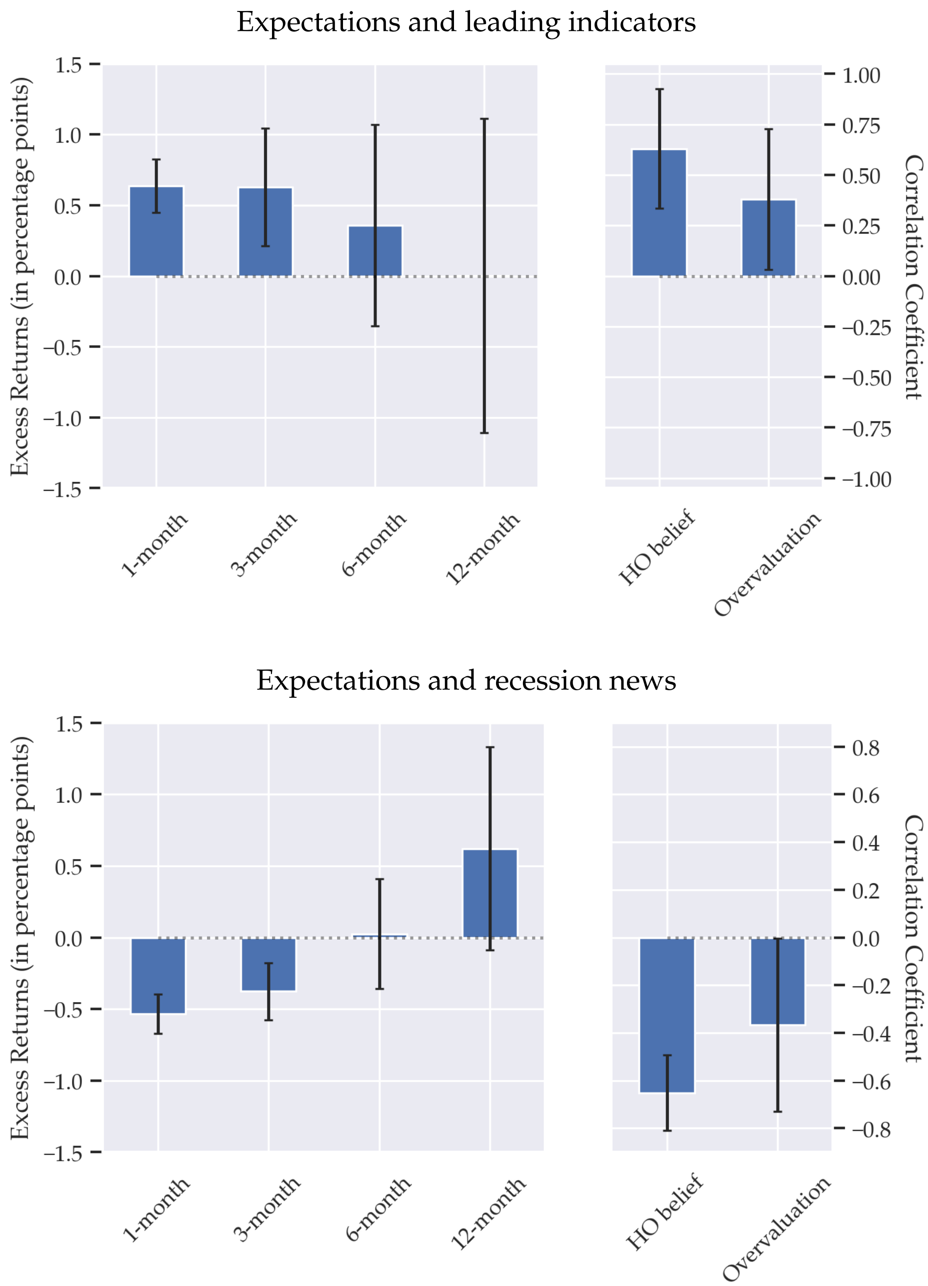

Figure C.3: Macroeconomic news and expectations (levels)

Note: The figure replicates Figure 2 using the levels of the dependent and independent variables, rather than changes in return expectations and innovations to the dependent variable.

Alt text: Bar graph depicting the coefficients from regressions of return expectations, *HO belief*, and *Overvaluation* on macroeconomic news measures, with 95% confidence intervals.

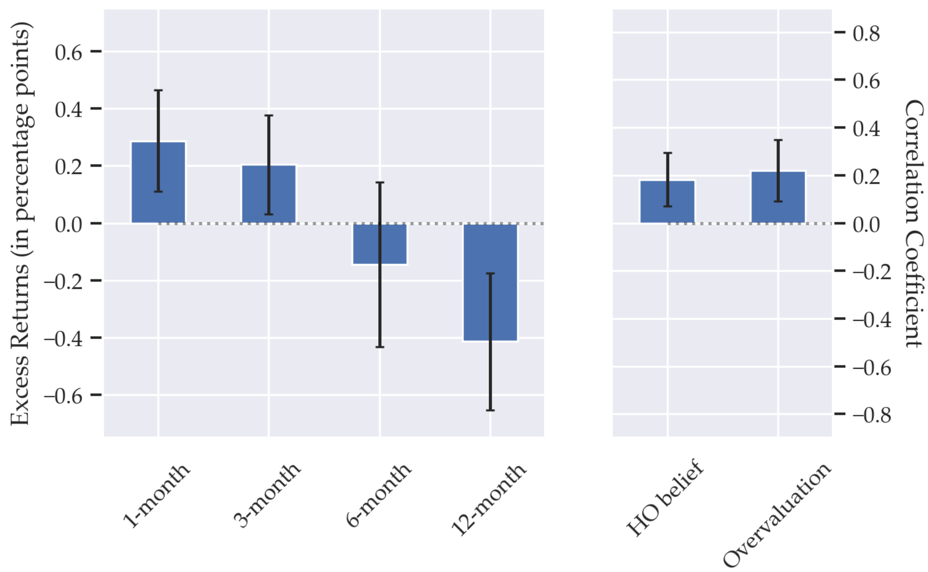

Figure C.4: Coincident indicators and expectations

Note: The figure replicates the top panel of Figure 2, using innovations to the Coincident macroeconomic indicators index from the Conference Board.

Alt text: Bar graph depicting the coefficients from regressions of return expectations, *HO belief*, and *Overvaluation* on macroeconomic news measures, with 95% confidence intervals.

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