The Role of Beliefs in Asset Prices: Evidence from Exchange Rates[*]

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Abstract

A long-standing question is why asset prices sometimes underreact and sometimes overreact to news. We explore this question in currency markets. We use survey data to estimate a model featuring investors with noisy private information and extrapolative beliefs about interest rates, and find the estimated model quantitatively matches patterns of initial underreaction and delayed overreaction of currencies in response to interest rate news. The model also helps explain changes in the time-series predictability of currency returns by interest rates in recent years, the term structure of UIP deviations, and additional features of beliefs in survey data. Our results highlight the role of investors' beliefs in asset price behavior.

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1 Introduction

Underreaction and overreaction to news are pervasive features of asset price behavior across a variety of settings, but a long-standing challenge remains understanding when and why asset prices sometimes appear to underreact and sometimes appear to overreact to news (Barberis (2018)). Models proposed to explain the facts have placed particular focus on incorrect beliefs by market participants as an explanation.[1] A growing body of work in macroeconomics and finance has focused on survey data as a means to understand the beliefs of forecasters and market participants, and has documented substantial deviations from the traditional Full-Information Rational Expectations (FIRE) paradigm (Mankiw, Reis and Wolfers (2003), Coibion and Gorodnichenko (2015), Bordalo et al. (2020*b*), Kohlhas and Walther (2020), and Angeletos, Huo and Sastry (2020)). Motivated by this work, we take seriously that survey data may capture the beliefs of market participants, and use survey data to qualitatively and quantitatively study the ability of imperfect investor expectations to explain asset price behavior.[2]

We use currency markets as a laboratory for our analysis. There are rich historical international survey data on expectations of exchange rates, interest rates, and related macroeconomic fundamentals.[3] Moreover, a substantial literature has documented puzzling patterns consistent with currencies underreacting and overreacting to interest rate news. In particular, increases in interest rates positively predict currency excess returns at short horizons (Eichenbaum and Evans (1995)), and higher interest rate differentials negatively predict currency excess returns at longer horizons (Bacchetta and Van Wincoop (2010)).[4] In a related vein, currency excess returns also exhibit momentum and reversal; they have positive autocorrelations at short horizons and negative autocorrelations at longer horizons (Moskowitz, Ooi and Pedersen (2012)). Existing models of exchange rate determination have struggled to reconcile these patterns (Engel (2016)).

We begin by presenting three pieces of motivating empirical analysis. First, we analyze the relationship between currency excess returns and survey-based consensus forecast errors of short-term interest rate differentials (forecast errors averaged across all individuals). We find a significant, positive relationship between forecast errors of interest rate differentials and currency excess returns, suggesting that understanding forecast errors of interest rate differentials may be useful

[1]Influential early models of underreaction and overreaction include Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Hong and Stein (1999). Barberis (2018) discusses underreaction and overreaction of asset prices in a recent survey.

[2]In this regard, our paper is highly related to a related strand of literature documents patterns consistent with extrapolation in survey data on expected stock market returns (e.g., Greenwood and Shleifer (2014)), and works to build models consistent with the survey evidence and observed asset price behavior across asset classes (e.g., Barberis et al. (2015), Glaeser and Nathanson (2017) , Jin and Sui (Forthcoming)).

[3]There is a notable literature using survey data to study expectations of exchange rates movements, both historical (Dominguez (1986), Frankel and Froot (1987, 1990), Froot and Frankel (1989), Ito (1990)), and more recent (Bacchetta, Mertens and Van Wincoop (2009), Stavrakeva and Tang (2020*a*,*b*) and Kalemli-Ozcan and Varela (2021)). A consensus emerges from this literature that deviations from full information rational expectations may play an important role in explaining exchange rate behavior.

[4]The initial underreaction of exchange rates to interest rate news is often referred to as the *delayed overshooting* puzzle. The negative predictability of interest rate differentials for currency excess returns at long horizons is often referred to as the *predictability reversal* puzzle.

for understanding currency behavior, and particularly the patterns of underreaction and overreaction of currency excess returns in response to interest rate news.

Our second analysis reveals that following news of higher interest rate differentials, consensus forecast errors of interest rate differentials are initially positive for a number of periods, indicating initial underreaction of expectations to the news. Subsequently, forecast errors become negative, indicating delayed overreaction of expectations to the news. Third, we find that the underreaction of survey-based forecasts to news is primarily a feature of *consensus* forecasts, and largely disappears when analyzing forecasts made by individuals. The second and third analyses reveal that forecasters make systematic and predictable errors in forecasting interest rates, with interest rate forecast errors exhibiting similar dynamics as exchange rates. The third piece of evidence also highlights that information heterogeneity plays a key role in underreaction to news.

We seek to construct a model consistent with the survey evidence, and evaluate its ability to quantitatively capture currency excess returns and their predictability by interest rate news. The model features overlapping-generations of investors in a small open-economy setting. In the model, the interest rate differential between countries is determined by macroeconomic fundamentals, which follow an exogenous AR(1) process. The equilibrium exchange rate is determined by short-lived investors' relative demand for home versus foreign currency bonds, which, in turn, is determined by investors' beliefs and higher-order beliefs about future interest rate differentials.

We introduce two key frictions that help capture the survey evidence. First, investors each receive noisy private signals about macroeconomic fundamentals, which they use to formulate their beliefs about future interest rate differentials. These noisy private signals can be literally thought of as corresponding with dispersed information (Lucas Jr (1972), Morris and Shin (2002)), or as stemming from inattention (Mankiw and Reis (2002), Sims (2003, 2010), Woodford (2003)). Second, investors are *extrapolative*; they uniformly overestimate the persistence of fundamentals. The frictions in the model, relative to a benchmark of Full Information Rational Expectations (FIRE), allow the model to capture the survey evidence. We estimate the model using the empirical evidence as target moments. We find the estimated model is able to qualitatively and quantitatively capture exchange rate behavior, and in particular the observed underreaction and overreaction to interest rate news.

Noisy private information plays the key role in allowing the model to capture the sluggish response of consensus expectations to news about higher future interest rate differentials. Consistent with the survey data, investors in the model do not underreact to the news that they observe. However, the noise in their signals prevents them from immediately observing and updating their beliefs in response to the 'true' interest rate news in a given period. Hence, the consensus expectation strongly underreacts to interest rate news.

The exchange rate in the model also displays initial underreaction to news of higher future interest rate differentials. A significant determinant of the exchange rate is the sum of future expected interest rate differentials; accordingly, the exchange rate inherits many properties of consensus interest rate forecasts, including their initial underreaction. A second driver of the exchange rate's underreaction is higher-order uncertainty. Investors only partially trade towards their fundamental valuation of the exchange rate, because of uncertainty regarding whether other investors agree with their beliefs. Our estimated model provides a tractable way to evaluate the relative importance of the different mechanisms. Doing so, we find that the sluggishness of the consensus belief regarding future interest rate differentials, rather than higher-order uncertainty or individual-level underreaction, is the primary driver of currency underreaction. The use of survey data to quantify the importance of higher order uncertainty versus the sluggishness of first -order beliefs is a novel result that provides empirical content to a primarily theoretical literature on asset pricing models featuring differences-of-information and higher-order uncertainty.[5]

Co-existing with underreaction driven by dispersed private information, investors' extrapolation leads them to overestimate the persistence of the interest rate differential. Once consensus expectations fully internalize past interest rate news, investors believe that the interest rate differential will remain at its current level longer than it actually does. This mistaken perception leads exchange rates to eventually overreact; currencies experience low excess returns several periods after they have high interest rates, corresponding with investors observing lower interest rate differentials than they expected. Given the relationship between exchange rates and interest rates, the above patterns also manifest in positive autocorrelations of currency excess returns at short-horizons (momentum) and negative autocorrelations at longer-horizons (reversal). Despite not using data on exchange rates in our estimation, we find that our estimated model is able to do a reasonable quantitative job of capturing underreaction and overreaction of exchange rates.

Having found that the model is able to capture asset price underreaction and overreaction in a quantitatively realistic way, we dig deeper into the implications of the model, and its ability to explain recent facts in the literature. First, the return predictability of interest rate differentials for currency excess returns has become substantially weaker, and has perhaps even reversed, in the post-financial crisis period (Bussiere et al. (2018); Engel et al. (2019); Engel, Kazakova and Wang (2021)). In our model, the predictability of currency excess returns by interest rate differentials is driven by underreaction to interest rate news, stemming from dispersed private information. As we reduce the dispersion of private information, extrapolation leads consensus expectations to overreact to interest rate news, reversing the sign of the predictability of interest rate differentials for currency excess returns. Consistent with this channel, we find suggestive evidence that belief dispersion about future interest rates has decreased in recent times, and consensus forecasts appear to overreact to news about interest rate differentials, in contrast with the earlier part of the sample. This result suggests that informational frictions (dispersed private information) and errors in belief formation (extrapolation) are *both* necessary ingredients to fully characterize the patterns over the full sample.

Second, a notable fact in the literature is that individual's subjective beliefs are highly persis-

[5] Singleton (1987) and Allen, Morris and Shin (2006) are prominent papers in the literature on beauty contests and higher-order beliefs in asset pricing. Similar to our paper, Bacchetta and Van Wincoop (2006) suggest a role for dispersed private information and higher-order expectations as drivers of exchange rate behavior, though they do not focus on the relationship between interest rates and exchange rates, nor do they use data on beliefs to estimate the model.

tent; pessimists are persistently pessimistic and optimists are persistently optimistic (Giglio et al. (2021)). In our model, because investors never observe the 'true' macroeconomic fundamentals, private information received in a given period influences investor beliefs for several subsequent periods. In turn, investors that receive a positive signal about the future interest rate differential, relative to investors that receive a negative signal, may hold onto the belief of relatively higher future interest rate differentials and exchange rates for several periods. In our estimated model, it takes more than two years for the average belief of investors in the top or bottom deciles of the belief distribution of exchange rates to converge to the average belief of the population. This result, which is not explicitly targeted in the model, is remarkably consistent with the behavior of interest rate forecasts from Consensus Economics and the Survey of Professional Forecasters, and suggests a potentially important role for dispersed information in the persistence of subjective beliefs. At the same time, we find evidence that some agents' beliefs converge more slowly than predicted by the model, suggesting features other than dispersed information may also be important for capturing persistent disagreement in asset valuations.

Third, a recently documented fact in the literature on exchange rates is that, while interest rate differentials positively predict currency excess returns, this return predictability is declining in the maturity of bonds used to borrow and lend in currency trades (Lustig, Stathopoulos and Verdelhan (2019)). In our model, the source of currency return predictability is the fact that when the interest rate differential increases, the home currency appreciates in the following period, as it sluggishly incorporates information of higher future interest rate differentials. However, as investors incorporate news of higher future interest differentials into asset prices, long maturity bonds of the home country earn lower returns than long maturity bonds of the foreign country, offsetting the exchange rate returns of borrowing in foreign currency and lending in home currency. Hence, our model is able to partially explain the declining return predictability of interest rate differentials for carry trades using longer maturity bonds.[6]

1.1 Relation to Existing Literature

Our paper relates to a previously mentioned literature in finance that documents and seeks to understand patterns of underreaction and overreaction in asset prices. Our work suggests that these phenomena may be partially reconciled by investors with noisy private information (which contributes to underreaction), who also believe that fundamentals are more persistent than they are in reality (which contributes to overreaction).[7] Our work carries the advantage of providing a quantitatively-realistic explanation for the behavior of exchange rates that is grounded in evi-

[6]The insight that underreaction to interest rate news may contribute to the downward-sloping term structure is shared with Granziera and Sihvonen (2021), who suggest more broadly that sluggish consensus expectations of short-rates may help explain why short rates and yield spreads predict bond and currency returns.

[7]Relatedly, Bordalo et al. (2020*a*) study a model where investors receive noisy private signals, which induces consensus underreaction, and also have *diagnostic expectations*, which induces them to update their belief too far in the direction of states of the world whose objective likelihood has increased the most due to recent news. Assuming investors overreact to news from multiple past periods, the two ingredients may lead to initial underreaction and delayed overreaction. The authors also include learning from prices and speculation in their model, and study bubbles and crashes.

dence found in macroeconomic survey data, and may have broader applications across different asset markets, where patterns of underreaction and overreaction of asset prices are ubiquitous.[8] Our paper is also closely related to a recent literature that places particular focus on forecasts (and errors in forecasts) of interest rates (Cieslak (2018), Brooks, Katz and Lustig (2019), Wang (2020), d'Arienzo (2020), Hanson, Lucca and Wright (2021), Granziera and Sihvonen (2021)). Our evidence of initial underreaction and delayed overreaction of interest rate expectations builds on the findings in these papers, and we extend the implications of such expectations in order to understand the behavior of exchange rates.

In the literature on exchange rates, the closest antecedent to our work is Gourinchas and Tornell (2004).[9] In their model, a representative agent believes that interest rates may temporarily deviate from their fundamental values. This belief induces the representative agent to underreact to interest rate changes due to confusion about whether such changes are persistent or transitory. In an independent and contemporaneous paper to our own, Candian and De Leo (2021) extend the model of Gourinchas and Tornell (2004) by assuming the representative agent over-extrapolates of the level of fundamentals that govern the interest rate, which enables their model to capture the delayed overreaction of exchange rates to interest rate news.[10] In related work, Granziera and Sihvonen (2021) suggest that a representative agent with sticky expectations of interest rates can help capture term structure facts about bond and currency returns, as well as the failure of UIP.

In contrast to these papers, our model does not employ a representative agent framework, but rather includes investors with heterogeneous beliefs. Our framework allows us to capture key insights relevant for asset price behavior, in a manner consistent with the survey data. For example, our paper not only suggests that investor underreaction to fundamental news may be important, but seeks to capture and quantitatively assess the underlying mechanisms behind this underreaction. Individual investors *do not* underreact to the information they receive about future interest rates, as the representative investor does in the other works. Instead, consensus underreaction stems from dispersion of information about macroeconomic fundamentals. This channel of underreaction is useful for understanding the failure of UIP over the full sample, where investor disagreement about interest rates has decreased in recent years (with rates close to the zero lower bound). Our model predicts that with lower dispersion of information (proxied by lower dis-

[8] In addition to underreaction and overreaction in first moments of asset prices, Lochstoer and Muir (2022) suggest that agents may also underreact and overreact to news about second moments. They suggest that investors may have sticky and extrapolative beliefs about stock volatility.

[9] Other behavioral models in the literature that reproduce some features of exchange rate dynamics include: Burnside et al. (2011), who focus on investor overconfidence, and suggest that overreaction to inflation news may drive the forward premium puzzle; Ilut (2012), who suggests that ambiguity aversion may help resolve the UIP puzzle and capture time-series momentum; and Molavi, Tahbaz-Salehi and Vedolin (2021), who argue that limited information-processing capacity, in the form of only being able to process k of $N > k$ factors that drive the true data generating process, leads some investors to misperceive the process that interest rate differentials follow, contributing to the failure of uncovered interest rate parity and predictability reversal puzzles. Relative to these papers, our study is consistent with additional survey evidence, and also explains additional puzzles.

[10] Candian and De Leo (2021) embed their framework into a two-country general equilibrium model that endogenizes the interest rate, and turn their focus to the relationship between consensus expectations of macroeconomic quantities and exchange rates.

agreement), overextrapolation of interest rates plays a dominant role, consensus beliefs *overreact* to interest rate news, and the sign of the failure of UIP reverses, just as we observe in the data.

Moreover, even when consensus beliefs deviate from FIRE, an important question is how such belief deviations are transmitted into asset prices. When investors have heterogeneous beliefs, other factors enter into their trading behavior, for example their higher-order beliefs about other investors' beliefs. A novel result in our paper is to use survey data to evaluate and quantitatively assess the potential importance of different channels that may matter for asset price behavior in models featuring differences-of-information.

Finally, relaxing the representative agent paradigm, we are able to study features of investor beliefs, such as the persistence of belief disagreement, an important reason that different investors hold different allocations. In addition to their relevance to facts in currency markets, these advantages of our framework are also more broadly applicable for understanding the dynamics of prices in other asset markets.

While our work suggests that expectational errors of macroeconomic fundamentals, and in particular of interest rates, may play an important role in explaining the underreaction and overreaction of exchange rates to interest rate and fundamental news, the channel we focus on may be complementary to other explanations for exchange rate behavior posed in the literature. For example, Bacchetta and Van Wincoop (2010, 2021) suggest that delayed portfolio adjustment might explain the initial underreaction and delayed overreaction of exchange rates in response to news, and other features of exchange rate behavior. Valchev (2020) suggests that endogenous time-varying convenience yield differentials between bonds of different currencies may help rationalize the predictability of currency excess returns by interest rate differentials.[11] Another body of work also suggests that imperfect financial markets may help explain the failure of UIP (Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021)), as well as the downward-sloping term structure of UIP violations (Greenwood et al. (2020), Gourinchas, Ray and Vayanos (2021)).[12] And finally, our model does not speak to the dynamics of investors' subjective return expectations (Bacchetta, Mertens and Van Wincoop (2009), Stavrakeva and Tang (2020*a*), Nagel and Xu (2022)), or to potential time-varying risk premia (Verdelhan (2010), Bansal and Shaliastovich (2013), Colacito and Croce (2013) Farhi and Gabaix (2016)), which may be important to exchange rate dynamics and facts beyond the particular puzzles of interest to us.

[11] Jiang, Krishnamurthy and Lustig (2021) discuss the relationship between convenience yields and exchange rates more generally.

[12] In particular, Greenwood et al. (2020) and Gourinchas, Ray and Vayanos (2021) focus on specialized bond investors that absorb demand for bonds (as in Vayanos and Vila (2021)), and also absorb currency risk (as in Gabaix and Maggiori (2015)). The downward-sloping term structure of UIP violations emerges in these models because, relative to the short-maturity UIP trade, the currency exposure of global bond investors in the long-maturity UIP trade helps offset the interest rate risk the investors face in long-term bonds. Greenwood et al. (2020) and Gourinchas, Ray and Vayanos (2021) also use their models to understand the relationship between exchange rates, term premia, and policies that affect term premia such as quantitative easing.

2 Motivating Empirical Evidence

We begin our analysis in the paper by presenting three facts using survey data on expectations. The facts motivate and guide the model that we present later in the paper.

2.1 Sample and Data Sources

The sample for our analysis consists of the G11 currencies: the Australian dollar, the Canadian dollar, the Danish krone, the Euro, the Japanese yen, the New Zealand dollar, the Norwegian Krone, the Swedish krona, the Swiss franc, and the British pound sterling. The sample period begins in the 1980s (differing slightly across analyses due to data availability) and ends in December 2007. We choose the end date for our sample based on the fact, noted by Bussiere et al. (2018), that some of the patterns in the data appear to reverse following the financial crisis. We explore this point in more detail later in the paper.[13]

Below we describe the data sources used in more detail.

Refinitiv Datastream

We obtain data on exchange rates for our entire sample from Refinitiv datastream.

Survey of Professional Forecasters

We use data on forecasts (and the corresponding realizations) of US Treasury Bill rates, US unemployment, and US inflation from the Survey of Professional Forecasters from the Philadelphia Fed, a commonly used data source to study macroeconomic forecasting. The data include quarterly data on forecasts and realizations of macroeconomic series.

FX4Casts

We obtain data on exchange rate forecasts for the full sample of countries from FX4casts. For each month, the dataset provides the average forecast of exchange rates and interest rates from a number of large financial institutions that actively participate in foreign exchange markets across the world.

The data on exchange rate forecasts include 1-, 3-, 6-, 12-, and 24-month ahead forecasts of the spot exchange rates for 32 currencies, along with the 5th and 95th percentile of the distribution of forecasts made for each currency at each point in time. The data begin in August 1986.

Consensus Economics

We obtain data on interest rate forecasts for all countries in our sample from Consensus Economics. For each month, the dataset provides forecasts of interest rates (and other macroeconomic

[13] We also report results for the motivating facts using a sample that ends in December 2019 in Internet Appendix IA.A. The patterns documented in the shorter sample are still largely present in the full sample.

quantities) from a number of large financial institutions across the world. The sample begins in October 1989.

The data on interest rate forecasts include forecaster level data for forecasts of the short-term interest rate three months from the forecast date and twelve months from the forecast date. We form consensus forecasts of interest rate differentials by taking the average forecast of each country at each point in time and subtracting the average forecast of the US interest rate in the same period. We also construct individual level interest rate differential forecasts for forecasters that make forecasts of *both* the US short-term interest rate and the short-term interest rate of a given country, and perform some analyses on these forecasts. However, the number forecasters that make forecasts for both the US and a foreign country's short-term interest rates is small, and is almost always less than ten for a given country.

Fact 1: Interest Rate Forecast Errors Correlate with Exchange Rate Behavior

Our first piece of motivating evidence is that survey-based measures of interest rate forecast errors are highly correlated with both currency excess returns and consensus forecast errors of exchange rates.

For currency j and quarter t, we run regressions of the form

$$y_{j,t} = \beta \left(i^d_{j,t} - \bar{\mathbb{E}}_{t-1} i^d_{j,t} \right) + \epsilon_{j,t}, \tag{1}$$

where $y_{j,t}$ is the dependent variable in the regression, $i^d_{j,t}$ is the short-term interest rate of currency j minus the US short-term interest rate, $\bar{\mathbb{E}}$ corresponds with the consensus expectation reported in survey data, and β is the coefficient of interest. We consider two dependent variables. The first is the excess returns of borrowing in USD and investing in currency j from quarter $t-1$ to quarter t, $s_{j,t} - s_{j,t-1} + i^d_t$, where $s_{j,t}$ is the log exchange rate expressed in units of USD per one unit of currency j. The second is the consensus exchange rate forecast error, $s_{j,t} - \bar{\mathbb{E}}_{t-1} s_{j,t}$. We also run versions of the regressions including the period $t-1$ expectation of the interest rate differential, $\mathbb{E}_{t-1}(i^d_{j,t})$, as a control. We standardize variables in the regression have zero mean and unit standard deviation for each country, so that β can be interpreted as a correlation. Standard errors in the regressions are clustered by currency and time period.

Table 1 reports the results from the regressions. In columns (1) and (2), with currency excess returns as the dependent variable, the coefficient on interest rate forecast errors is 0.19 in a univariate regression, and 0.20 when controlling for interest rate forecasts, with t-statistics of 2.02 and 2.15. The evidence indicates a strong relationship between consensus interest rate forecast errors and currency excess returns, with currencies appreciating when interest rates are higher than forecasted. Columns (3) and (4) of the table feature consensus exchange rate forecast errors as the dependent variable. The coefficient on interest rate forecast errors is 0.17 in the univariate regression and 0.18 in the multivariate regression, with t-statistics of 1.83 and 1.91. These results suggest that the currency excess returns associated with interest rate forecast errors largely stem from unexpected currency appreciations, as measured using survey-based expectations of exchange rates. Appendix Table IA.A.1 reports regression results with the sample extended through the end of 2019. The estimated regression coefficients on interest rate forecast errors are similar in the full sample, with t-statistics above 2 in each of the regressions. The regression evidence also suggests that lagged forecast errors of interest rate differentials have some return predictability for currency excess returns, though this result is not a particular focus of ours. This result can be explained by fact that interest rate differentials exhibit substantial time-series persistence in conjunction with the well-known empirical failure of UIP.

The regression results suggest that forecast errors of interest rate differentials contemporaneously correspond with currency movements. While we explicitly focus on interest rates in our facts here, the results are also intuitively linked to the fact that survey-based forecast errors of other macroeconomic variables, which we expect are related to short-term interest rates, also have explanatory power for contemporary exchange rate movements (Engel et al. (2007), Stavrakeva and Tang (2020*b*)).[14]

Given the empirical facts suggesting that exchange rates underreact and overreact to interest rate news, the evidence suggests that better understanding forecast errors of interest rate differentials may shed additional light on exchange rate behavior. Hence, we next dig deeper into the behavior of interest rate forecast errors.

Fact 2: Initial Underreaction and Delayed Overreaction in Consensus Interest Rate Expectations

Our second piece of motivating empirical evidence is that in response to interest rate news, consensus expectations of short-term interest rates initially underreact and subsequently overreact. In particular, following the arrival of news indicating higher short-term interest rates, survey-based forecasts are lower than realized interest rates for an initial period, indicating *underreaction*. Following this initial underreaction, in subsequent periods, forecasts of interest rates are higher than realized interest rates, indicating *overreaction*.

To capture the arrival of interest rate news, we use a time-series of interest rate news shocks constructed by Angeletos, Collard and Dellas (2020).[15] The shocks are constructed by running a VAR of ten US macroeconomic variables, including the US Federal Funds rate, and extracting the linear combination of residuals in the VAR that explains the most quarterly variation of the federal funds rate for 6 to 32 quarters ahead.

[14]The fact that survey-based measures of macroeconomic surprises are correlated with exchange rate movements is perhaps surprising, given the common finding that macroeconomic variables themselves do not appear to be correlated with exchange rates (the exchange rate disconnect, Meese and Rogoff (1983)). Using a VAR, Stavrakeva and Tang (2020*b*) find survey-based measures of macroeconomic surprises may explain nearly half of nominal exchange rate variation. Our theoretical framework in this paper, which suggests that incorrect beliefs about macroeconomic fundamentals and interest rates are important for understanding exchange rate movements, suggests a way to reconcile why there is such a strong relationship between survey-based measures of macroeconomic news and exchange rates, despite the exchange rate disconnect.

[15]We download data on shocks from George-Marios Angeletos' website.

Figure 1 plots impulse response functions, at the quarterly frequency, of US Treasury Bill rates, consensus forecasts of US Treasury Bill rates from the Survey of Professional Forecasters from four quarters prior, and consensus forecast errors of US Treasury Bill rates. The impulse response functions are estimated from regressions of the form

$$x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma_h C_t + u_{t+h} \tag{2}$$

where x_{t+h} is the variable of interest, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are the interest rate shocks. The variables of interest are i_{t+h} (the Treasury Bill rate h quarters after the shock), $\bar{\mathbb{E}}_{t+h-4} i_{t+h}$ (the period $t+h-4$ consensus forecast of the period $t+h$ Treasury Bill rate rate), and $i_{t+h} - \bar{\mathbb{E}}_{t+h-4} i_{t+h}$ (the consensus forecast error of the interest rate). The sample for the analysis runs from 1981 to 2007. The figure also plots plus and minus one standard error for the impulse response functions.

The impulse response functions reveal that, for four to six quarters after the arrival of an interest rate news shock, consensus forecasts of interest rates are persistently lower than the realized interest rate, indicating underreaction to interest rate news. However, for seven to eighteen quarters after the shock, forecasted interest rates exceed the realized interest rate, indicating the subsequent overreaction of interest rate forecasts. These patterns are captured by initial positive forecast errors, followed by negative forecast errors.

For exchange rates, the behavior of the differentials between the US short-term interest rate and the short-term interest rates of other currencies is of particular interest. Using Equation (2), we estimate impulse response functions where the variables of interest are interest rate differentials (the US interest rate minus the foreign interest rate), consensus forecasts of interest rate differentials, and consensus forecast errors of interest rate differentials. The data on interest rate differential forecasts and realizations span all the countries in our sample and are from Consensus Economics. The sample for the analysis runs from October 1989 through December 2007.

Figure 2 plots the impulse response functions. The figure reveals a similar pattern of initial underreaction and subsequent overreaction of forecasts to positive US interest rate news shocks. The consistent initial underreaction and subsequent overreaction in survey-based forecasts of interest rate differentials indicates the potential importance of these features for understanding the behavior of exchange rates.

In Internet Appendix IA.B, we analyze the patterns of underreaction and overreaction in a number of different ways than presented here. This includes using shocks following the methodology in Romer and Romer (2004) (compiled by Wieland and Yang (2020)), computing bias coefficients following the methodology proposed by Kucinskas and Peters (2019) (using both data on US and foreign interest rate forecasts), and tests for underreaction and overreaction suggested by Coibion and Gorodnichenko (2015) and Kohlhas and Walther (2020). Across all of our tests, we find consistent evidence of underreaction and overreaction of survey-based consensus expectations to interest rate news.

Other work has shown that short-term interest rate forecasts reported in surveys underreact to interest rate news, both in the US and in other countries (e.g., see Cieslak (2018), Brooks, Katz and Lustig (2019), Schmeling, Schrimpf and Steffensen (2020) and Wang (2020)). Underreaction to interest rate news also serves as the motivation for Gourinchas and Tornell (2004) in explaining the failure of UIP in exchange rates. But the result on overshooting of interest rates expectations following interest rate news shocks is new. The broader patterns of initial underreaction and subsequent overreaction of expectations are consistent with similar patterns in survey-based expectations of macroeconomic variables found in other work (see, e.g., Angeletos, Huo and Sastry (2020)).

Fact 3: Underreaction of Interest Rate Forecasts is Primarily a Consensus Phenomenon

Our third piece of motivating empirical evidence is that the underreaction of short-term interest rate forecasts to news appears to be a phenomenon primarily found in consensus forecasts; when focusing on individual forecasts, underreaction to interest rate news is much less pronounced, and largely disappears.

We regress forecast errors on forecast revisions, using both consensus-level observations (as in Coibion and Gorodnichenko (2015)) and individual forecaster-level observations (as in Bordalo et al. (2020*b*)). In particular, regressions are of the form

$$x_{t+k} - \bar{\mathbb{E}}_t x_{t+k} = \alpha + \beta_{CG}(\bar{\mathbb{E}}_t x_{t+k} - \bar{\mathbb{E}}_{t-k} x_{t+k}) + \epsilon_{t+k} \tag{3}$$

$$x_{t+k} - \mathbb{E}_{i,t} x_{t+k} = \alpha + \beta_{BGMS}(\mathbb{E}_{i,t} x_{t+k} - \mathbb{E}_{i,t-k} x_{t+k}) + \epsilon_{i,t+k} \tag{4}$$

where x_{t+k} is the variable of interest, $\bar{\mathbb{E}}_t x_{t+k}$ is the period t consensus expectation of x in period $t+k$, $\mathbb{E}_{i,t} x_{t+k}$ is forecaster i's period t expectation of x in period $t+k$, and β_{CG} and β_{BGMS} are the coefficients of interest in the regressions. We show regression results where the variables of interest are US Treasury Bill rates, short-term interest rates for foreign countries, and interest rate differentials between foreign and US rates. As Coibion and Gorodnichenko (2015) note, $\beta > 0$ corresponds to forecasters underreacting to information that arrives between period $t-k$ and period t, and $\beta < 0$ corresponds to forecasters overreacting to information that arrives between $t-k$ and t, with larger magnitude coefficients indicating more underreaction or overreaction. A positive coefficient indicates that the forecast error is positively correlated with changes in forecasters' expectations from $t-k$ to t. This reflects that forecasters' beliefs did not move sufficiently to capture information they observed, consistent with underreaction. Conversely, a negative coefficient indicates that forecasters' beliefs moved too much, consistent with overreaction.

Figure 3 plots coefficients from the regressions. For all of the variables, the coefficients estimated using consensus-level observations have more positive coefficients than the observations estimated using individual forecaster-level observations, indicating that underreaction is substantially more pronounced at the consensus level than it is as the individual level. Focusing on the regression for Treasury Bills, the regression coefficients for one-, two-, and three-quarter ahead forecast errors are (0.18, 0.31, 0.59) at the consensus level, while they are (-0.02, 0.08, 0.16) at the individual level, suggesting that while underreaction is substantial in consensus-level forecasts, it is much more muted in individual-level forecasts. For non-US interest rate forecasts and interest rate differential forecasts, we find similar results. The regression coefficients for one-quarter ahead consensus forecasts of interest rates and interest rate differentials are (0.17, 0.30) versus (0.01, 0.02) for individual forecasts.[16]

The regression results indicate that underreaction is much more pronounced at the consensus level than at the individual level. In Appendix Table IA.B.2, we follow an approach similar to Angeletos, Huo and Sastry (2020), and run multivariate regressions of individual forecast errors on consensus and individual forecast revisions in the SPF data. That analysis similarly reveals that underreaction is primarily a feature of consensus expectations.

The fact that underreaction is primarily a feature of consensus expectations, and not individual expectations, suggests that information heterogeneity across forecasters may play an important role in explaining underreaction, as argued by Bordalo et al. (2020*b*).[17] This fact indicates that information heterogeneity may be an important feature to consider in order to understand *why* exchange rates appear to underreact to news.

3 Model

We construct a model of exchange rate determination, which features agents with noisy private information and potentially biased beliefs about the macroeconomic fundamentals that determine interest rates. Our goal is to explain exchange rate behavior in a manner consistent with the motivating empirical evidence. We estimate the model using data on interest rate forecasts and interest rates. We evaluate the model based on its ability to explain the behavior of exchange rates, and find that the frictions we introduce are able to qualitatively and quantitatively reproduce the patterns of interest in currency markets.

3.1 Preliminaries

Time is discrete and is indexed by $t \in \{0, 1, 2, ...\}$. There are two countries, the Home country and the Foreign country; variables from the latter are starred. We assume a small open-economy

[16] Because we only have one- and four-quarter ahead forecasts from Consensus Economics, forecast revisions for the international sample are calculated as the difference between the period t and period $t-3$ forecasts of the period $t+1$ interest rate, which is slightly different than the expressions given in Equation (3) and (4).

[17] Bordalo et al. (2020*b*) generally find evidence that individual expectations *overreact* to macroeconomic news. However, for news about short-term interest rates, we (and, in fact, they) find evidence that individual expectations, may, if anything, slightly *underreact*, though in a less pronounced way than consensus expectations. Consensus underreaction to interest rate news is consistent with evidence in other work (e.g., Cieslak (2018); Wang (2020); Schmeling, Schrimpf and Steffensen (2020)). One rationalization for underreaction to interest rate news present in some of these papers is that forecasters did not have knowledge of Central Banks' reaction functions, and in particular, underestimated how quickly central banks have been willing to cut interest rates in recessionary periods or following poor stock market performance. While this likely contributes to underreaction, we note that our results suggest that heterogeneous private information also plays an important role in underreaction to news about short-term interest rates over the sample period.

setting, where the Home country is large and the Foreign country is infinitesimally small. The log nominal exchange rate between the two countries in period t is denoted as s_t, expressed in units of foreign currency per one unit of home currency.

There are two assets, a one period bond for each country, which are both in zero net supply. Investors may take short positions (borrow) or take long positions (lend) in each of the bonds. The interest rates of the bonds are given by i_t and i_t^*. We denote the interest rate differential between the two countries as $i_t^d = i_t - i_t^*$. The interest rate differential is generated by a macroeconomic fundamental, which is unobserved. The fundamental follows an $AR(1)$ process,

$$\xi_t = \rho\xi_{t-1} + \eta_t \text{ or } \xi_t = \frac{1}{1-\rho L}\eta_t, \text{ where } \eta_t \sim \mathcal{N}(0,1). \tag{5}$$

While we do not specify details of the fundamental, it can be thought of as reflecting the interest rate differential that would be set by monetary authorities given macroeconomic fundamentals in each economy (e.g., growth and inflation). The interest differential is equal to the fundamental plus an idiosyncratic error term.

$$i_t^d = \xi_t + \sigma_\varepsilon \varepsilon_t, \text{ where } \varepsilon_t \sim \mathcal{N}(0,1). \tag{6}$$

Because the Foreign country is infinitesimal, only the Home country investors matter for the bond market equilibrium. Each period, a unit mass of short-lived, Home country investors with exponential utility is born, indexed by $i \in [0,1]$. Each investor i receives a noisy private signal about the fundamental in period t,[18] given by

$$x_{it} = \xi_t + \sigma_u u_{it}, \text{ where } u_{it} \sim \mathcal{N}(0,1).$$

Investors born in period t receive a unit endowment, which they invest. In period $t+1$, each investor i consumes her investment return, passes on her private information to the new investor i born in that period, and dies. Investor i's problem is given by

$$\begin{aligned} &\max_{\alpha^i} -\mathbb{E}_{i,t}(e^{-\gamma c^i_{t+1}}) \\ &\text{subject to} \quad c^i_{t+1} = \alpha^i(-s_{t+1} + s_t + i_t^*) + (1-\alpha^i)(1+i_t), \end{aligned} \tag{7}$$

where α^i is her allocation to the foreign bond, and $\mathbb{E}_{i,t}$ captures her subjective expectations. Solving Equation (7), investor i's demand for the foreign bond is

$$\alpha_i = \frac{\mathbb{E}_{i,t}(-s_{t+1}) + s_t - i_t^d}{\gamma\sigma_t^2}, \tag{8}$$

[18]Noisy private signals can be interpreted literally as corresponding with dispersed information (as in Lucas Jr (1972), Morris and Shin (2002)), or as emerging from rational inattention (Mankiw and Reis (2002), Sims (2003, 2010), Woodford (2003)).

where σ_t^2 is the conditional variance of next period's exchange rate, which is the same for all investors in equilibrium. Each investor's demand for foreign currency bonds is proportional to her expected returns, which are comprised of two components: expectations of foreign currency appreciation, $\mathbb{E}_{i,t}(-s_{t+1}) + s_t$, and the interest rate differential, i_t^d. Investor i's expectation of currency appreciation depends upon her expectation of next period's exchange rate, which is a function of the foreign currency bond demand of every other investor. Accordingly, higher-order beliefs about other investors' beliefs enter into her and every other investor's demand in equilibrium.

We assume that investors *do not* extract information about fundamentals from the equilibrium exchange rate, s_t, when formulating their demand. In the context of our estimated model, which treats beliefs reported in surveys as investors' true beliefs, this assumption means that any learning from prices on the part of investors is considered part of their noisy private signals. The assumption that investors do not learn from prices also has other motivations in both the noisy rational expectations literature and the behavioral economics literature.[19]

Additionally, we permit investors' beliefs to deviate from the standard framework in the following way. Investors may perceive $\hat{\rho}$ and $\hat{\sigma}_\varepsilon$, rather than the true parameter values (ρ and σ_ε), and all investors share the same (potentially distorted) belief about these parameters. $\hat{\rho} > \rho$ indicates that investors are extrapolative, and believe the interest rate differential is more persistent than it is in reality, which we find when we estimate the model.[20] The assumption that investors may perceive $\hat{\sigma}_\varepsilon$ differently than the true σ_ε is used to provide scope for investors' private signals to matter for their valuations. In our calibration, we estimate $\sigma_\varepsilon \approx 0$ and $\hat{\sigma}_\varepsilon > \sigma_\varepsilon$, indicating that investors believe the interest rate differential may deviate from the fundamentals in a transitory way each period. This incorrect belief may stem, for example, from market participants disagreeing with central banks about the state of the economy, and hence the future path of interest rates (e.g., as discussed in Caballero and Simsek (2020)), or relatedly, from investors not understanding central banking authorities' reaction functions.[21]

Defining the precision of the innovations as $\tau_\varepsilon = \sigma_\varepsilon^{-2}$ and $\tau_u = \sigma_u^{-2}$ (with corresponding hatted variables indicating investors' perceived precisions), we can write the investors' perceived processes for variables in the economy as

$$\begin{bmatrix} \hat{i}_t^d \\ \hat{x}_{it} \end{bmatrix} = \begin{bmatrix} \hat{\tau}_\varepsilon^{-1/2} & 0 & \frac{1}{1-\hat{\rho}L} \\ 0 & \tau_u^{-1/2} & \frac{1}{1-\hat{\rho}L} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_{it} \\ \eta_t \end{bmatrix}.$$

[19]This type of assumption is motivated in the noisy rational expectations literature by introducing noise traders or noisy asset supply (Allen, Morris and Shin (2006)), or corresponding with privately informed investors submitting market orders to a centralized limit order book (as in Bacchetta and Van Wincoop (2006)). An alternative motivation for this assumption from behavioral economics is that investors may be "cursed"; they do not fully appreciate that they can invert prices to learn other investors' information (Eyster and Rabin (2005); Eyster, Rabin and Vayanos (2019)).

[20]This form of misperception of persistence is also used and discussed in Gabaix (2016, 2019) and Angeletos, Huo and Sastry (2020).

[21]Given that $\sigma_\varepsilon \approx 0$, a seemingly obvious simplification would be to remove the ε_t from the model. However, consensus expectations in such a model would only be able to exhibit initial underreaction and delayed overreaction if i_t^d is unobserved. This would be counterfactual.

The market clearing condition for foreign bonds is

$$0 = \int \alpha_i di \propto \int \mathbb{E}_{i,t}\left[-s_{t+1}|\mathcal{I}_{i,t}\right] + s_t - i_t^d, \tag{9}$$

which in turn yields

$$s_t - i_t^d = \bar{\mathbb{E}}_t[s_{t+1}], \tag{10}$$

where $\bar{\mathbb{E}}_t$ is the average belief across all agents. Note that because each investor's demand for foreign bonds is linear in her expected returns, the equilibrium condition coincides with *uncovered interest rate parity* (UIP) holding for consensus expectations. We make this assumption for simplicity, though the model can also be written down to accommodate potentially time-varying risk premia.[22] Solving for the equilibrium exchange rate amounts to solving for the average expectation of next period's exchange rate across investors.

3.2 Interest Rate Expectations and Forecast Errors in the Model

Before solving the model for the equilibrium exchange rate, we discuss the behavior of interest rate expectations in the model, which are key to understanding the behavior of equilibrium exchange rates. The proofs for all propositions are presented in the appendix.

Proposition 1 (Investors' Expectations of Fundamentals). *Investor i's expectation of the fundamental in period t is*

$$\mathbb{E}_{i,t}\left[\xi_t\right] = \lambda \mathbb{E}_{i,t-1}\left[\xi_{t-1}\right] + \left(1 - \frac{\lambda}{\hat{\rho}}\right)\xi_t + \frac{\lambda \hat{\tau}_\varepsilon \sigma_\varepsilon}{\hat{\rho}\left(1-\hat{\rho}\lambda\right)}\varepsilon_t + \frac{\lambda \tau_u \sigma_u}{\hat{\rho}\left(1-\hat{\rho}\lambda\right)} u_{i,t}, \tag{11}$$

and the consensus expectation of the fundamental in period t is

$$\bar{\mathbb{E}}_t\left[\xi_t\right] = \lambda \bar{\mathbb{E}}_{t-1}\left[\xi_{t-1}\right] + \left(1 - \frac{\lambda}{\hat{\rho}}\right)\xi_t + \frac{\lambda \hat{\tau}_\varepsilon \sigma_\varepsilon}{\hat{\rho}\left(1-\hat{\rho}\lambda\right)}\varepsilon_t, \tag{12}$$

where λ is defined as

$$\lambda = \frac{1}{2}\left(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}} - \sqrt{\left(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}}\right)^2 - 4}\right). \tag{13}$$

In period t, investor i's expectation of the period $t+k$ interest rate differential is $\hat{\rho}^k \mathbb{E}_{i,t}\left[\xi_t\right]$, and the consensus expectation of the period $t+k$ interest rate differential is $\hat{\rho}^k \bar{\mathbb{E}}_t\left[\xi_t\right]$.

Proposition 1 illustrates how frictions enter into investor expectations of future interest rates. Under FIRE, $\lambda = 0$, $\hat{\rho} = \rho$, and investors hold accurate expectations regarding fundamentals.

[22] While not crucial for our model's insights, in Internet Appendix IA.B.1, we evaluate the extent to which Equation (10) holds in survey data of exchange rate expectations. We find evidence that Equation (10) holds on average in the pre-2008 sample.

However, $\lambda \neq 0$ corresponds with information processing frictions entering into investors' beliefs. Investors place a weight of $1 - \frac{\lambda}{\hat{\rho}}$ (their Kalman gain) on the true period t fundamental, but also (imperfectly) incorporate their present and past private signals, as well as past interest rate differentials, into their expectations. At the consensus level, private information cancels out to zero across investors.

To better understand the influence of the frictions we introduce on interest rate forecasts, we focus on the consensus forecast error of interest rate differentials, defined as $FE_{t,t+1} \equiv i^d_{t+1} - \bar{\mathbb{E}}_t [\xi_{t+1}]$. We can study the consensus forecast error to understand *underreaction* and *overreaction* to interest rate news in the model. In the model, the interest rate news that arrives in period t that is relevant to future interest rates is $\eta_t = \xi_t - \rho\xi_{t-1}$, the persistent shock to fundamentals. A positive relationship between the period t consensus forecast error and news that arrived δ periods previously, $\eta_{t-\delta}$, indicates *underreaction* to the past news, while a negative relationship indicates *overreaction*, using similar logic as the tests we implement in the previous section.

The covariance between the period $t+1$ forecast error and period $t-\delta$ interest rate news, $\eta_{t-\delta}$, is

$$cov(FE_{t,t+1}, \eta_{t+1-\delta}) = \lambda^\delta + (\rho - \hat{\rho})\frac{\lambda^\delta - \rho^\delta}{\lambda - \rho} \tag{14}$$

Under FIRE, $\lambda = 0$ and $\rho = \hat{\rho}$, so Equation (14) reduces to zero, i.e., forecast errors are unforecastable by past news. More generally, however, Equation (14) tells us that interest rate forecast errors *are* predictable. For $\delta = 1$, Equation (14) reduces to

$$cov(FE_{t,t+1}, \eta_t) = \underbrace{\lambda}_{\text{Information Frictions}} - \underbrace{(\hat{\rho} - \rho)}_{\text{Extrapolation}} . \tag{15}$$

Ceteris paribus, increased extrapolation ($\hat{\rho} > \rho$) generates *overreaction* to period t interest rate news, as $\frac{\partial cov(FE_{t,t+1},\eta_t)}{\partial \hat{\rho}} < 0$. Noisy private information, on the other hand, generates underreaction of the consensus interest rate expectation to news. In particular, increasing the dispersion of investors' private signals (smaller τ_u) or the perceived noise in interest rate differentials relative to fundamentals (smaller τ_ε) increases underreaction to short-term interest rate news, as $\frac{\partial cov(FE_{t,t+1},\eta_t)}{\partial \tau_u} < 0$ and $\frac{\partial cov(FE_{t,t+1},\eta_t)}{\partial \tau_\xi} < 0$. Whether overreaction or underreaction dominates depends upon the relative strength of extrapolation versus investors' informational frictions.

Equation (14) also suggests that interest rate expectations may display initial underreaction (to news that arrived in period $t-\delta$ for small values of δ) and delayed overreaction (for larger values of δ), as we empirically observe in the data. The covariance between past news and forecast errors in Equation (14) has indeterminate sign when investors are extrapolative ($\hat{\rho} > \rho$), and can change sign for different values of δ for a given parametrization.

Proposition 2 (Initial Underreaction and Delayed Overreaction). *The consensus interest rate expectation displays initial underreaction and delayed overreaction for $\hat{\rho} - \lambda < \rho < \hat{\rho}$ and $\rho \neq \lambda$, where initial underreaction indicates $cov(FE_{t,t+1}, \eta_t) > 0$, and delayed overreaction indicates $cov(FE_{t,t+1}, \eta_{t+1-\bar{\delta}}) < 0$,* *for some* $\bar{\delta} > 1$.

Proposition 2 formally provides conditions under which consensus interest rate expectations underreact to interest rate news that arrived in the recent past, and overreact to interest rate news that arrived further in the past. In particular, extrapolation ($\hat{\rho} > \rho$) helps to generate overreaction to news, and can co-exist with underreaction to recent interest rate news as long as it is not so strong as to dominate the influence of informational frictions in the model.

Lastly, we can also analyze how individual and consensus expectations respond to news by analyzing the relationship between forecast errors and forecast revisions at the consensus and individual levels.

Proposition 3 (**Individual- and Consensus-level Underreaction**). *If* $\lambda > 0$ *and* $\beta_{CG} > 0$*, then* $\beta_{CG} > \beta_{BGMS}$*, where* β_{CG} *and* β_{BGMS} *are coefficients from regressions of forecast errors on forecast revisions at the consensus and individual levels, i.e.,*

$$\underbrace{\xi_{t+1} - \bar{\mathbb{E}}_t\left[\xi_{t+1}\right]}_{\text{consensus forecast error}} = \alpha + \beta_{CG}\underbrace{\left(\bar{\mathbb{E}}_t\left[\xi_{t+1}\right] - \bar{\mathbb{E}}_{t-1}\left[\xi_{t+1}\right]\right)}_{\text{consensus forecast revision}} + e_{t+1},$$

$$\underbrace{\xi_{t+1} - \mathbb{E}_{i,t}\left[\xi_{t+1}\right]}_{\text{individual forecast error}} = \alpha + \beta_{BGMS}\underbrace{\left(\mathbb{E}_{i,t}\left[\xi_{t+1}\right] - \mathbb{E}_{i,t-1}\left[\xi_{t+1}\right]\right)}_{\text{individual forecast revision}} + e_{i,t+1}.$$

Proposition 3 indicates that whenever consensus expectations underreact, as measured by the regression coefficient of forecast errors on lagged forecast revisions (Coibion and Gorodnichenko (2015)), consensus expectations underreact *more* than individual expectations do. The intuition behind this result is simple, and follows the discussion in Bordalo et al. (2020*b*) and Angeletos, Huo and Sastry (2020). Individuals each underreact modestly to the information they receive (or perhaps even overreact). However, the noise in their signals prevents them from immediately observing, and updating their beliefs in response to, the true news in a given period. Hence, at the consensus level, where noisy private signals cancel out, we observe strong underreaction.

3.3 Exchange Rates in the Model

Investors' demand for foreign bonds, and accordingly the equilibrium exchange rate, depend both on investors' beliefs about the future path of interest rate differentials (captured by their beliefs about fundamentals, ξ_t), and, because of their short investment-horizons, also upon their higher order uncertainty regarding other investors' beliefs. To separately understand the influence of belief biases about future interest rate differentials and of higher-order uncertainty, we first present a solution for the exchange rate in the absence of higher-order uncertainty. This would be the prevailing exchange rate if investors traded fully towards their beliefs about fundamentals. Then we proceed to the solution for the equilibrium exchange rate in the model.

Proposition 4. *The equilibrium exchange rate in the absence of higher-order uncertainty, denoted as* $\tilde{s}_t$*, is the consensus expected sum of all future interest rate differentials. This log exchange rate can be expressed* *as*

$$\begin{aligned}\tilde{s}_t =& i_t^d + \frac{\hat{\rho}}{1-\hat{\rho}} \bar{\mathbb{E}}_t[\xi_t] \\ =& i_t^d + \frac{\hat{\rho}}{1-\hat{\rho}} \left(1 - \frac{\lambda}{\hat{\rho}}\right) \frac{1}{1-\lambda L} \xi_t + \frac{\lambda \hat{\tau}_\varepsilon \sigma_\varepsilon}{(1-\hat{\rho})(1-\lambda L)(1-\hat{\rho}\lambda)} \varepsilon_t.\end{aligned}$$

Proposition 4 is noteworthy for two reasons. First, it describes how (errors in) expected interest rate differentials enter into exchange rates. In the absence of higher-order uncertainty, the exchange rate is the sum of expected interest rates differentials, and accordingly inherits the properties of interest rate forecast errors discussed in the previous section. Extrapolation ($\hat{\rho} > \rho$) will tend to generate overreaction of exchange rates to news, and noisy private information (smaller τ_u and $\hat{\tau}_\varepsilon$) will tend to generate underreaction, just as they do for interest rate expectations. Second, the proposition illustrates that in the absence of higher-order uncertainty, the exchange rate can be written as a sum of this period's interest rate differential, past fundamentals (ξ_t), and past transitory wedges between the interest rate differential and macroeconomic fundamentals (ε_t). This analytical expression is useful for understanding the influence of interest rate expectations and higher-order uncertainty in the model, as we discuss further.

We next present the solution for the unique equilibrium exchange rate in the model. To solve for the exchange rate, we broadly follow the methodology outlined in Huo and Takayama (2018) for solving models with dispersed information and strategic complementarity. Relative to previous work solving similar models, which adapts the solution method in Townsend (1983), this solution method has the advantage of providing an exact analytical solution.[23]

Proposition 5 (Equilibrium Exchange Rate). *The log exchange rate in the model is*

$$s_t = i_t^d + \frac{\hat{\rho}}{1-\hat{\rho}} \left(1 - \frac{\vartheta}{\hat{\rho}}\right) \frac{1}{1-\vartheta L} \xi_t + \frac{\hat{\tau}_\epsilon \sigma_\epsilon \vartheta}{(1-\hat{\rho})\,(1-\vartheta L)(1-\hat{\rho}\vartheta)} \varepsilon_t, \tag{16}$$

where ϑ^{-1} is the outside root of the equation

$$\lambda \hat{\rho} k^2 - \hat{\rho}(1+\lambda^2)k + \lambda \tau_u + \lambda \hat{\rho} = 0.$$

Proposition 5 shows that the expression for the equilibrium exchange rate presented in Equation (16) is *exactly* identical to the consensus expectation of the sum of all future interest rate differentials (the exchange rate in Proposition 4), except for the fact that λ is replaced by ϑ, where $\vartheta > \lambda$. As with λ, ϑ also increases with noisy private information (it is decreasing in $\hat{\tau}_\varepsilon$ and τ_u), as

[23] Townsend (1983) points out that a difficulty in solving these types of models is the 'infinite regress' problem, where, due to the role of higher order beliefs, if an agent believes that other agents keep track of n state variables, she, in turn, must keep track of $n+1$ state variables. Iterating ad infinitum, there is no finite-state representation of the equilibrium policy rule. Townsend (1983) deals with this problem by assuming that information becomes common knowledge after a (small) number of periods, a strategy followed and built upon in other subsequent works, including work on asset pricing (e.g., Singleton (1987) and Bacchetta and Van Wincoop (2006)). However, the infinite regress problem can be avoided by transforming the problem into a tractable problem of finding analytic functions. This is the approach taken by Kasa, Walker and Whiteman (2014) and Huo and Takayama (2018), the latter whom we follow in our solution.

we show in the appendix.

The similarity of the expressions for exchange rates in Propositions 4 and 5 suggests that even in the presence of higher-order uncertainty, exchange rates by-and-large inherit the properties of interest rate expectations. They similarly may overreact to news due to the role of extrapolation, and underreact due to noisy private information. Because both expressions represent exchange rates as a function of the current interest rate differential, past fundamentals, and past transitory wedges between the interest rate differential and fundamentals, the inclusion of higher order uncertainty primarily influences the speed with which information is incorporated into prices.

In Proposition 4, the coefficient on ξ_t can be expressed as

$$\underbrace{\frac{\hat{\rho}}{1-\hat{\rho}}}_{\text{Perceived Fundamental Persistence}} \times \underbrace{\left(1-\frac{\lambda}{\hat{\rho}}\right)}_{\text{Kalman Gain of Fundamental}} \times \underbrace{\frac{1}{1-\lambda L}}_{\text{Reaction to Older Fundamental News}} . \tag{17}$$

Substituting ϑ for λ can be understood in the context of Equation (17). Because $\vartheta > \lambda$, the Kalman gains from current news are smaller, and the reaction to older news that arrived in the past is stronger. Put differently, higher-order uncertainty induces a more sluggish exchange-rate reaction to news about future interest rate differentials, consistent with the long-standing results found in other work regarding the role of higher-order uncertainty.

To summarize, the equilibrium exchange rate in the model has two drivers: (1) investors' expectations of the sum of all future interest rate differentials and (2) each investor's higher order uncertainty regarding all other investors' (higher-order) beliefs about future interest rate differentials. Because of (1), investors may underreact to recent interest rate news (because of noisy private information), but may also overreact to older interest rate news by overestimating the persistence of interest rate differentials. Higher-order uncertainty in (2) induces further sluggishness of exchange rates to interest rate news. The relative importance of interest rate forecast errors versus higher-order uncertainty is an empirical question that we evaluate. We find that errors in consensus expectations of interest rates play a substantially larger quantitative role than higher-order uncertainty.

3.4 Model Estimation

With the model solution in hand, we next turn to estimating the model. The model has four parameters: ρ, $\hat{\rho}$, $\hat{\sigma}_\epsilon$, and σ_u. We estimate these parameters to match the dynamics of the interest rate process and survey-based forecasts of interest rates. We use quarterly data, so that one period in the model corresponds with one quarter.

We estimate ρ to match the impulse response function of interest rate differentials to the interest rate news shocks from Angeletos, Collard and Dellas (2020). We estimate $\hat{\rho}$ and $\hat{\sigma}_\epsilon$ to match the impulse responses of consensus forecast errors of interest rate differentials to interest rate news shocks, and to match the coefficient in individual-level regressions of interest rate differential forecast errors on forecast revisions (the *BGMS* coefficient).[24] The minimization problems are given by

$$\min_{\rho} \left(\hat{\Phi} - \Phi(\rho)\right)' \Omega_{\Phi} \left(\hat{\Phi} - \Phi(\rho)\right), \tag{18}$$

$$\min_{\hat{\rho},\hat{\sigma}_{\epsilon}} \left(\hat{\Theta} - \Theta(\hat{\rho}, \hat{\sigma}_{\epsilon})\right)' \Omega_{\Theta} \left(\hat{\Theta} - \Theta(\hat{\rho}, \hat{\sigma}_{\epsilon})\right), \tag{19}$$

where Ω_{Φ} and Ω_{Θ} are diagonal matrices containing the weights of the target moments,[25] $\Phi(\rho)$ is a function that maps ρ to model-implied impulse responses of interest rate differentials, $\hat{\Phi}$ is a vector of empirical interest rate differential impulse responses, $\Theta(\hat{\rho}, \hat{\sigma}_{\epsilon})$ is a function that maps $\hat{\rho}$ and $\hat{\sigma}_{\epsilon}$ to a vector containing the model-implied impulse responses of forecast errors and the model-implied *BGMS* coefficient, and $\hat{\Theta}$ is a vector containing consensus forecast errors' impulse responses to interest rate news shocks and the empirical *BGMS* coefficient. The system is overidentified, as we use impulse responses from 4 to 20 periods after an interest rate news shock, meaning there are 17 target moments to estimate the parameters.

The estimated parameters are $(\rho, \hat{\rho}, \hat{\sigma_{\epsilon}}, \sigma_u) = (0.93, 0.96, 3.5, 2.75)$. With the estimated parameters in hand, we evaluate how well the model matches our motivating empirical evidence, and find that the model does a reasonably good job of matching the targeted moments.[26]

The first panel of Figure 4 plots the model-implied impulse response function of interest rate forecast errors in response to an interest rate news shock, computed by simulating 5000 economies for 144 periods and taking the average IRF computed for each simulation, compared with the same impulse response function estimated directly in the data. The model forecast errors capture the pattern of initial underreaction and subsequent overreaction reported in our second motivating fact. Almost all points of the model's IRF are inside the one-standard error confidence interval of the IRF estimated from the data.

The second panel in Figure 4 plots the model-implied regression coefficients for regressions of forecast errors of period $t+1$ interest rates on forecast revisions from period $t-3$ to period t. We plot the coefficients alongside the same coefficients estimated using Consensus Economics forecasts of interest rate differentials, as reported in Figure 3, with positive coefficient values indicating underreaction. As in the data, the model-implied regression coefficients suggest substantially stronger underreaction at the consensus level than at the individual level, indicating that the model also captures our third motivating piece of evidence. The model-implied coefficients sug-

[24] This approach broadly follows Christiano, Eichenbaum and Evans (2005).

[25] For the impulse responses, weights are inversely proportional to the sample variances of the empirical impulse responses of interest rate differentials and forecast errors of interest rate differentials. For Ω_{Θ}, there is an additional element in the matrix, corresponding with the weight given to the *BGMS* coefficient. We specify this weight as the same as the weight given to the impulse responses.

[26] Rather than estimating ρ to match impulse response functions, we can also estimate ρ assuming that fundamentals follow an AR(1) process (we estimate a persistence of 0.95 for interest rate differentials). However, that alternative approach does not have a direct analog for estimating the perceived $\hat{\rho}$. Matching impulse response functions provides us a direct way to recover ρ and $\hat{\rho}$ using the same methodology.

gest slightly stronger levels of underreaction at the consensus level to the amount we find in the data, while they almost exactly match the individual level underreaction in the data (as targeted).

We provide some intuition on the behavior of interest rate forecast errors and the ability of the model to match the motivating evidence, which is also relevant for understanding the behavior of the exchange rate in the model. Consensus forecast errors are initially positive in response to an interest rate news shock, due to the relatively high values of $\hat{\sigma}_\epsilon$ (the amount of perceived noise driving a wedge between interest rate differentials and fundamentals) and σ_u (the amount of noise in signals investors receive). Each investor's belief does not underreact to the news she observes. However, noise in private signals leads consensus expectations to substantially underreact, because it prevents investors from immediately observing, and updating their beliefs in response to, the 'true' news that arrives in a given period. Over time, investors observe subsequent realizations of interest rate differentials, and consensus expectations adjust to reflect the interest rate news that arrived in the past. Eventually, consensus forecast errors switch to being negative, as over-extrapolation of fundamentals ($\hat{\rho} > \rho$) begins to dominate the initial underreaction, and investors believe that the higher interest rate differential will last longer than it does in the data. While the estimated $\rho = 0.93$ implies a very persistent process for the fundamentals that govern the interest rate differential, our estimate of $\hat{\rho} = 0.96$ suggests that, on average, investors believe the fundamental process is even more persistent.

In Figure 5, we illustrate how each of the ingredients of the model contributes to the dynamics of interest rate differential forecasts and forecast errors. In the figure, we plot the impulse response functions of interest rate differential forecasts and forecast errors in the model to a one standard deviation shock to fundamentals in four scenarios: (1) full information rational expectations (FIRE), (2) a version of the model where there is no noise in investors' private signals (all investors' signals correspond with the true fundamental) (3) a version of the model where $\hat{\rho} = \rho$ (there is no extrapolation) (4) the fully estimated model. Under FIRE, investors perfectly forecast the interest rate differential process, and understand that interest rate shocks eventually mean-revert via standard autoregressive dynamics. Introducing noisy private information, the consensus forecast of interest rate differentials underreacts to the shock to fundamentals, and converges to FIRE, but never overreacts (interest rate differential forecasts are never higher than the realized value in the next period). In the third scenario, where investors are extrapolative but do not receive noisy private signals, investors overreact to the shock to fundamentals, and consistently overestimate next period's interest rate differential. The model does not capture the initial underreaction of the consensus interest rate forecast in this scenario. Combining noisy private information and extrapolation in the full estimated model, the consensus belief about the interest rate differential initially underreacts and then subsequently overreacts, consistent with the data.

4 Exchange Rate Dynamics in the Model

With the estimated model in hand, we next turn to evaluate the model's ability to explain exchange rate dynamics. Statistics in the model are calculated by simulating the model 5000 times for 144 periods.

4.1 Underreaction and Overreaction of Exchange Rates

4.1.1 The Response of Exchange Rates to Interest Rate Shocks

Figure 6 plots the model-implied impulse response function of exchange rates and of home currency excess returns in response to a one standard deviation shock to the interest rate differential. The exchange rate appreciates for five quarters after a one standard deviation shock, after which it begins to depreciate. This behavior of exchange rates results in positive excess returns associated with borrowing in foreign currency bonds and investing in home currency bonds for five quarters after the shock.

These predictable excess returns of currencies capture initial underreaction and delayed overreaction. They can be drawn in contrast with the behavior we would expect under FIRE. Under FIRE, we expect the home currency exchange rate to appreciate when home interest rates increase relative to foreign interest rates, but the home currency should subsequently depreciate, and there should be no excess returns from investing in the home currency.

Internet Appendix Figure IA.C.1 presents impulse response functions of exchange rates and excess returns in response to a shock to the interest rate differential when turning off different frictions in the model. The figure reveals that noisy private information is the primary driver of underreaction, and that extrapolation is the driver of overreaction. This is intuitive; noisy private information leads interest rate expectations to underreact to interest rate news, which, in turn leads exchange rates to underreact to interest rate news. Eventually, exchange rates overreact to past interest rate news, and excess returns are negative, as investors observe interest rate differentials that are less persistent than they expected.

4.1.2 A Quantitative Assessment via UIP Regressions

We can quantitatively assess the estimated model's ability to capture exchange rate dynamics by running time-series regressions of currency excess returns in period $t+k$ on the interest rate differential in period t. The regressions are of the form

$$\lambda_{t+k} = \alpha_k + \beta_k i_t^d + \epsilon_{t+k}. \tag{20}$$

where λ_{t+k} is the excess return from borrowing in foreign currency bonds and investing in home currency bonds from period $t+k-1$ to $t+k$, and i_t^d is the period t interest rate differential. Regressions of this form are common in the literature on exchange rates, dating back to Fama

(1984), who documents deviations from uncovered interest rate parity stemming from the fact that $\beta_1 > 0$.

In the context of our model, any predictable excess returns stem from deviations from FIRE. A coefficient $\beta_k > 0$ indicates that when the interest rate differential is higher than its long-term average, currency excess returns for the home country are higher than average k periods in the future. This corresponds with underreaction to interest rate news, as it indicates the exchange rate continuing to impound past news of high interest rate differentials, with the home currency earning positive excess returns as it does. By similar logic, $\beta_k < 0$ corresponds with overreaction to interest rate news.

Figure 7 plots the model-implied values of β_k for different values of k, alongside the same values estimated in the data. In the model, $\beta_k > 0$ for $k < 5$, and $\beta_k < 0$ for $k > 5$, indicating initial underreaction and delayed overreaction to interest rate news. Coefficients peak at 1.5 ($k = 1$), and the lowest β_k is -0.6 (for $k = 7$). The magnitude of these coefficients matches the range of coefficients observed in the data (-0.8 to 1.5). This is noteworthy, in that it suggests the model is able to quantitatively capture the magnitude of exchange rate underreaction and overreaction, despite not using any data on exchange rates in the calibration. However, Figure 7 also reveals a dimension along which the model does not quite match the data. While the β_k values switch from being positive to being negative for $k > 5$ in the model, this switch occurs for $k = 12$ in the data.

4.1.3 Time-Series Momentum and Reversal

Another way that the model captures initial underreaction and delayed overreaction is the fact that currency excess returns exhibit time-series momentum and reversal. Figure 8 plots regression betas computed from regressing period t returns on period $t-k$ returns for $k \in \{1, \ldots, 20\}$. The figure reveals strong evidence of time-series momentum and reversal. Excess returns from one to four quarters prior are strongly positively correlated with quarterly returns; and excess returns from more than five quarters prior are negatively correlated with quarterly returns. The especially strong performance of time-series momentum using a look-back period of one quarter, and the return predictability of past returns using lookback periods of up to four quarters are remarkably consistent with the evidence reported by Moskowitz, Ooi and Pedersen (2012).

Time-series momentum and reversal are natural features of the model, given the relationship between exchange rates and interest rate differentials. In our simple model, beliefs about future interest rate differentials are the sole driver of exchange rates in the model. Because exchange rates largely reflect the expected sum of future interest rate differentials, increases in expected future interest rate differentials correspond with positive excess returns for the home currency. Consensus expectations are slow to reflect news about future interest rate differentials, so changes in expectations of future interest rate differentials are positively autocorrelated at short-horizons, leading to positive autocorrelations in currency excess returns. At longer horizons, changes in expectations of future interest rate differentials are negatively autocorrelated. This is because extrapolation leads consensus expectations to eventually reflect the belief that an increased interest rate differential will last for longer than it does; this belief is revised downwards in the future when investors eventually observe lower interest rate differentials. In turn, currency excess returns are negatively autocorrelated at longer horizons.

4.2 Decomposing the Source of Underreaction

Our estimated model provides a tractable way to decompose the contribution of various sources to exchange rate underreaction. A first potential source of asset price underreaction is that individuals each underreact to the news they observe. However, as already discussed, the survey evidence (and our estimated model) indicate that individuals do not underreact to the news they observe. Rather, the source of asset price underreaction is from dispersed private information.

We can further decompose the impact of dispersed private information on exchange rate underreaction into two channels. The first is that consensus expectations of interest rate differentials sluggishly incorporate interest rate news, as investors react to their own private signals rather than the true interest rate news. The second is that because investors have short investment horizons and have private information that is not revealed by (or learned from) asset prices, investors' higher-order uncertainty about other investors' beliefs leads them to only trade partially towards their beliefs of fundamentals.[27] This trading behavior further contributes to exchange rate underreaction. While both channels are theoretically known to contribute to asset price underreaction, there has been little work to understand their relative importance. A novel result in our paper is to document their relative importance in the context of our estimated model.

Proposition 4 provides an expression for the log exchange rate in our model in the absence of higher-order uncertainty, denoted as $\tilde{s_t}$. The dynamics of $\tilde{s_t}$ capture underreaction and overreaction corresponding with the underreaction and overreaction of consensus beliefs about interest rate differentials. We can quantitatively assess the importance of higher-order uncertainty by comparing comparing the results from the model using $\tilde{s_t}$ with those using the log nominal exchange rate in the model, s_t.

The first panel of Figure 9 plots the behavior of s_t and $\tilde{s_t}$ in response to a one-standard deviation shock to interest rate differentials in period $t = 0$. The second panel in the figure plots regression coefficients from UIP regressions of period $t + k$ currency excess returns on the period t interest rate differential, where currency excess returns are computed using s_t and $\tilde{s_t}$. The behavior of exchange rates is largely identical with and without higher-order uncertainty, and the relationship between interest rate differentials and subsequent currency excess returns is also similar. Qualitatively, the patterns of initial underreaction and delayed overreaction persist, even when eliminating the influence of higher-order uncertainty. Higher-order uncertainty induces slightly stronger initial underreaction to interest rate news, and slightly weaker delayed overreac-

[27] The fact that higher-order beliefs adjust more sluggishly than first-order beliefs is discussed by Woodford (2003), Morris and Shin (2006), and Angeletos and Huo (2021). Allen, Morris and Shin (2006) discuss the sluggishness of higher-order beliefs in the context of asset pricing. Bacchetta and Van Wincoop (2008) denote the wedge between equilibrium asset price and the asset price that would prevail in the absence of higher-order uncertainty as the *higher-order wedge*, and study its theoretical properties.

tion. The quantitative magnitude of these effects is small, however. For example, we can compare regression coefficients in the UIP regressions, for $k = 1$. The coefficient with higher-order uncertainty is 1.49, while it is 1.28 in the absence of higher-order uncertainty. This comparison suggests that approximately 15% of the initial underreaction of currencies to interest rate news stems from higher order uncertainty in the model, while the rest stems from the sluggishness of consensus expectations of interest rates.

The evidence indicates that in our stark model, where short-lived investors only derive utility from next period's return, higher-order uncertainty has limited quantitative ability to explain the strong underreaction of exchange rates to interest rate news. We expect the relative influence of higher-order uncertainty to be even weaker if investors have longer investment horizons and holding periods. A model where there is no higher-order uncertainty, which corresponds with investors fully trading towards their beliefs about fundamentals each period, produces asset prices with similar dynamics as our full model.[28]

5 Other Implications of the Model

Having found that our model is able to capture initial underreaction and delayed overreaction in a quantitatively realistic manner consistent with the survey data, we explore some further implications of the model. First, we analyze the failure of UIP following the financial crisis, where patterns in the data appear to have reversed from the earlier part of the sample. Then, we analyze the ability of our model to explain facts about the persistence of subjective beliefs and the term structure of exchange rates. In all three cases, we find evidence that our model is helpful in explaining the data.

5.1 The Failure of UIP During and After the Financial Crisis

Recent empirical work has documented that the time-series relationships between exchange rates and interest rate differentials has substantially attenuated, and even reversed, in developed markets in recent years, when interest rates have been at the zero lower bound (Bussiere et al. (2018), Engel, Kazakova and Wang (2021)). We more closely study some of the empirical evidence, and seek to understand the facts through the lens of our model.

We regress realized and forecasted monthly excess returns on the lagged interest rate differential from January 2008 through the end of our sample for each country. Panel A of Table 2 reports the average coefficients across countries. Consistent with Bussiere et al. (2018), with realized excess returns as the dependent variable, we find a negative coefficient for developed market

[28] Our analysis here only speaks to the direct influence of higher-order uncertainty on the behavior of exchange rates. In reality, higher-order uncertainty may indirectly influence exchange rates by influencing people's behavior in other areas of macroeconomic relevance, which in turn may affect the behavior of macroeconomic fundamentals relevant for exchange rates. For example, Angeletos and Huo (2021) study the general equilibrium macroeconomic effects of higher-order uncertainty. We treat macroeconomic fundamentals as exogenous, and accordingly do not factor in the potential effect of higher-order uncertainty on macroeconomic fundamentals.

countries (-1.66), substantially lower than the full sample coefficients of 1.5 (as β_1 reported in Figure 7). The coefficient can also be drawn in contrast to the average coefficients where forecasted excess returns are the dependent variable, which is 0.75. The positive coefficient suggests that in post-crisis period, forecasters may have learned from the historical data, and now believe that interest rate differentials positively forecast currency excess returns. However, the results indicate that the realized relationship between currency excess returns and interest rate differentials is more negative than the forecasted relationship. Put differently, the results suggest that forecast errors of exchange rates are negatively related to interest rate differentials in recent times, while they were previously positively related.

How can our model help understand these facts? The reason we have positive coefficients in the UIP regressions (and more positive coefficients where realized rather than forecasted excess returns are the dependent variable) is that information frictions leads consensus expectations of interest rates, and hence, exchange rates, to *underreact* to interest rate news, e.g., as discussed in Proposition 2. When we reduce the dispersion in private information in the model, investors *overreact* to interest rate news rather than underreacting, as extrapolation of the level of interest rate differentials dominates (Figure 5), and the coefficient in the UIP regression flips sign to being negative. Therefore, our model may rationalize the changing sign of the UIP regression in recent times, if investors have begun to overreact to news about short-term interest rates, stemming from reduced dispersion of beliefs about interest rate differentials. The empirical facts emphasize that both informational frictions (noisy private information) and behavioral frictions (extrapolation) are ingredients required to characterize the data over the full sample, and suggests that the reason for the reversal of the failure of UIP may have to do with the changing information environment associated with low short-term interest rates.

We find evidence of overreaction to interest rate news in the post-GFC sample, with less dispersion in beliefs reported in surveys. Panel B of Table 2 reports the average regression coefficient from regressing consensus forecast errors of interest rates and interest rate differentials on the previous period's forecast revision, using Consensus Economics data from January 2008 through December 2019. The regression coefficient for interest rate levels is 0.09, and the regression coefficient for interest rate differentials is -0.27. The evidence is consistent with consensus forecasts overreacting to news about interest rate differentials in the later part of the sample.

In addition to overreaction to news about interest rate differentials, there is also less dispersion of beliefs about interest rates in the later part of the sample as well. Figure 11 plots the cross-sectional standard deviation of forecasts of short-term interest rates 1- and 4-quarters ahead, from Consensus Economics, averaged across countries at each point in time. For both forecast horizons, we observe a secular decline in forecast dispersion, and a particularly low dispersion in forecasts of short-term interest rates in the post-financial crisis period.

Taken together, our model and these additional facts suggest a potential resolution to the behavior of exchange rates in the post financial crisis period, when US and other developed market interest rates have been at the zero lower bound - there is less dispersion in information about interest rates. Accordingly, interest rate forecasts and exchange rates may overreact to interest rate news, leading the relationship between interest rate differentials and subsequent currency excess returns to reverse.[29]

This analysis also speaks to a point raised in Engel, Kazakova and Wang (2021); when running UIP regressions, the coefficients in the regressions tend to vary over time and across different countries. While the focus of this paper is not to dig more deeply into this idea, our results do suggest that the relative magnitude of private information about interest rates, which investors may overreact to, versus public information, which consensus expectations may underreact to, may be useful for better understanding time-variation in the relationship between interest rate differentials and currency excess returns.

5.2 Persistence of Subjective Beliefs and Belief Convergence

A notable fact in survey data of individual investors is the persistence of subjective beliefs. Optimists are persistently optimistic, and pessimists are persistently pessimistic (Giglio et al. (2021)).

This feature arises in our model due to the persist impact of private information. A private signal received in a given period remain important for an investor's beliefs for several periods, because individual investors never observe the true fundamentals underlying interest rates. To better understand this feature in the model, we simulate the estimated model 5,000 times and record the beliefs of 1000 investors in the model in each period in each simulation. We rank investors based on their beliefs about the fundamental ξ_t in each period. Each investor's expected interest rate differential, and expected returns from borrowing in foreign bonds and purchasing home bonds, are determined by ξ_t, so this ranking also ranks investors on the basis of their beliefs about fundamentals and expected returns.[30]

The first two panels in Figure 10 take investors ranked in the top and bottom quartiles based on their beliefs in period zero, and plot the average percentile rank of these investors in subsequent periods. For example, in period one, the panels plot the average percentile rank of investors whose beliefs ranked in the top quartile in period zero, and also plot the average percentile rank of investors whose belief ranked in the bottom quartile in period zero. A value of 0.5 indicates that the average belief of investors in a particular group are at the average of the overall population, and values greater than or less than 0.5 indicate that the average investor in the group has a higher or lower belief about ξ than the average investor in the population. The panels reveal that there is substantial persistence of individual beliefs. It takes more than two years for the average belief of investors in the top and bottom quartiles of the belief distribution in period zero to converge to the average belief of the population.

To better understand the driver of the dynamics of beliefs, the third panel in Figure 10 plots

[29] Candian and De Leo (2021) suggest a distinct rationale for the observed reversal of UIP violations post-GFC. Their explanation is based on the interaction between interest rates and exchange rate forecast errors under a monetary policy rule that no longer satisfies the Taylor principle.

[30] Here, we refer to investor i as being the same investor over time, given that i's belief is based on the sequence of signals observed by all agents i in the past.

the model-based impulse response function of subjective expectations of the interest rate four periods ahead, $\mathbb{E}_{i,t-4}i_t^d$, in response to a one standard deviation shock to the fundamental, η_t, and in response to a one-standard deviation private information shock, $u_{i,t}$. η shocks are commonly observed across all agents, and influence expectations of interest rate differentials for several quarters, consistent with the high degree of persistence of interest rate differentials. Private information shocks are specific to individual investors, and drive disagreement. A one standard deviation private information shock initially influences an investor's beliefs more than a one standard deviation shock to fundamentals; however, the importance of the private information shock fades bit more quickly. Private information shocks cease to become important after 15 quarters. This is consistent with the time it takes the beliefs of optimists and pessimists to converge towards the average belief in the first panel.

How does the degree of persistence of individual beliefs in the model compare with the data? Alongside the model-generated values, Figure 10 also plots corresponding values based on interest rate forecasts in the Survey of Professional Forecasters and Consensus Economics data. The model based values track the values in the data reasonably well. However, notably, in the data, even after five years, the average belief of period-zero optimists and pessimists does not converge completely towards the population average. The difference between the model and data suggests that while noisy private information may help explain the persistence of subjective beliefs in the data, other features may also play a role.

Our results suggest that noisy private information may play an important role in explaining the persistence of individual beliefs and disagreement. However, we also highlight that while our estimated model captures one dimension of the persistence of beliefs, it does not capture other features that have been documented elsewhere. Giglio et al. (2021) document that individual fixed effects explain a substantial amount of belief disagreement about stock market returns. This is true in our model in short samples, but does not hold over longer samples, given that our model predicts the average beliefs of optimists and pessimists eventually converge to the population average. This difference may stem from a few different sources. First, we focus on beliefs about interest rate differentials, which may be more fast-moving than investors' beliefs about stock market returns in Giglio et al. (2021). Second, our focus is on survey-based expectations of professional forecasters, while Giglio et al. (2021) study survey-based expectations of retail investors; it is possible that beliefs may be slower moving for the latter group versus the former. Third, other features that we do not capture in our model, for example the importance of individual experiences for beliefs, may be highly relevant for explaining the importance of individual fixed effects in ways that our model does not capture.[31]

[31] Malmendier and Nagel (2011) document the role of experience effects in individual beliefs about stock market returns and portfolio allocations to the stock market. The importance of past experiences for belief formation about financial variables may be grounded in psychological evidence, particularly the availability heuristic (Tversky and Kahneman (1974)), the tendency of people to overweight information that is most readily 'available' to them when making forecasts. Barberis and Jin (2021) suggest that a commonly used framework in psychology and neuroscience based on "model-free" and "model-based" learning can help capture the role of individual fixed effects in explaining variation in investor beliefs.

5.3 The Term Structure of UIP Violations

In recent work, Lustig, Stathopoulos and Verdelhan (2019) find that while it is profitable to borrow at short-maturities in foreign currency bonds to invest in short-maturity US bonds when the US interest rate is higher than foreign interest rates, the one-period return of executing this trade is decreasing with maturity. That is, the term structure of UIP violations is downwards sloping. For example, it is less profitable to borrow with 10-year maturity foreign bonds and invest in 10-year Treasury bonds when US interest rates are high than to execute a similar trade by borrowing and lending at 3-month interest rates. As Lustig, Stathopoulos and Verdelhan (2019) document, leading no-arbitrage models in international finance are unable to match this downward-sloping term structure. We introduce additional bonds of longer maturity into our model in order to understand the model's ability to explain the term structure of UIP violations.

5.3.1 Preliminaries

The structure of the extended model is identical to the baseline model, except for the traded assets. Each country offers n-period maturity zero coupon bonds for $n = 1, \ldots, N$, which each pay off one unit of local currency at maturity and are each in zero net supply. We denote the log price of the n-period home country bond in period t as $p_t^{(n)}$, and the one period return from holding this bond as $r_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)}$. Starred variables represent the corresponding quantities for the foreign bond (expressed in foreign currency).

Investor i's problem is given by

$$\begin{aligned} &\max_{\boldsymbol{\alpha}^i} -\mathbb{E}_{i,t}(e^{-\gamma c_{t+1}^i}) \\ &\text{subject to} \quad c_{t+1}^i = \sum_{n=1}^{N} \alpha_i^{(n)} r_{t+1}^{(n)} + \sum_{n=1}^{N} \alpha_i^{(n)*} \left(r_{t+1}^{(n)*} - (s_{t+1} - s_t) \right) \end{aligned} \tag{21}$$

where $\boldsymbol{\alpha}^i = \left[\alpha_i^{(1)}, \ldots, \alpha_i^{(N)}, \alpha_i^{(1)*}, \ldots, \alpha_i^{(N)*}\right]^T$ is a $2N \times 1$ vector of her asset allocations, and $\mathbb{E}_{i,t}$ captures her subjective expectations. Solving Equation (21), investor i's allocations are given by:

$$\boldsymbol{\alpha}_i = \frac{\mathbb{E}_{i,t}\mathbf{r_t}\, \Sigma_t^{-1}}{\gamma} \tag{22}$$

where $\mathbf{r_t} = \left[r_{t+1}^{(1)}, \ldots, r_{t+1}^{(N)}, r_{t+1}^{(1)*}, \ldots, r_{t+1}^{(N)*}\right]$ is a $2N \times 1$ vector capturing the returns from investing in each of the available bonds, and Σ is the $2N \times 2N$ covariance matrix of returns (which all investors agree on).

Because each investor has a unit endowment, $1 = \sum_{n=1}^{N}(\alpha_i^{(n)} + \alpha_i^{(n)*})$. We express $\alpha_i^{(1)} = 1 - \sum_{n=2}^{N} \alpha_i^{(n)} - \sum_{n=1}^{N} \alpha_i^{(n)*}$ for each investor.

5.3.2 Equilibrium Exchange Rate and Bond Prices

Equilibrium consists of the market clearing exchange rate, s_t, prices for each home currency bond, $\left\{p_t^{(n)}\right\}_{n=1}^{N}$, and for each foreign currency bond, $\left\{p_t^{(n)*}\right\}_{n=1}^{N}$. The market clearing condition for home country bonds is

$$\begin{aligned} 0 &= \int \alpha_i^{(n)} di \\ &\propto \int \mathbb{E}_{i,t} r_{t+1}^{(n)} - r_{t+1}^{(1)} di \\ &= \bar{\mathbb{E}}_t(p_{t+1}^{(n-1)}) - p_t^{(n)} - i_t \end{aligned}$$

where $\bar{\mathbb{E}}$ is the average expectation across investors. This yields the market clearing prices

$$p_t^{(n)} = \bar{\mathbb{E}}_t(p_{t+1}^{(n-1)}) - i_t \tag{23}$$

The market clearing condition for foreign country bonds is

$$\begin{aligned} 0 &= \int \alpha_i^{(n)*} di \\ &\propto \int \mathbb{E}_{i,t} r_{t+1}^{(n)*} - (s_{t+1} - s_t) - r_{t+1}^{(1)} \\ &= \int \mathbb{E}_{i,t} p_{t+1}^{(n)*} - p_t^{(n-1)*} - (s_{t+1} - s_t) - i_t \end{aligned}$$

yielding the market clearing prices $p_t^{(n)*} = \bar{\mathbb{E}}_t p_{t+1}^{(n-1)*} - i_t - (\bar{\mathbb{E}}_t s_{t+1} - s_t)$. Solving for the local currency price of the one period foreign currency bond yields the expression $s_t - i_t^d = \bar{\mathbb{E}}_t s_{t+1}$, which is exactly the same UIP condition as in the baseline model, and yields the same expression for exchange rates. We use the UIP condition to re-write the market clearing price of the foreign currency bond as

$$p_t^{(n)*} = \bar{\mathbb{E}}_t p_{t+1}^{(n-1)*} - i_t^* \tag{24}$$

To compute bond prices using Equations (23) and (24), we use a recursive computation method that we outline in the Appendix A.3.[32] We use the same estimated parameters as the baseline model. We analyze the term structure of UIP violations by simulating the model. In each simulation, we simulate two independent economies, a home and foreign economy, where shocks in both economies have the same magnitude, but are scaled such that the distribution of the difference of shocks in the economies matches the distribution of shocks in the baseline model.

[32]The expressions for bond prices in Equations (23) and (24) are the same as those in Barillas and Nimark (2017). The recursive solution we use is similar in spirit to the approach they follow, though our solution method differs.

5.3.3 The Downward-Sloping Term Structure of UIP Violations in the Model

The one period return to a UIP trade that borrows in units of foreign currency and invests in units of home currency using n-period maturity bonds can be written as

$$r^{(n)}_{UIP,t+1} = \underbrace{r^{(n)}_{t+1} - r^{(n)*}_{t+1} + i^d_t}_{\text{Bond Excess Return Differential}} + \underbrace{(s_{t+1} - s_t)}_{\text{Exchange Rate Change}} \tag{25}$$

The returns of the trade consist of two pieces: the excess return differential of n-period maturity bonds, and the change in exchange rates.

Figure 12 plots the model-implied regression coefficient from regressions of the form

$$r^{(n)}_{UIP,t+1} = \gamma^{(n)} + \beta^{(n)} i^d_t + \epsilon^{(n)}_t \tag{26}$$

with $\beta^{(n)}$ on the y-axis, and bond-maturities, n, on the x-axis. The regressions are calculated by simulating the model 5,000 times for 144 periods. The regression coefficients are decreasing with maturity. This captures the downward-sloping term structure of UIP violations found in Lustig, Stathopoulos and Verdelhan (2019) - a higher US short-term interest rate relative to foreign interest rates positively predicts the returns to borrowing in foreign bonds and investing in US bonds, but this predictability is declining in the maturity of bonds used for borrowing and lending.

The exchange rate component of the UIP trades is the same regardless of the maturity of the traded bonds. Hence, the downward-sloping term structure of UIP violations is primarily driven by the fact that the short-term interest rate differential has more return predictability for the home minus foreign return differential of short-maturity bonds versus long-maturity bonds.

Why is this the case? The return predictability of interest rate differentials for home minus foreign bond return differentials primarily stems from dispersed private information, as displayed in Internet Appendix Figure IA.C.4. When interest rate differentials increase, consensus expectations are slow to internalize that they will remain high in the short-term future. In subsequent periods, the consensus belief adjusts to reflect the high interest rate differential. As this happens, the returns of the higher interest rate home currency bonds exceed the returns of the lower interest rate foreign currency bonds. However, this effect is weaker for long maturity bonds. This is because the prices of longer maturity bonds are less sensitive to movements in the consensus belief about short-term interest rate differentials than the prices of shorter maturity bonds, as investors expect interest rate differentials to revert to the mean over long horizons. This results in more muted return predictability for interest rate differentials for UIP trades in long-maturity bonds.

We can contrast the model predictions with the predictions under FIRE. When investors all have accurate beliefs about the current interest rate differential and the future path of interest rate differentials, there is no excess return predictability in bonds of any maturity. The term structure of excess returns for the UIP trade is flat at zero.

One point to make note of is that in our model, the difference in returns between short- and long-maturity bonds transpires entirely because of changes in expectations of future short rates;

there are no *term premia*. Accordingly our model presents a distinct, but complementary explanation for the downward-sloping term structure of UIP violations to Greenwood et al. (2020) and Gourinchas, Ray and Vayanos (2021), who focus on the potential role of term premia for explaining the facts.

6 Conclusion

In this paper, we propose an explanation for the underreaction and overreaction of exchange rates to news, guided by survey evidence. Our focus on survey-based forecasts of interest rates as a proxy for market participants' beliefs is motivated by the strong relationship between survey-based forecast errors of interest rates, currency excess returns, and survey-based forecast errors of exchange rates. Focusing on interest rate forecasts, we document that consensus forecasts of interest rates and interest rate differentials initially underreact, and subsequently overreact to interest rate news. Moreover, the underreaction of forecasts to interest rate news is primarily a feature of consensus forecasts, and is substantially muted when we analyze individual forecaster level data.

We propose a parsimonious model that matches the facts that we document in survey data, with investors who each extrapolate the level of interest rates and receive noisy private signals about future interest rates. We find that the model can qualitatively and quantitatively match the initial underreaction and delayed overreaction of exchange rates in response to news, in a manner consistent with the survey evidence. Our estimated model enables us to distinguish between reasons for asset price underreaction. We find that sluggish consensus expectations, stemming from dispersed private information, play the primary role, with a limited role for higher-order uncertainty and individual-level underreaction. We also use our model to study the persistence of subjective beliefs, and recently documented facts in the exchange rate literature, and find it useful for understanding the facts.

We conclude with some thoughts on further directions for work suggested by our analysis. Our paper highlights dispersed private information about the future path of interest rates as playing an important role in explaining exchange rate behavior. But we do not take a stance on the source of this dispersed private information. A deeper understanding and analysis of *when* and *why* investors disagree about interest rates may help us further understand patterns in exchange rates. Such an understanding of the nature of dispersed information can be applied more broadly, towards further understanding the well-established but still puzzling fact that asset prices sometimes appear to underreact to information and sometimes appear to overreact to information.

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Tables and Figures

TABLE 1: SURVEY-BASED INTEREST RATE FORECAST ERRORS AND EXCHANGE RATES

The table reports results regressions of currency excess returns and exchange rate forecast errors on interest rate forecasts errors. Regressions are of the form $y_{j,t} = \beta(i^d_{j,t} - \bar{\mathbb{E}} i^d_{j,t}) + \epsilon_{j,t}$, where $i^d_{j,t}$ is the short-term interest rate of currency j in period t minus the US short-term interest rate in period t, $y_{j,t}$ is the dependent variable, $\bar{\mathbb{E}}$ corresponds with consensus expectations reported in survey data, and β is the coefficient of interest. The dependent variables used are currency j's $(s_{j,t} - s_{j,t-1} + i^d_{j,t})$, and the unexpected currency appreciation of currency j, $s_{j,t} - \bar{\mathbb{E}}_{t-1} s_{j,t}$. Variables in the regressions are standardized to have zero mean and unit standard deviation for each currency, so that the regression coefficients may be interpreted as correlations. Observations are sampled quarterly. Standard errors are clustered by currency and time period. The table reports t-statistics in parentheses. The sample runs from October 1989 through December 2007.

	Currency Excess Returns $y = s_{j,t} - s_{j,t-1} + i^d_{j,t}$		Currency Forecast Errors $y = s_{j,t} - \bar{\mathbb{E}}_{t-1} s_{j,t}$	
	(1)	(2)	(3)	(4)
IR Differential Forecast Error	0.19	0.20	0.17	0.18
$i^d_{j,t} - \bar{\mathbb{E}}_{t-1}(i^d_{j,t})$	(2.02)	(2.15)	(1.83)	(1.91)
IR Differential Forecast		0.16		0.10
$\bar{\mathbb{E}}_{t-1}(i^d_{j,t})$		(2.53)		(1.72)

TABLE 2: THE FAILURE OF UIP IN THE POST-FINANCIAL CRISIS ERA

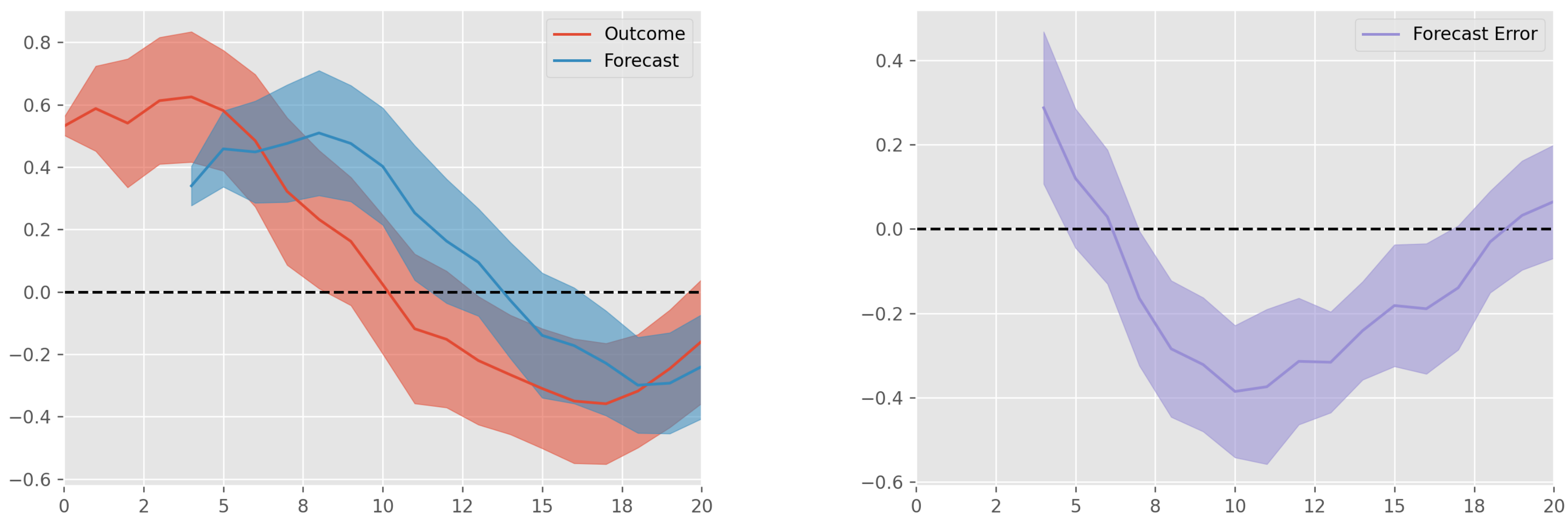
Panel A reports results from time-series regressions of the form $\lambda_{t+1} = \alpha + \beta_{UIP} i_t^d + \epsilon_t$, where λ_{t+1} is either the realized or forecasted excess returns for borrowing in a foreign currency and purchasing US bonds, and i_t^d is the interest rate differential. The panel reports the average coefficient for regressions across individual countries. Panel B reports results from regressions of the form $x_{t+1} - \bar{\mathbb{E}}_t x_{t+1} = \alpha + \beta_{CG}(\bar{\mathbb{E}}_t x_{t+1} - \bar{\mathbb{E}}_{t-3} x_{t+1}) + \epsilon_{t+1}$, where $\bar{\mathbb{E}}$ is the consensus expectation, and x_t are interest rate levels and interest rate differentials. In both panels, the sample consists of quarterly observations from January 2008 through December 2019. Standard errors are HAC-Panel standard errors and are reported in parentheses.

Panel A: UIP Regressions		
	Realized	Forecasted
β_{UIP}	-1.66	0.75
	(0.80)	(0.33)

Panel B: Interest Rate Expectations		
	Interest Rate Levels	Interest Rate Differentials
β_{CG}	0.09	-0.27
	(0.06)	(0.16)

Figure 1: Underreaction and Overreaction in US Interest Rate Response to Interest Rate News Shocks

The figure plots impulse response functions (IRFs) of US Treasury Bill rates, US Treasury Bill rate consensus forecasts, and US Treasury Bill rate consensus forecast errors in response to interest rate news shocks. The IRFs are estimated from regressions of the form $x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma_h C_t + u_{t+h}$, where $x_{t+h} \in (i_{t+h}, \bar{\mathbb{E}}_{t+h} i_{t+h+k}, i_{t+h+k} - \bar{\mathbb{E}}_{t+h} i_{t+h+k})$, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are the estimated interest rate news shocks. The estimated interest rate news shocks come from Angeletos, Collard and Dellas (2020). Forecast data are from the Survey of Professional Forecasters and the sample runs from 1981 to 2007.

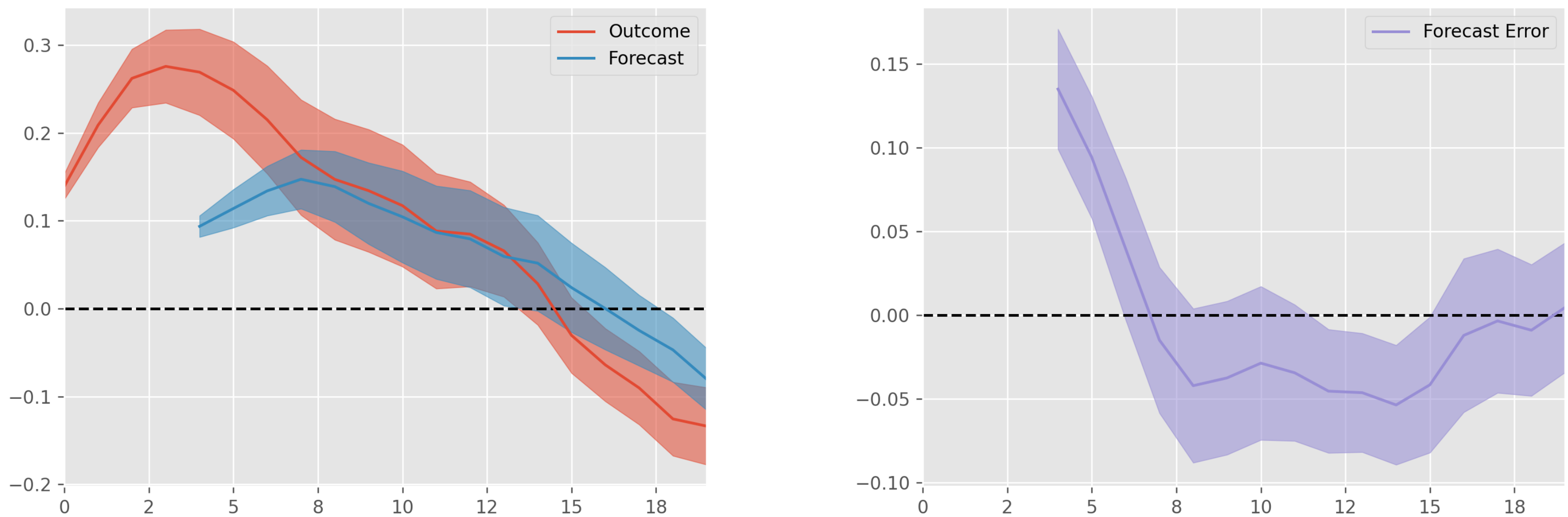


(a) US interest rate IRF to interest rate news

(b) US interest rate forecast errors IRF to interest rate news

FIGURE 2: UNDERREACTION AND OVERREACTION IN INTEREST RATE DIFFERENTIAL RESPONSE TO INTEREST RATE NEWS SHOCKS

The figure plots impulse response functions (IRFs) of interest rate differentials, interest rate differential consensus forecasts, and interest rate differential forecast errors between the US and international countries, in response to interest rate news shocks. The IRFs are estimated from regressions of the form $x_{j,t+h} = \alpha_{j,h} + \beta_h \epsilon_t + \gamma C_{j,t} + u_{j,t+h}$, j corresponds with a specific country, $x_{j,t+h} \in (i_{j,t+h}, \bar{\mathbb{E}}_{j,t+h} i_{j,t+h+k}, i_{j,t+h+k} - \bar{\mathbb{E}}_{j,t+h} i_{j,t+h+k})$, $C_{j,t}$ are lagged values of forecasts and outcomes used as controls, and ϵ_t are the estimated interest rate news shocks. The estimated interest rate news shocks come from Angeletos, Collard and Dellas (2020). Data on interest differential forecasts are from Consensus Economics. The sample consists of G11 currencies and runs from October 1989 through December 2007.

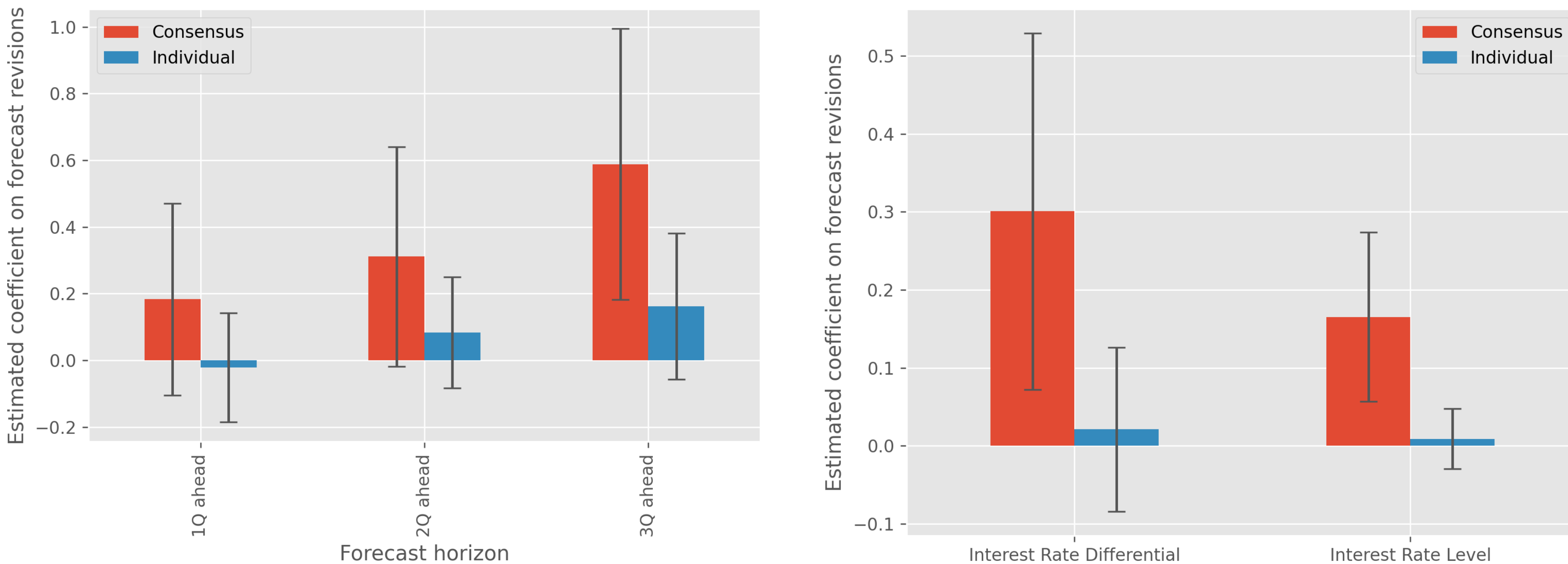


(a) Interest rate differential IRF to interest rate news

(b) Interest rate differential forecast errors IRF to interest rate news

FIGURE 3: CONSENSUS VS. INDIVIDUAL DEVIATIONS FROM FIRE

The figure plots regression coefficients from regressions of forecast errors of interest rates on forecast revisions of interest rates. The red bars are for regressions where observations correspond with consensus forecasts and the blue bars are for regressions where observations correspond with individual forecasts. The first panel in the figure reports regression coefficients from regressions of the form $x_{t+k} - \mathbb{E}_t x_{t+k} = \alpha + \beta(\mathbb{E}_t x_{t+k} - \mathbb{E}_{t-k} x_{t+k}) + \epsilon_{t+k}$, where x_{t+k} is the US Treasury Bill rate in period $t+k$, and $\mathbb{E}_t x_{t+k}$ is the forecast at period t of the realized outcome at period $t+k$ from the Survey of Professional Forecasters. The second panel in the figure reports regression coefficients from regressions of the form $x_{t+1} - \mathbb{E}_t x_{t+1} = \alpha + \beta(\mathbb{E}_t x_{t+1} - \mathbb{E}_{t-3} x_{t+1}) + \epsilon_{t+1}$, where x_t is the interest rate differential (or interest rate level) for a given foreign currency, and $\mathbb{E}$ is the forecast at period t of the realized outcome in $t+1$ from Consensus Economics. Standard errors for panel regressions are two-way clustered by forecaster and time period. Lines denoting plus or minus two standard errors are included in the plots. The sample consists of quarterly observations from 1969 through 2007 from the Survey of Professional Forecasters, and from 1989 to 2007 from Consensus Economics.

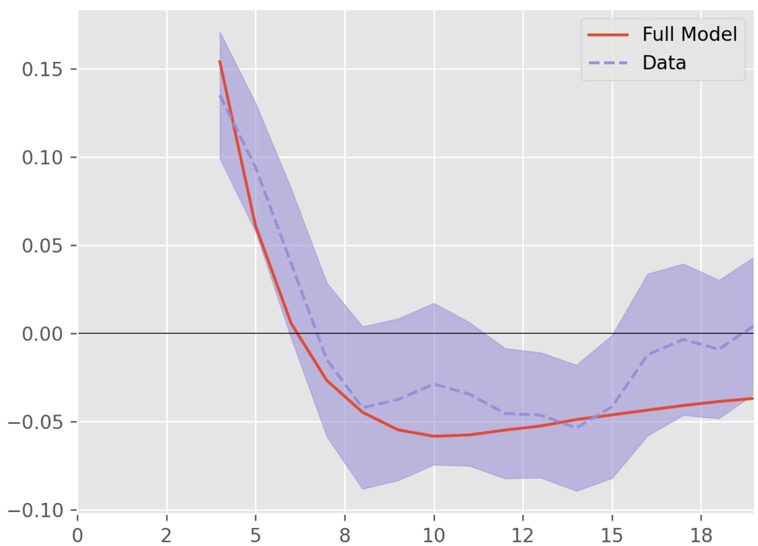


(a) T-Bill (SPF)

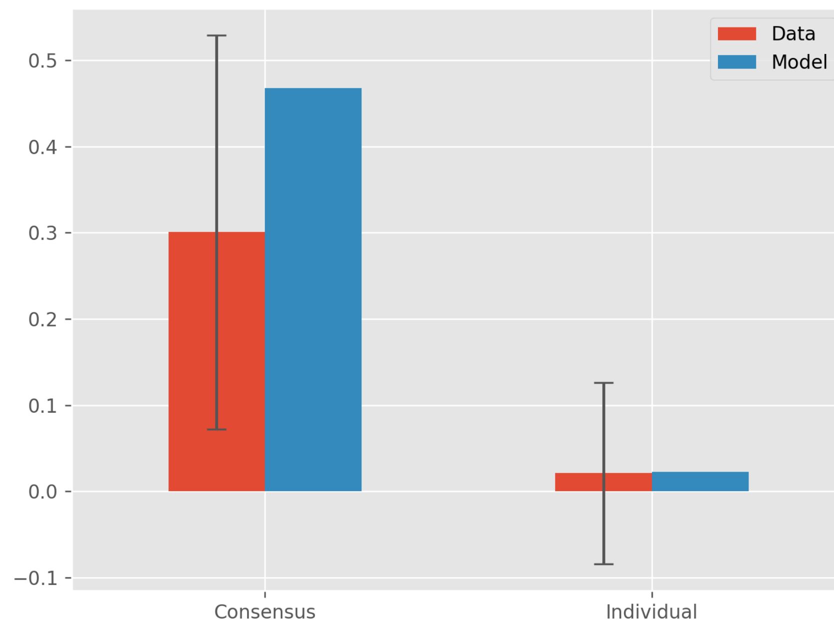
(b) Foreign Short-Rates (Consensus Economics)

Figure 4: Model Calibration of Interest Rate Forecast Errors

The figure displays information about the model calibration of interest rate forecast errors. The first panel plots impulse response functions (IRFs) of forecast errors generated by the estimated model (Full Model) and compares it with the empirical IRF (Data). The Full Model IRF is computed by simulating 5,000 economies for 144 periods, computing the IRF for each simulated economy, and computing the average across each simulation. The second panel plots regression coefficients from regressions of the form $x_{t+1} - \mathbb{E}_t x_{t+1} = \alpha + \beta(\mathbb{E}_t x_{t+1} - \mathbb{E}_{t-3} x_{t+1}) + \epsilon_{t+3}$, where x is the variable of interest, $\mathbb{E}$ captures (subjective) expectations, and each time period corresponds with one quarter. The Data bars, in red and blue, corresponds with regression coefficients estimated using interest rate forecast data from Consensus Economics. The Model bars, in purple, correspond with regression coefficients implied by the model calibration for the interest rate differential. The panel presents regression coefficients where observations are at the consensus forecast level (averaged across individuals), as well as regression coefficients where observations are at the individual forecaster level.



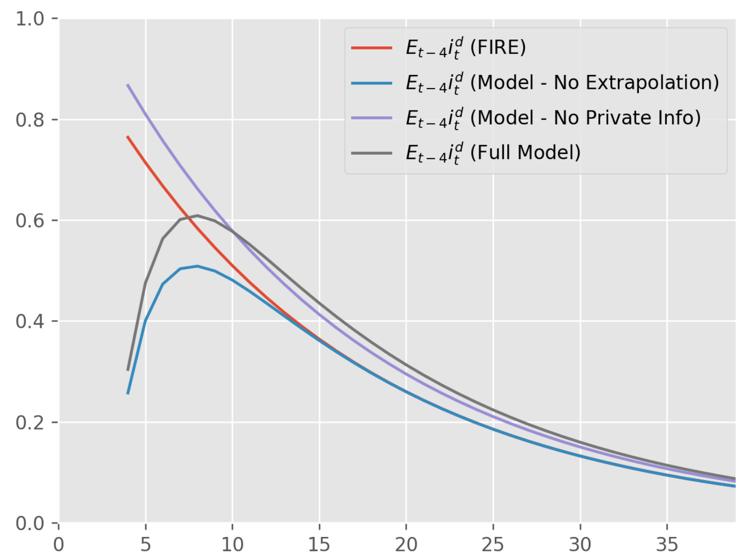
(a) Forecast Error IRFs



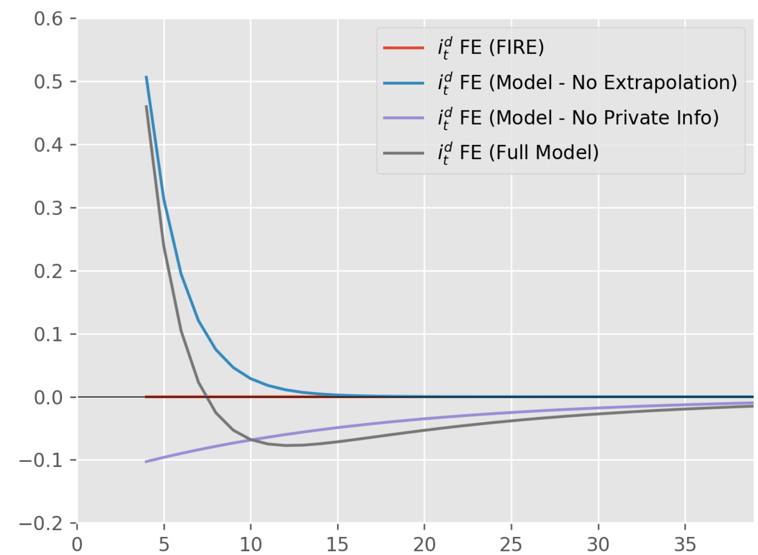
(b) Model Bias Coefficients

Figure 5: Model Impulse Response Functions to a Fundamental Shock

The first panel in the figure plots the model-implied impulse response function of the estimated model for consensus interest rate differential forecasts ($\bar{\mathbb{E}}_{t-4} i_t^d$) in response to a one standard deviation shock to fundamentals, ξ_t. The second panel in the figure plots the model-implied impulse response function of interest rate differential forecast errors ($i_t^d - \bar{\mathbb{E}}_{t-4} i_t^d$) in response to a one standard deviation to fundamentals, ξ_t. Both panels include IRFs corresponding with the full estimated model, as well as IRFs for models with a subset of frictions included.



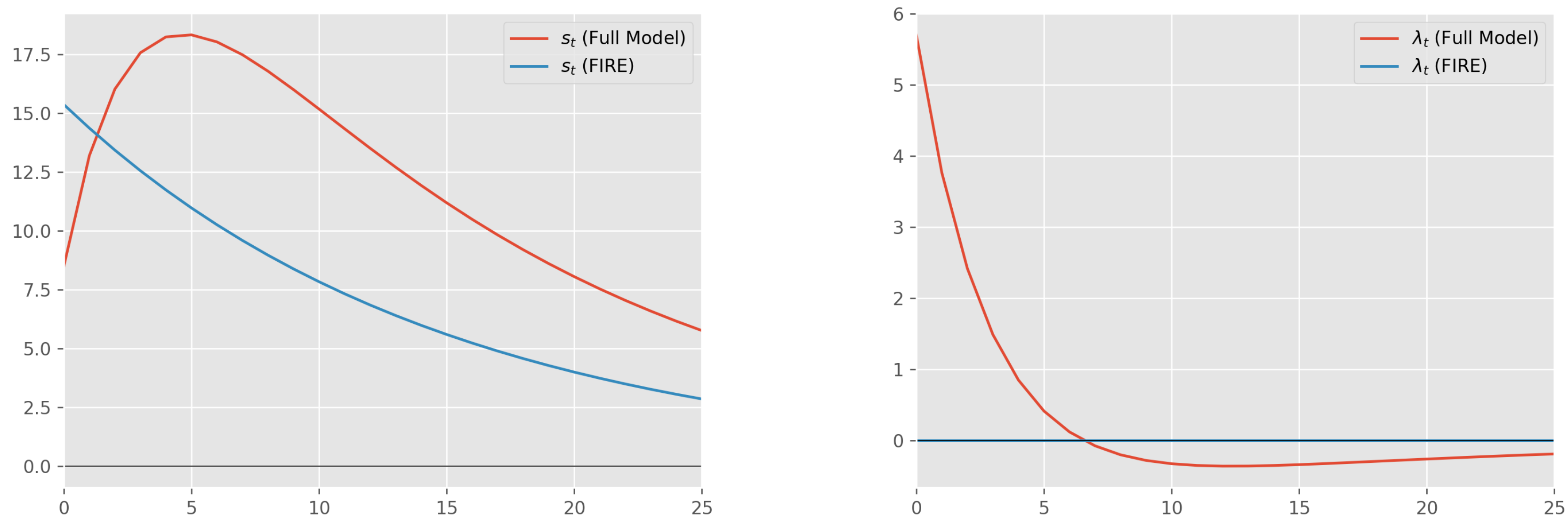
(a) Interest Rate Differential Forecast



(b) Interest Rate Differential Forecast Error

FIGURE 6: EXCHANGE RATE RESPONSE TO INTEREST RATE SHOCKS IN THE MODEL

The figure plots exchange rates and currency excess returns in response to a one standard deviation shock to the fundamental process, ξ_t, as implied by the fully estimated model. For comparison, the figure also includes exchange rates and currency excess returns in a Full-Information Rational Expectations model.



(a) Exchange Rate

(b) Excess Return

FIGURE 7: UIP REGRESSIONS IN THE MODEL AND DATA

The figure reports model UIP regression for different k-period ahead horizons. We simulate the estimated model 5,000 times for 144 periods. For each simulation and k-period ahead horizon, we estimate the following regression $\lambda_{t+k} = \alpha_k + \beta_k i_t^d + \epsilon_{t+k}$, where λ_{t+k} is the excess return between period $t+k-1$ and $t+k$, and i_t^d is the interest rate differential at period t. We report the average regression coefficient of all simulations.

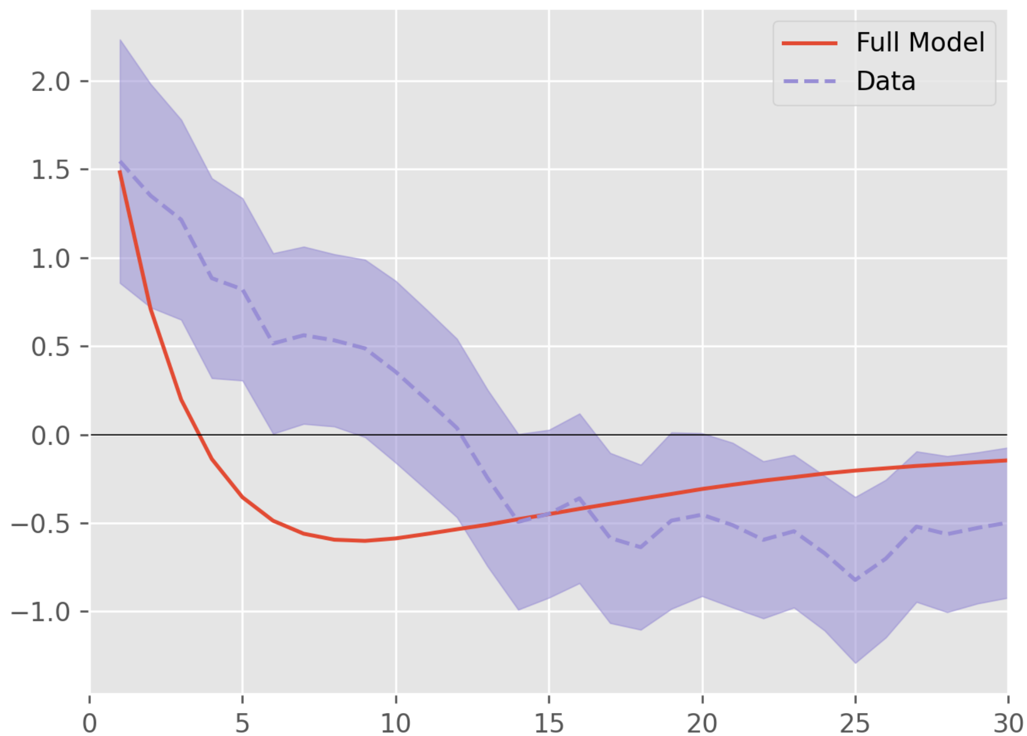


Figure 8: Time-Series Momentum and Reversal

The figure plots autocorrelations of currency excess returns in the model. The k-period autocorrelation is calculated by simulating the estimated model 5,000 times for 144 periods, and taking an average autocorrelation of currency excess returns with k-period lagged excess returns in each simulated sample.

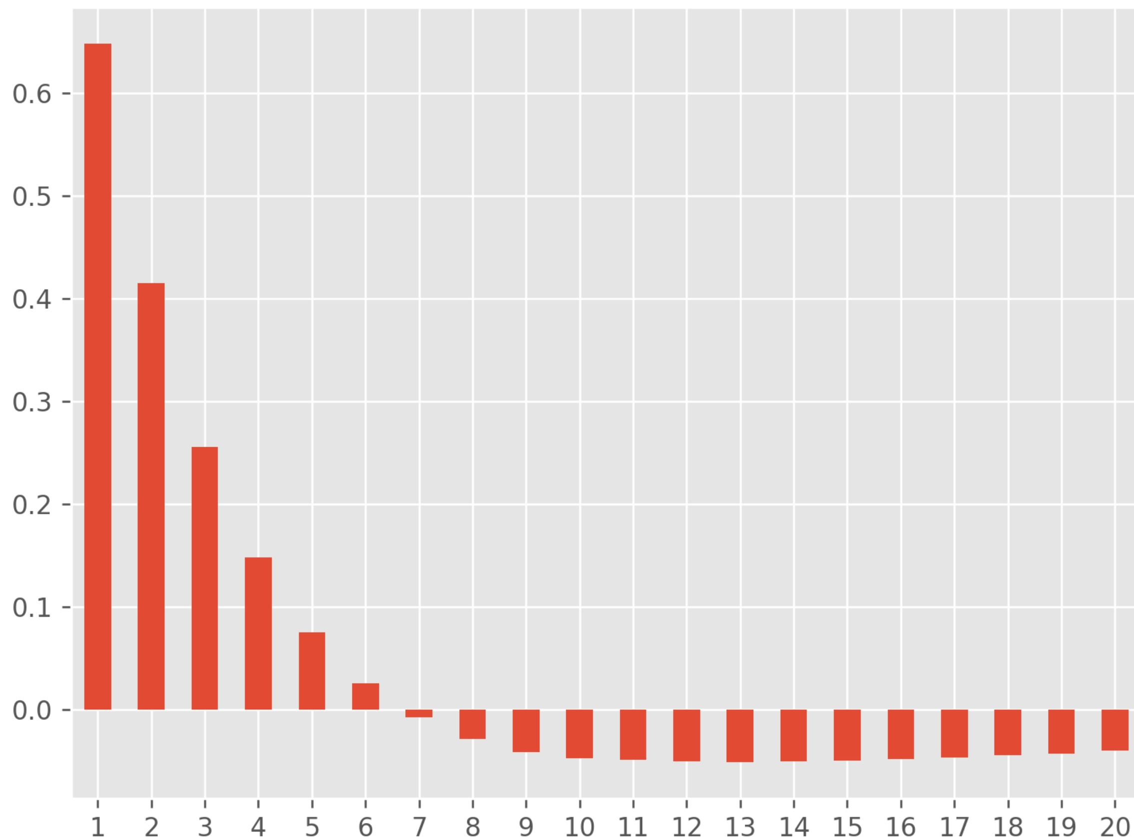
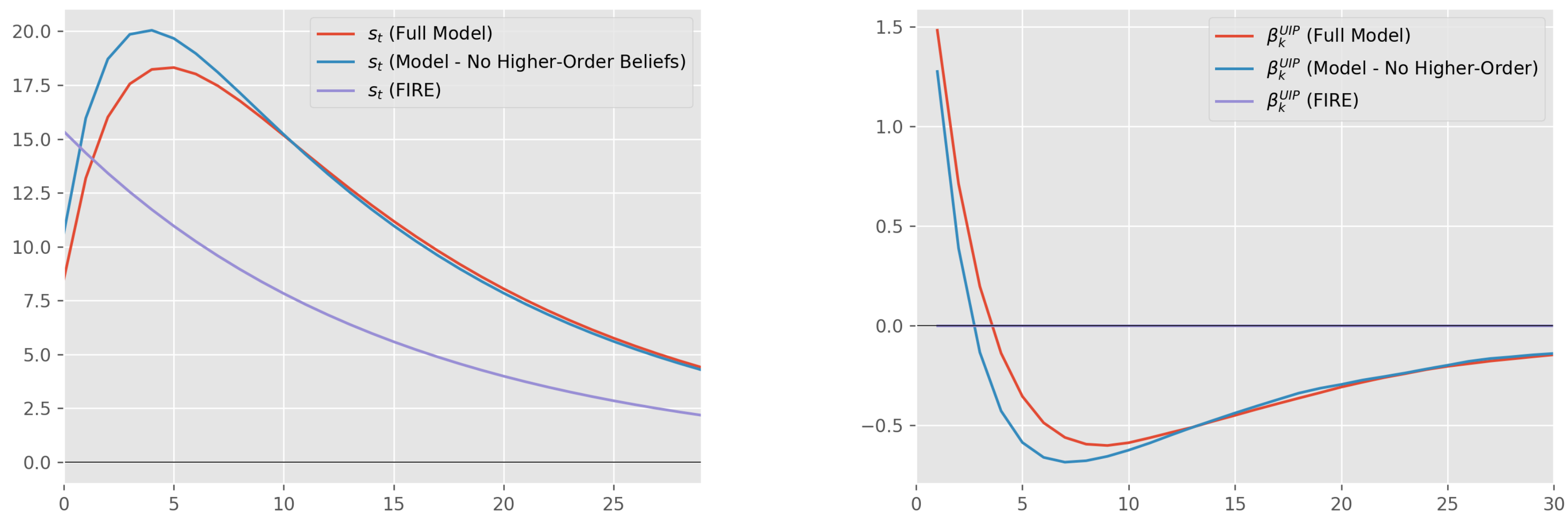


FIGURE 9: THE INFLUENCE OF HIGHER-ORDER UNCERTAINTY

The first panel in the figure plots s, the log exchange rate in the model, and $\tilde{s}$, the log exchange rate in the model in the absence of higher-order uncertainty, in response to an interest rate shock in period 0. The second panel in figure plots regression coefficients from UIP regressions of the period $t+k$ currency excess returns on the period t interest rate differential. These coefficients are the β_k values from regressions of the form $\lambda_{t+k} = \alpha_k + \beta_k i^d_{t+k} + \varepsilon_{t+k}$ for $k = 0, 1, \ldots, 30$, where λ_{t+k} is the excess return from borrowing in foreign currency bonds and investing in home currency bonds from period $t+k-1$ to $t+k$. The figure also plots coefficients from regressions computing currency excess returns using the log exchange rate in the absence of higher-order uncertainty.The values are computed by simulating the model 5,000 times for 144 periods.

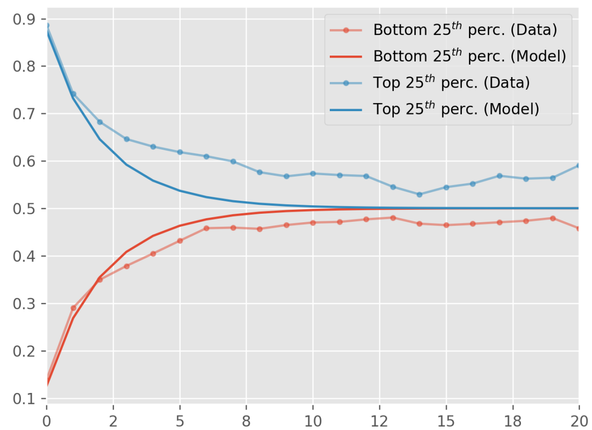


(a) Exchange Rate

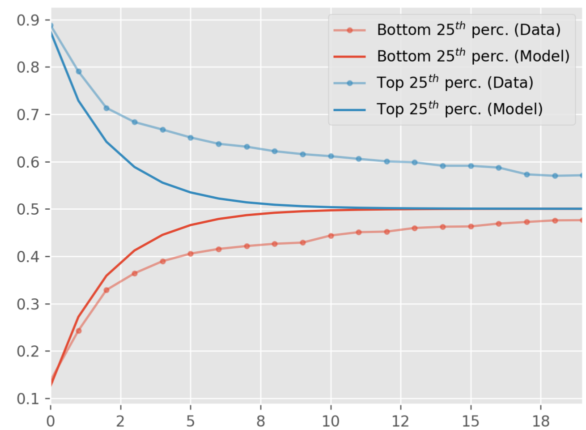
(b) Excess Return

FIGURE 10: PERSISTENCE OF SUBJECTIVE BELIEFS

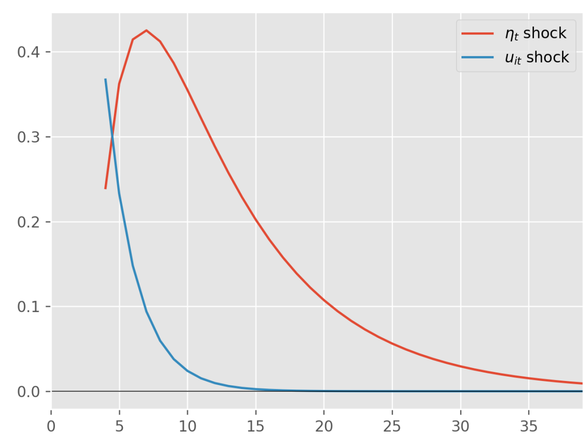
The first two panels in the figure plot the persistence of subjective beliefs in the model and in the data. The model results are computed by simulating the model 5,000 times for 144 periods. For each simulation, we compute the beliefs of 1000 investors in each period of the simulation. We rank investors based on their beliefs about the fundamental, ξ_t, in each period. The panels also plot the average percentile ranks of investors in the top and bottom quartile of the belief distribution in subsequent periods. The data lines in the panels are computed by ranking forecasters in the Consensus Economics and SPF data based on their beliefs about short-term interest rates for a given country, and computing the average percentile rank of the forecasters in the top and bottom quartiles of the belief distribution in subsequent periods. The third panel in the figure plots the impulse response function of expected interest rate differentials four periods ahead, $\mathbb{E}_{i,t-4}i_t^d$, in response to a one-standard deviation shock to private information ($u_{i,t}$), and in response to a one-standard deviation shock to fundamentals (η_t).



(a) SPF



(b) Consensus Economics



(c) η_t and $u_{i,t}$ shocks

Figure 11: Dispersion of Beliefs About Interest Rates Over Time

The figure plots the cross-sectional standard deviation of forecasts of the short-term interest rate at each point in time, averaged across the countries in our sample. The red line corresponds with forecasts of the short-term interest rate one quarter ahead, and the blue line corresponds with forecasts of the short-term interest rate four quarters ahead.

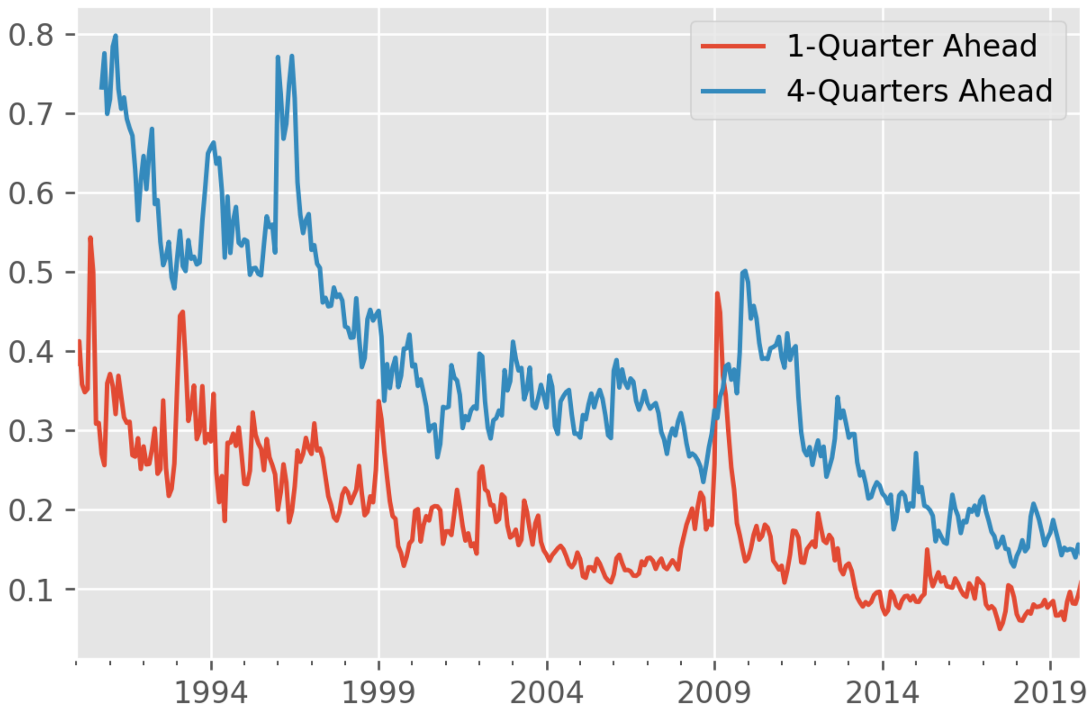
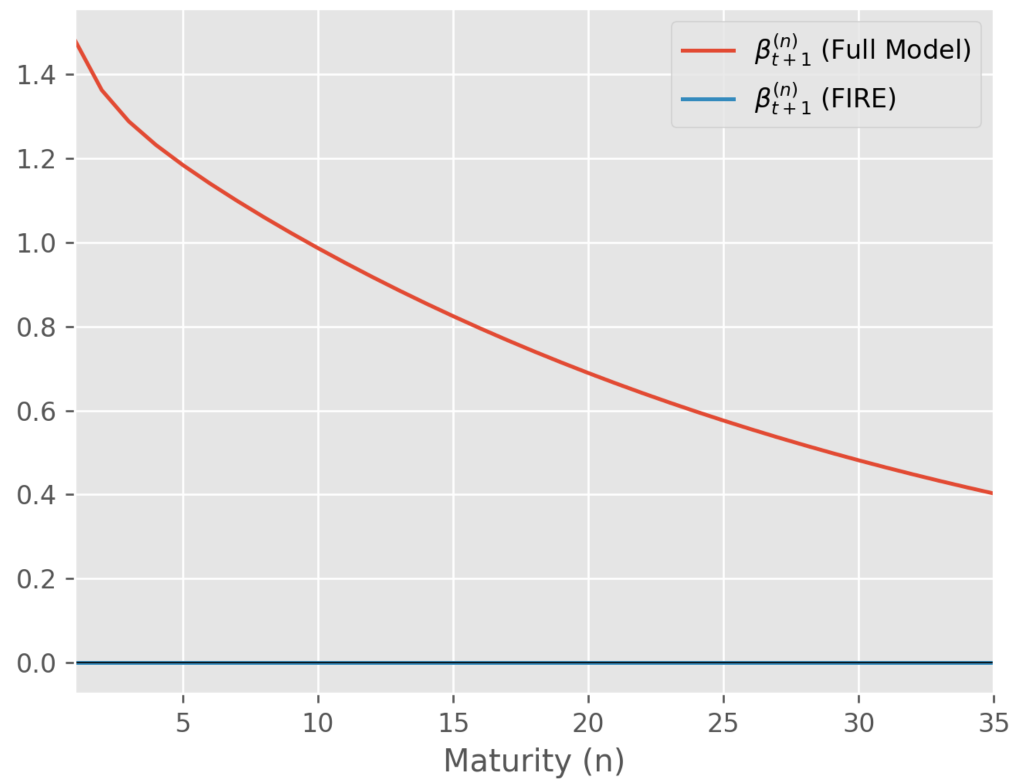


FIGURE 12: THE DOWNWARD-SLOPING TERM STRUCTURE OF UIP VIOLATIONS

The figure plots the model-implied regression coefficients from regressing the returns to borrowing in n-period maturity foreign bonds and investing in n-period maturity home country bonds on the interest rate differential (the home currency interest rate minus the foreign country interest rate), for different values of n. The coefficients are computed by simulating the model 5,000 times for 144 periods. The figure plots regression coefficients for the full estimated model and under FIRE.



A Proofs and Derivations

The Wold representation theorem and the Wiener-Hopf prediction theorem are used to prove the propositions in the paper; they can be found in Huo and Takayama (2018). We reproduce the details below.

Signal Process. The signals observed by investor i follow

$$\boldsymbol{x}_t = \begin{bmatrix} \hat{i}_t^d \\ \hat{x}_{it} \end{bmatrix} = \begin{bmatrix} \hat{\tau}_\varepsilon^{-1/2} & 0 & \frac{1}{1-\hat{\rho}L} \\ 0 & \tau_u^{-1/2} & \frac{1}{1-\hat{\rho}L} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_{it} \\ \eta_t \end{bmatrix} = \boldsymbol{M}(L)\boldsymbol{e}_t.$$

Wold Representation. Suppose the signals' state-space representation is

$$\mathbf{x}_t = \mathbf{H}\boldsymbol{\xi}_t + \mathbf{R}\mathbf{u}_t \text{ and } \boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{Q}\boldsymbol{\nu}_t,$$

where $\boldsymbol{v}_t$ and $\boldsymbol{u}_t$ are standard normal shocks. If all the eigenvalues of $\boldsymbol{F}$ lie inside the unit circle, the Wold representation is

$$\mathbf{x}_t = \boldsymbol{B}(L)\boldsymbol{w}_t.$$

$\boldsymbol{B}(L)$ is given by

$$\mathbf{B}(L) = \mathbf{I} + \mathbf{H}(\mathbf{I} - \mathbf{FL})^{-\mathbf{1}}\mathbf{FKL},$$

the inverse of $\mathbf{B}(L)$ is

$$\mathbf{B}(L)^{-1} = \mathbf{I} - \mathbf{H}[\mathbf{I} - (\mathbf{F} - \mathbf{FKH})\mathbf{L}]^{-\mathbf{1}}\mathbf{FKL},$$

and the co-variance matrix $\boldsymbol{V}$ is

$$\boldsymbol{V} = \mathbf{HPH}' + \mathbf{RR}',$$

where define the $\mathbf{P}$ matrix as the one solves

$$\mathbf{P} = \mathbf{F}[\mathbf{P} - \mathbf{PH}'(\mathbf{HPH}' + \mathbf{RR}')^{-\mathbf{1}}\mathbf{HP}]\mathbf{F}' + \mathbf{QQ}'.$$

The Kalman gain matrix is $\mathbf{K} = \mathbf{PH}'(\mathbf{HPH}' + \mathbf{RR}')^{-\mathbf{1}}$.

Wiener-Hopf Prediction. Suppose the original representation of the signal process is $\boldsymbol{x}_t = \boldsymbol{M}(L)\boldsymbol{e}_t$, and a stationary process $\boldsymbol{f}_t = \varphi(L)\boldsymbol{e}_t$, then the prediction formula is

$$\mathbb{E}[\boldsymbol{f}_t|\boldsymbol{x}_t] = \left[\varphi(L)\boldsymbol{M}'(L^{-1})\boldsymbol{B}'(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}\boldsymbol{x}_t.$$

In order to apply the prediction formula, we need to find the Wold representation of our signal process. Define

$$\lambda = \frac{1}{2}\left(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}} - \sqrt{(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}})^2 - 4}\right).$$

In our setting,

$$\boldsymbol{B}^{-1}(L) = \frac{1}{1-\lambda L}\begin{bmatrix} 1 - \frac{\hat{\tau}_\varepsilon \hat{\rho} + \lambda \tau_u}{\hat{\tau}_\varepsilon + \tau_u} L & \frac{\tau_u(\lambda - \hat{\rho})}{\hat{\tau}_\varepsilon + \tau_u} L \\ \frac{\hat{\tau}_\varepsilon(\lambda - \hat{\rho})}{\hat{\tau}_\varepsilon + \tau_u} L & 1 - \frac{\tau_u \hat{\rho} + \lambda \hat{\tau}_\varepsilon}{\hat{\tau}_\varepsilon + \tau_u} L \end{bmatrix},$$

and

$$\boldsymbol{V}^{-1} = \frac{\hat{\tau}_\varepsilon \tau_u}{\hat{\rho}(\hat{\tau}_\varepsilon + \tau_u)}\begin{bmatrix} \frac{\tau_u \hat{\rho} + \lambda \hat{\tau}_\varepsilon}{\tau_u} & \lambda - \hat{\rho} \\ \lambda - \hat{\rho} & \frac{\hat{\tau}_\varepsilon \hat{\rho} + \lambda \tau_u}{\hat{\tau}_\varepsilon} \end{bmatrix}.$$

A.1 Interest Rate Expectations and Errors

We use the Wiener-Hopf prediction formula to derive the expectation of the fundamental variable (and accordingly, expectations of future interest rate differentials), which we in turn use to prove Proposition 1.

$$\begin{aligned}
\mathbb{E}_{it}[\xi_t|\mathcal{I}_{it}] =& [\begin{bmatrix} 0 & 0 & \frac{1}{1-\hat{\rho}L} \end{bmatrix} \boldsymbol{M}'(L^{-1})\boldsymbol{B}'(L^{-1})^{-1}]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}\begin{bmatrix} i_t^d \\ x_{it} \end{bmatrix} \\
=& \frac{\lambda(\hat{\tau}_\varepsilon i_t^d + \tau_u x_{it})}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)}
\end{aligned}$$

Therefore $\mathbb{E}_{it}[\xi_{t+j}|\mathcal{I}_{it}] = \hat{\rho}^{j-1}\frac{\lambda(\hat{\tau}_\varepsilon i_t^d + \tau_u x_{it})}{(1-\lambda L)(1-\hat{\rho}\lambda)}$. At the consensus level

$$\bar{\mathbb{E}}_t[\xi_t] = \int \mathbb{E}_{it}[\xi_t|\mathcal{I}_{it}]di = \frac{\lambda\hat{\tau}_\varepsilon \varepsilon_t + \lambda(\hat{\tau}_\varepsilon + \tau_u)\xi_t}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)} \text{ and } \bar{\mathbb{E}}_t[\xi_{t+j}] = \hat{\rho}^{j-1}\frac{\lambda\hat{\tau}_\varepsilon \varepsilon_t + \lambda(\hat{\tau}_\varepsilon + \tau_u)\xi_t}{(1-\lambda L)(1-\hat{\rho}\lambda)}.$$

Proof of Proposition 1

Proof. The period t expectation of the fundamental variable in $t+1$ can be written as

$$\bar{\mathbb{E}}_t[\xi_{t+1}] = \hat{\rho}\left(1 - \frac{\lambda}{\hat{\rho}}\right)\frac{1}{1-\lambda L}\xi_t + \frac{\lambda\hat{\tau}_\varepsilon \sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.$$

The forecast error is therefore

$$FE_{t,t+1} = i_{t+1}^d - \hat{\rho}(1 - \frac{\lambda}{\hat{\rho}})\frac{1}{1-\lambda L}\xi_t - \frac{\lambda\hat{\tau}_\varepsilon \sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t$$

$$=\frac{1-\hat{\rho}L}{1-\lambda L}\xi_{t+1}+\sigma_\varepsilon\varepsilon_{t+1}-\frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.$$

□

Proof of Proposition 2

Proof. To have initial under-reaction, we only need the covariance term to be positive when $\delta = 1$. This holds when $\rho > \hat{\rho} - \lambda$. To have delayed overreaction, it's sufficient to show that the Equation (27) has a root in the interval $(0, +\infty)$.

$$g(\delta) = \lambda^\delta(\hat{\rho}-\lambda) + \rho^\delta(\rho-\hat{\rho}). \tag{27}$$

Suppose such a root exits, and is denoted as $\bar{\delta}$. We know

$$(\lambda/\rho)^{\bar{\delta}} = \frac{\hat{\rho}-\rho}{\hat{\rho}-\lambda}.$$

For such a $\bar{\delta}$ to exist, we must have that $\hat{\rho} > \rho$. When $\lambda < \rho$ ($\lambda > \rho$), the LHS is smaller (greater) than one, and correspondingly, the RHS is also smaller (greater) than one. So $\rho < \hat{\rho}$ is a sufficient condition to have a finite $\bar{\delta}$. □

Proof of Proposition 3

Proof. Denote $\theta = \frac{\lambda}{1-\hat{\rho}\lambda}$, we write the individual level and consensus level forecast as

$$\begin{aligned}
\mathbb{E}_{it}[\xi_{t+1}] =& \frac{\hat{\rho}-\lambda}{1-\lambda L}\xi_t + \frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t + \frac{\theta\tau_u\sigma_u}{1-\lambda L}u_{it},\\
\bar{\mathbb{E}}_t[\xi_{t+1}] =& \frac{\hat{\rho}-\lambda}{1-\lambda L}\xi_t + \frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t.
\end{aligned}$$

The individual and consensus forecast errors are

$$\begin{aligned}
FE_{it,t+1} =& \frac{1-\hat{\rho}L}{1-\lambda L}\xi_{t+1} + \sigma_\varepsilon\varepsilon_{t+1} - \frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t - \frac{\theta\tau_u\sigma_u}{1-\lambda L}u_{it}\\
FE_{t,t+1} =& \frac{1-\hat{\rho}L}{1-\lambda L}\xi_{t+1} + \sigma_\varepsilon\varepsilon_{t+1} - \frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t.
\end{aligned}$$

The individual and consensus forecast revisions are

$$\begin{aligned}
FR_{it,t+1} =& \frac{(\hat{\rho}-\lambda)(1-\hat{\rho}L)}{1-\lambda L}\xi_t + \frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon(1-\hat{\rho}L)}{1-\lambda L}\varepsilon_t + \frac{\theta\tau_u\sigma_u(1-\hat{\rho}L)}{1-\lambda L}u_{it}\\
FR_{t,t+1} =& \frac{(\hat{\rho}-\lambda)(1-\hat{\rho}L)}{1-\lambda L}\xi_t + \frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon(1-\hat{\rho}L)}{1-\lambda L}\varepsilon_t.
\end{aligned}$$

Denote $\kappa_1 = (\hat{\rho}-\lambda)\frac{\lambda}{1-\lambda^2}$, $\kappa_2 = (\hat{\rho}-\lambda)(\rho-\hat{\rho})\frac{(1+\lambda^2)(1-\rho^2)+(\lambda+\rho)(\rho-\hat{\rho})}{(1-\lambda^2)(1-\rho^2)(1-\rho\lambda)}$. The covariance between the consensus forecast error and forecast revision can be written as

$$\begin{aligned}
cov(FR_{t,t+1}, FE_{t,t+1}) =& cov(\frac{1-\hat{\rho}L}{(1-\lambda L)(1-\rho L)}\eta_{t+1}, \frac{(\hat{\rho}-\lambda)(1-\hat{\rho}L)}{(1-\lambda L)(1-\rho L)}\eta_t) \\
& - \theta^2\hat{\tau}_\varepsilon^2\sigma_\varepsilon^2 cov(\frac{1}{1-\lambda L}\varepsilon_t, \frac{1-\hat{\rho}L}{1-\lambda L}\varepsilon_t) \\
=& \kappa_1 + \kappa_2 - \theta^2\hat{\tau}_\varepsilon^2\sigma_\varepsilon^4\frac{1-\hat{\rho}\lambda}{1-\lambda^2}.
\end{aligned}$$

The covariance between individual forecast errors and forecast revisions can be written as

$$cov(FR_{it,t+1}, FE_{it,t+1}) = \kappa_1 + \kappa_2 - \theta^2\hat{\tau}_\varepsilon^2\sigma_\varepsilon^4\frac{1-\hat{\rho}\lambda}{1-\lambda^2} - \theta^2\tau_u^2\sigma_u^4\frac{1-\hat{\rho}\lambda}{1-\lambda^2}.$$

The variance of the forecast revisions is

$$\begin{aligned}
var(FR_{t,t+1}) =& (\hat{\rho}-\lambda)^2\frac{1-\hat{\rho}}{(1-\lambda)(1-\rho)} + \theta^2\hat{\tau}_\varepsilon^2\sigma_\varepsilon^4\frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2} \\
var(FR_{it,t+1}) =& (\hat{\rho}-\lambda)^2\frac{1-\hat{\rho}}{(1-\lambda)(1-\rho)} + \theta^2\hat{\tau}_\varepsilon^2\sigma_\varepsilon^4\frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2} + \theta^2\tau_u^2\sigma_u^4\frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2}.
\end{aligned}$$

Therefore, we have

$$\frac{cov(FR_{it,t+1}, FE_{it,t+1}) + \theta^2\tau_u^2\sigma_u^4\frac{1-\hat{\rho}\lambda}{1-\lambda^2}}{var(FR_{it,t+1}) - \theta^2\tau_u^2\sigma_u^4\frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2}} = \frac{cov(FR_{t,t+1}, FE_{t,t+1})}{var(FR_{t,t+1})}.$$

As long as $cov(FR_{t,t+1}, FE_{t,t+1}) > 0$, we have

$$\frac{cov(FR_{t,t+1}, FE_{t,t+1})}{var(FR_{t,t+1})} > \frac{cov(FR_{it,t+1}, FE_{it,t+1})}{var(FR_{it,t+1})}$$

provided that $\tau_u \neq 0$, $\sigma_u \neq 0$ and $\hat{\sigma}_\varepsilon \neq 0$. $\square$

A.2 Exchange Rates

Proof of Proposition 4

Proof. The average expectation of the fundamental is

$$\bar{\mathbb{E}}_t[\xi_t] = \frac{\lambda(\hat{\tau}_\varepsilon + \tau_u)}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)}\xi_t + \frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.$$

We have

$$\frac{\lambda(\hat{\tau}_\varepsilon + \tau_u)}{(1-\hat{\rho})(1-\lambda L)(1-\hat{\rho}\lambda)}\xi_t = \frac{\hat{\rho}}{1-\hat{\rho}}(1-\frac{\lambda}{\hat{\rho}})\frac{1}{1-\lambda L}\xi_t$$

by using the identity

$$\lambda + \lambda^{-1} = \hat{\rho} + \hat{\rho}^{-1} + (\hat{\tau}_\varepsilon + \tau_u)\hat{\rho}^{-1}.$$

$\square$

Proof of Proposition 5

Proof. We conjecture the exchange rate takes the form $s_t = g(L)\xi_t + h_1(L)\sigma_\varepsilon\varepsilon_t$. Therefore

$$\begin{aligned} s_{t+1} =& g(L)/L\xi_t + h_1(L)\sigma_\varepsilon/L\varepsilon_t \\ =& \frac{g(L)L^{-1}}{1-\hat{\rho}L}\eta_t + \tau_\varepsilon^{-1/2}L^{-1}h_1(L)\varepsilon_t. \end{aligned}$$

Defining $h_2(L) = g(L) - h_1(L)$, applying the Wiener-Hopf prediction formula, and incorporating investors' subjective beliefs, we have that

$$\begin{aligned} \mathbb{E}_{it}[s_{t+1}|\mathcal{I}_{it}] =& \left[\begin{bmatrix} \hat{\tau}_\varepsilon^{-1/2}L^{-1}h_1(L) & 0 & \frac{g(L)L^{-1}}{1-\hat{\rho}L} \end{bmatrix} \boldsymbol{M}'(L^{-1})\boldsymbol{B}'(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}\begin{bmatrix} i_t^d \\ x_{it} \end{bmatrix} \\ =& \begin{bmatrix} -\frac{\lambda(1-\hat{\rho}L)\tau_\varepsilon h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} - \frac{(1-\lambda L)\tau_u + (1-\hat{\rho}L)\tau_\varepsilon}{L(\tau_\varepsilon+\tau_u)(1-\lambda L)}h_1(0) + \frac{h_1(L)}{L} + \frac{\lambda\tau_\varepsilon h_2(L)}{\hat{\rho}(L-\lambda)(1-\lambda L)} \\ -\frac{\lambda(1-\hat{\rho}L)\tau_u h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} + \frac{(\hat{\rho}-\lambda)\tau_u h_1(0)}{(\tau_u+\tau_\varepsilon)(1-\lambda L)} + \frac{\lambda\tau_u h_2(L)}{\hat{\rho}(L-\lambda)(1-\lambda L)} \end{bmatrix}' \begin{bmatrix} i_t^d \\ x_{it} \end{bmatrix} \\ \equiv& q_1(L)i_t^d + q_2(L)x_{it}. \end{aligned}$$

As a result, we can express the consensus expectation of the period $t+1$ exchange rate as

$$\int \mathbb{E}_{it}[s_{t+1}|\mathcal{I}_{it}]di = \left(q_1(L) + q_2(L)\right)\xi_t + q_1(L)\sigma_\varepsilon\varepsilon_t.$$

Recall the equilibrium condition for the exchange rate:

$$s_t - i_t^d = \int \mathbb{E}_{it}[s_{t+1}|\mathcal{I}_{it}]di.$$

We can re-write this condition as

$$g(L)\xi_t + h_1(L)\sigma_\varepsilon\varepsilon_t - \xi_t - \sigma_\varepsilon\varepsilon_t = \left(q_1(L) + q_2(L)\right)\xi_t + q_1(L)\sigma_\varepsilon\varepsilon_t.$$

Matching coefficients on ξ_t and ε_t yields

$$g(L) - 1 = q_1(L) + q_2(L) \text{ and } h_1(L) - 1 = q_1(L)$$

which can be written as the following functional equations in matrix form

$$\boldsymbol{A}(L)\begin{bmatrix} h_1(L) \\ h_2(L) \end{bmatrix} = \boldsymbol{d}(L)$$

where

$$\boldsymbol{A}(L) = \begin{bmatrix} 1-L^{-1} & -\frac{\lambda\hat{\tau}_\varepsilon}{\hat{\rho}(L-\lambda)(1-\lambda L)} \\ 0 & 1-\frac{\lambda\tau_u}{\hat{\rho}(L-\lambda)(1-\lambda L)} \end{bmatrix}$$

$$\boldsymbol{d}_1(L) = -\frac{\lambda(1-\hat{\rho}L)\hat{\tau}_\varepsilon h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} - \frac{(1-\lambda L)\tau_u + (1-\hat{\rho}L)\hat{\tau}_\varepsilon}{L(\hat{\tau}_\varepsilon+\tau_u)(1-\lambda L)}h_1(0) + 1$$

and

$$\boldsymbol{d}_2(L) = -\frac{\lambda(1-\hat{\rho}L)\tau_u h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} + \frac{(\hat{\rho}-\lambda)\tau_u}{(\tau_u+\hat{\tau}_\varepsilon)(1-\lambda L)}h_1(0).$$

The determinant of $\boldsymbol{A}(L)$ is given by

$$det(\boldsymbol{A}(L)) = \frac{(L-1)(-\lambda\hat{\rho}L^2 + \hat{\rho}(1+\lambda^2)L - \lambda\tau_u - \lambda\hat{\rho})}{\hat{\rho}L(L-\lambda)(1-\lambda L)} = \frac{-\lambda(L-1)(L-\omega)(L-\vartheta^{-1})}{L(L-\lambda)(1-\lambda L)}$$

which has three roots, 1, ω and ϑ^{-1} with $|\omega| < |\vartheta^{-1}|$. The following two identifies hold,

$$\omega\vartheta^{-1} = 1 + \frac{\tau_u}{\hat{\rho}} \text{ and } \omega + \vartheta^{-1} = \lambda + \frac{1}{\lambda} = \hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\tau_u + \hat{\tau}_\varepsilon}{\hat{\rho}}$$

We need to solve two unknowns

$$\varphi_1 = -\frac{\lambda h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)} \text{ and } \varphi_2 = \frac{h_1(0)}{\tau_\varepsilon + \tau_u}.$$

By Cramer's rule, we know

$$h_1(L) = \frac{\begin{vmatrix} d_1(L) & A_{12}(L) \\ d_2(L) & A_{22}(L) \end{vmatrix}}{det(\boldsymbol{A}(L))} \text{ and } h_2(L) = \frac{\begin{vmatrix} A_{11}(L) & d_1(L) \\ A_{21}(L) & d_2(L) \end{vmatrix}}{det(\boldsymbol{A}(L))}.$$

We choose φ_1 and φ_2 to remove the inside poles of $h_1(L)$. This leads to the following system of equations

$$\begin{aligned} \varphi_1 =& \frac{(\omega-\lambda)(\lambda-\hat{\rho})}{1-\omega\hat{\rho}}\varphi_2 \\ \varphi_2 =& \frac{\omega\left(\lambda\tau_u + \hat{\rho}\left((\omega-\lambda)(\lambda\omega-1) - \hat{\tau}_\varepsilon\varphi_1\right) + \hat{\rho}^2\hat{\tau}_\varepsilon\omega\varphi_1\right)}{\tau_u\left(\lambda\tau_u + \hat{\rho}(\omega-\lambda)(\lambda\omega-1)\right) + \hat{\tau}_\varepsilon\left(\lambda\tau_u + \hat{\rho}(\omega-\lambda)\left(\hat{\rho}\omega-1\right)\right)}. \end{aligned}$$

The policy functions are

$$
\begin{aligned}
h_1(L) =& -\frac{\omega\left(\left(\hat{\rho}-1\right)\left(\tau_u\left(1-\hat{\rho}(L+\omega-1)\right)+\hat{\rho}\left(L\hat{\rho}-1\right)\left(\hat{\rho}\omega-1\right)+\tau_u^2\right)-\hat{\tau}_\varepsilon\left(\hat{\rho}+\tau_u\right)\right)}{\left(\hat{\rho}-1\right)\left(\hat{\rho}+\tau_u-\hat{\rho}^2\omega\right)\left(\hat{\rho}(L\omega-1)-\tau_u\right)} \\
=& 1+\frac{\hat{\tau}_\varepsilon\vartheta}{(1-\hat{\rho})(1-\vartheta L)(1-\hat{\rho}\vartheta)} \\
h_2(L) =& -\frac{\hat{\rho}\tau_u\omega^2\left(-\hat{\rho}\left(\tau_u+\omega+1\right)+\tau_u+\tau_\varepsilon+\hat{\rho}^2\omega+1\right)}{\left(\hat{\rho}-1\right)\left(\hat{\rho}\omega-1\right)\left(\hat{\rho}+\tau_u-\hat{\rho}^2\omega\right)\left(\hat{\rho}(L\omega-1)-\tau_u\right)} \\
=& \frac{\tau_u\vartheta(1-\vartheta)}{(1-\vartheta L)(1-\hat{\rho}\vartheta)(1-\hat{\rho})}
\end{aligned}
$$

And $g(L)=h_1(L)+h_2(L)$ is

$$
g(L) = 1+\frac{\hat{\rho}-\vartheta}{(1-\vartheta L)(1-\hat{\rho})}
$$

□

Comparative Statics of ϑ

We prove the following comparative statistics,

$$
\frac{\partial\vartheta}{\partial\hat{\rho}}>0, \frac{\partial\vartheta}{\partial\hat{\tau}_\varepsilon}<0 \text{ and } \frac{\partial\vartheta}{\partial\tau_u}<0.
$$

Proof. Note ω and ϑ^{-1} are defined as the roots of the following quadratic equation

$$
L^2-(\lambda+\frac{1}{\lambda})L+(1+\frac{\tau_u}{\hat{\rho}})=0.
$$

Therefore we have

$$
\begin{aligned}
\omega\vartheta^{-1} &= 1+\frac{\tau_u}{\hat{\rho}} \\
\omega+\vartheta^{-1} &= \lambda+\frac{1}{\lambda}
\end{aligned}
$$

which implies that $0<\omega<1<\vartheta^{-1}$. Define the following function

$$
g(x)=x^2-(\lambda+\frac{1}{\lambda})x+(1+\frac{\tau_u}{\hat{\rho}}).
$$

We first observe that the following holds

$$
\frac{\partial\vartheta}{\partial\hat{\rho}}=-\vartheta^2\frac{\partial\vartheta^{-1}}{\partial\hat{\rho}}
$$

where

$$\frac{\partial \vartheta^{-1}}{\partial \hat{\rho}} = -\frac{\partial g(\vartheta^{-1})/\partial \hat{\rho}}{\partial g(\vartheta^{-1})/\partial \vartheta^{-1}}.$$

Using the identify $\lambda + \lambda^{-1} = \hat{\rho} + \hat{\rho}^{-1} + (\hat{\tau}_\varepsilon + \tau_u)\hat{\rho}^{-1}$, we then prove the following

$$\begin{aligned}\frac{\partial g(\vartheta^{-1})}{\partial \hat{\rho}} &= -(1 - \frac{1}{\hat{\rho}^2} - \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}^2})\vartheta^{-1} - \frac{\tau_u}{\hat{\rho}^2} > 0 \\ \frac{\partial g(\vartheta^{-1})}{\partial \vartheta^{-1}} &= 2\vartheta^{-1} - (\lambda + \frac{1}{\lambda}) > 0.\end{aligned}$$

To prove that $-(1 - \frac{1}{\hat{\rho}^2} - \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}^2})\vartheta^{-1} - \frac{\tau_u}{\hat{\rho}^2} > 0$, it is equivalent to showing the following holds,

$$\begin{aligned} & (\hat{\rho}^2 - 1 - \hat{\tau}_\varepsilon - \tau_u) < \tau_u \vartheta \\ \Rightarrow\ & \hat{\rho}^2 < 1 + \hat{\tau}_\varepsilon + \tau_u + \tau_u \vartheta,\end{aligned}$$

which holds because $\hat{\rho}^2 < 1$. To prove $\frac{\partial g(\vartheta^{-1})}{\partial \vartheta^{-1}} > 0$, note

$$2\vartheta^{-1} > \omega + \vartheta^{-1} = \lambda + \frac{1}{\lambda}.$$

As a result, $2\vartheta^{-1} > \lambda + \frac{1}{\lambda}$. Therefore

$$\frac{\partial \vartheta^{-1}}{\partial \hat{\rho}} = \frac{2\hat{\rho}\vartheta^{-1} - \vartheta^{-2} - 1}{2\hat{\rho}\vartheta^{-1} - (\lambda + \lambda^{-1})\hat{\rho}} < 0 \text{ and } \frac{\partial \vartheta}{\partial \hat{\rho}} > 0.$$

Similarly, we have

$$\frac{\partial g(\vartheta^{-1})}{\partial \hat{\tau}_\varepsilon} = -\frac{\vartheta^{-1}}{\hat{\rho}} < 0 \text{ and } \frac{\partial g(\vartheta^{-1})}{\partial \tau_u} = \frac{1 - \vartheta^{-1}}{\hat{\rho}} < 0.$$

Therefore

$$\frac{\partial \vartheta}{\partial \hat{\tau}_\varepsilon} < 0 \text{ and } \frac{\partial \vartheta}{\partial \tau_u} < 0.$$

□

A.3 Term Structure of UIP Violations

To compute bond prices in the model, we implement an iterative procedure. In particular, we assume that bond prices follow the general relation that $p_t^{(n)} = p_t^{(n-1)} + g(t)$. Then, we know that

$$p_t^{(n+1)} = p_t^{(n)} + \int_i \mathbb{E}_{it}[g(t+1)]di$$

where $g(t+1)$ is a function of f_{t+1} and i^d_{t+1}, where f_t is the average expectation of the fundamental in period t across agents. With this idea in hand, we implement the following steps:

(i) Start from $p_t^{(1)} = -i_t^d = g(t)$. The next period's price is

$$p_t^{(2)} = p_t^{(1)} + \int_i \mathbb{E}_{it}[g(t+1)]di.$$

In computing $\mathbb{E}_{it}[g(t+1)]$, expectation of future interest rates uses f_{it} and past information is perfectly observed. We have

$$p_t^{(2)} = p_t^{(1)} - \hat{\rho} f_t.$$

(ii) Computing $p_t^{(3)}$, the second term is the average forecast of the one-period-forward second term in the last price

$$\int_i \mathbb{E}_{it}[-\hat{\rho} f_{t+1}]di.$$

When computing the integrand, we use the following equation to first replace future forecast

$$f_{t+1} = \lambda f_t + a_\varepsilon i^d_{t+1} + a_u \xi_{t+1}.$$

The expectation of i^d_{t+1} and ξ_{t+1} is easily to derive. For $\mathbb{E}_{it}[f_t]$, we replace f_t by its state-space representation, i.e.,

$$f_t = \frac{a_\varepsilon i_t^d + a_u \xi_t}{1 - \lambda L}$$

and then we use

$$\begin{aligned} &\mathbb{E}_{it}[\frac{\xi_t}{(1-\lambda L)^k}] \\ =&\frac{L(1-\hat{\rho}\lambda)(1-\lambda^2)^k - \lambda(1-\hat{\rho}L)(1-\lambda L)^k}{(L-\lambda)(1-\lambda L)^k(1-\lambda^2)^k} f_{it}. \end{aligned}$$

(iii) Repeat the procedure, forward one-period, collect future average forecast, use the process of f_t to replace and apply the above prediction formula to aggregate information.

Internet Appendix For

The Role of Beliefs in Asset Prices: Evidence from Exchange Rates

Joao Paulo Valente, Kaushik Vasudevan, Tianhao Wu

April 23, 2022

IA.A Full Sample Regression Results for Motivating Evidence

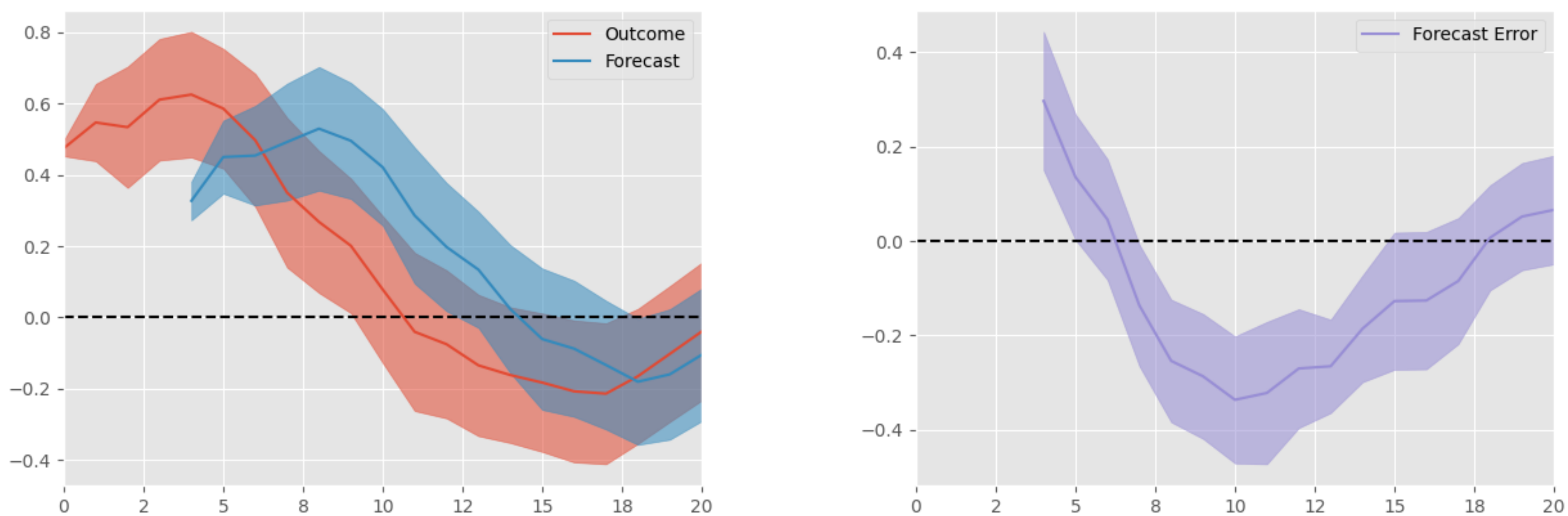
TABLE IA.A.1: SURVEY-BASED INTEREST RATE FORECAST ERRORS AND EXCHANGE RATES

The table reports results regressions of currency excess returns and exchange rate forecast errors on interest rate forecasts errors. Regressions are of the form $y_{j,t} = \beta(i^d_{j,t} - \bar{\mathbb{E}} i^d_{j,t}) + \epsilon_{j,t}$, where $i^d_{j,t}$ is the short-term interest rate of currency j in period t minus the US short-term interest rate in period t, $y_{j,t}$ is the dependent variable, $\bar{\mathbb{E}}$ corresponds with consensus expectations reported in survey data, and β is the coefficient of interest. The dependent variables used are currency j's $(s_{j,t} - s_{j,t-1} + i^d_{j,t})$, and the unexpected currency appreciation of currency j, $s_{j,t} - \bar{\mathbb{E}}_{t-1} s_{j,t}$. Variables in the regressions are standardized to have zero mean and unit standard deviation for each currency, so that the regression coefficients may be interpreted as correlations. Observations are sampled quarterly. Standard errors are clustered by currency and time period. The table reports t-statistics in parentheses. The sample runs from October 1989 through December 2019.

	Currency Excess Returns $y = s_{j,t} - s_{j,t-1} + i^d_{j,t}$		Currency Forecast Errors $y = s_{j,t} - \bar{\mathbb{E}}_{t-1} s_{j,t}$	
	(1)	(2)	(3)	(4)
IR Differential Forecast Error	0.20	0.22	0.19	0.20
$i^d_{j,t} - \bar{\mathbb{E}}_{t-1}(i^d_{j,t})$	(2.28)	(2.41)	(2.09)	(2.16)
IR Differential Forecast		0.16		0.10
$\bar{\mathbb{E}}_{t-1}(i^d_{j,t})$		(2.88)		(1.70)

FIGURE IA.A.1: UNDERREACTION AND OVERREACTION IN US INTEREST RATE RESPONSE TO INTEREST RATE NEWS SHOCKS

The figure plots impulse response functions (IRFs) of US Treasury Bill rates, US Treasury Bill rate consensus forecasts, and US Treasury Bill rate consensus forecast errors in response to interest rate news shocks. The IRFs are estimated from regressions of the form $x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma_h C_t + u_{t+h}$, where $x_{t+h} \in (i_{t+h}, \bar{\mathbb{E}}_{t+h} i_{t+h+k}, i_{t+h+k} - \bar{\mathbb{E}}_{t+h} i_{t+h+k})$, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are the estimated interest rate news shocks. The estimated interest rate news shocks come from Angeletos, Collard and Dellas (2020). Forecast data are from the Survey of Professional Forecasters and the sample runs from 1981 to 2019.

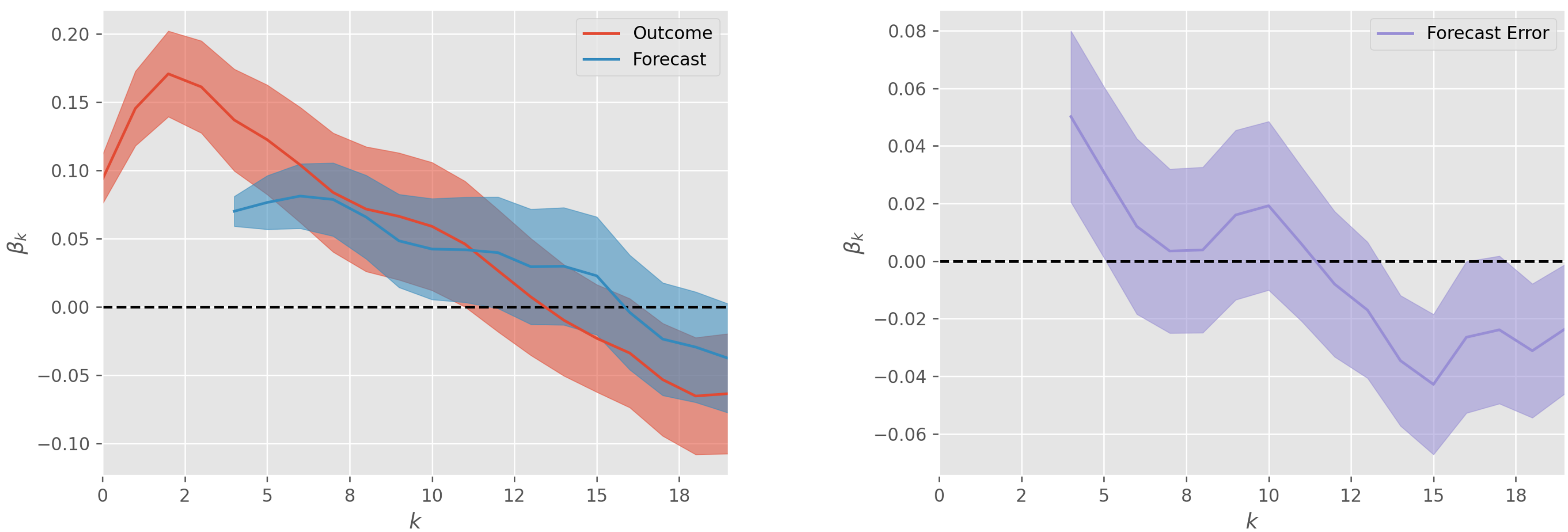


(a) US interest rate IRF to interest rate news

(b) US interest rate forecast errors IRF to interest rate news

Figure IA.A.2: Underreaction and Overreaction in Interest Rate Differential Response to Interest Rate News Shocks

The figure plots impulse response functions (IRFs) of interest rate differentials, interest rate differential consensus forecasts, and interest rate differential forecast errors between the US and international countries, in response to interest rate news shocks. The IRFs are estimated from regressions of the form $x_{j,t+h} = \alpha_{j,h} + \beta_h \epsilon_t + \gamma C_{j,t} + u_{j,t+h}$, j corresponds with a specific country, $x_{j,t+h} \in (i_{j,t+h}, \bar{\mathbb{E}}_{j,t+h} i_{j,t+h+k}, i_{j,t+h+k} - \bar{\mathbb{E}}_{j,t+h} i_{j,t+h+k})$, $C_{j,t}$ are lagged values of forecasts and outcomes used as controls, and ϵ_t are the estimated interest rate news shocks. The estimated interest rate news shocks come from Angeletos, Collard and Dellas (2020). Data on interest differential forecasts are from Consensus Economics. The sample consists of G11 currencies and runs from October 1989 through December 2019.

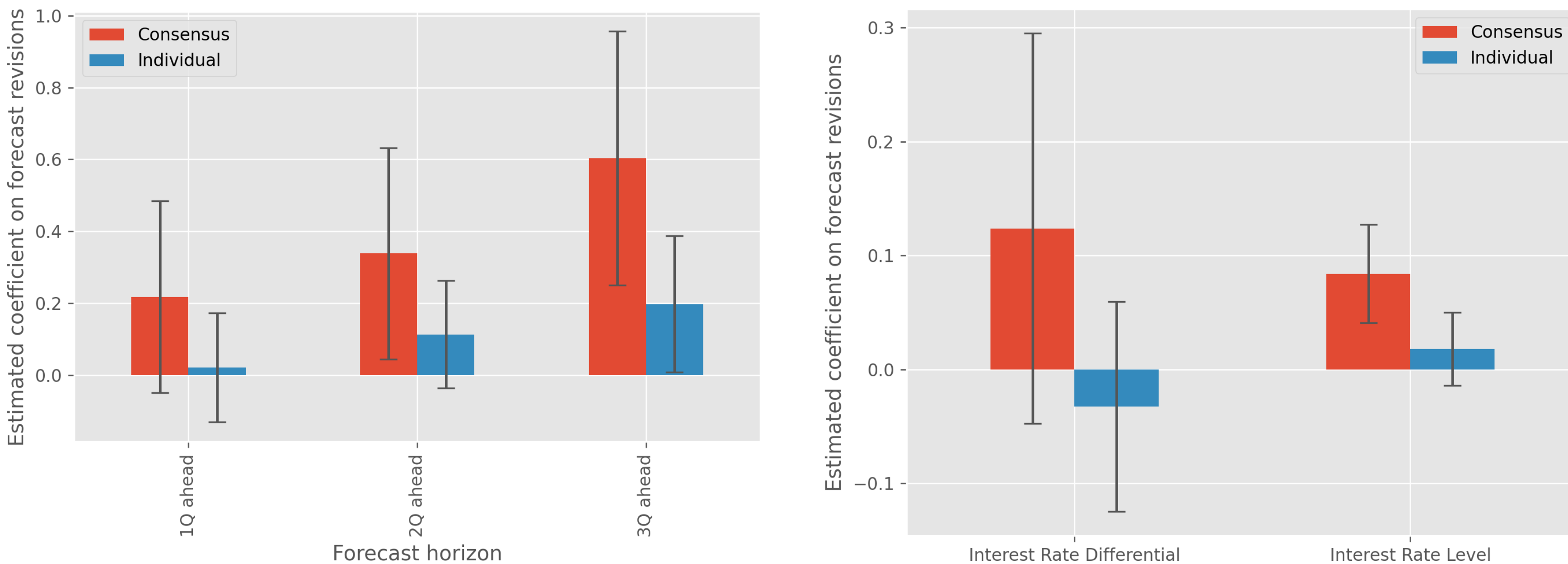


(a) Interest rate differential IRF to interest rate news

(b) Interest rate differential forecast errors IRF to interest rate news

FIGURE IA.A.3: CONSENSUS VS. INDIVIDUAL DEVIATIONS FROM FIRE

The figure plots regression coefficients from regressions of forecast errors of interest rates on forecast revisions of interest rates. The red bars are for regressions where observations correspond with consensus forecasts and the blue bars are for regressions where observations correspond with individual forecasts. The first panel in the figure reports regression coefficients from regressions of the form $x_{t+k} - \mathbb{E}_t x_{t+k} = \alpha + \beta(\mathbb{E}_t x_{t+k} - \mathbb{E}_{t-k} x_{t+k}) + \epsilon_{t+k}$, where x_{t+k} is the US Treasury Bill rate in period $t+k$, and $\mathbb{E}_t x_{t+k}$ is the forecast at period t of the realized outcome at period $t+k$ from the Survey of Professional Forecasters. The second panel in the figure reports regression coefficients from regressions of the form $x_{t+1} - \mathbb{E}_t x_{t+1} = \alpha + \beta(\mathbb{E}_t x_{t+1} - \mathbb{E}_{t-3} x_{t+1}) + \epsilon_{t+1}$, where x_t is the interest rate differential (or interest rate level) for a given foreign currency, and $\mathbb{E}$ is the forecast at period t of the realized outcome in $t+1$ from Consensus Economics. Standard errors for panel regressions are two-way clustered by forecaster and time period. Lines denoting plus or minus two standard errors are included in the plots. The sample consists of quarterly observations from 1981 through 2019 from the Survey of Professional Forecasters, and from 1989 to 2019 from Consensus Economics.



(a) T-Bill (SPF)

(b) Foreign Short-Rates (Consensus Economics)

IA.B Additional Analysis of Underreaction and Delayed Overreaction

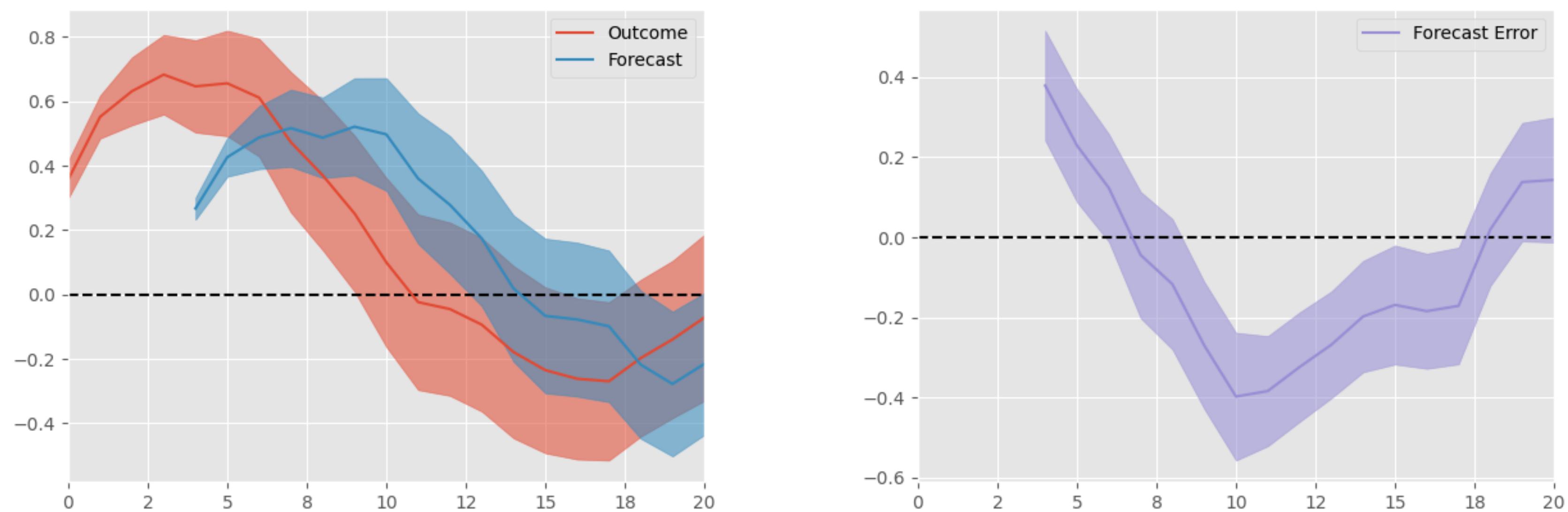
TABLE IA.B.1: UNDERREACTION AND OVERREACTION IN HYBRID REGRESSIONS

This table reports results from hybrid Coibion and Gorodnichenko (2015) and Kohlhas and Walther (2020) regressions for T-Bill (3mo) forecasts (SPF) from January 1985 to December 2007. Coibion and Gorodnichenko (2015) regressions are given by $x_{t+k} - F_t x_{t+k} = \alpha + \beta_{CG}(F_t x_{t+k} - F_{t-k} x_{t+k}) + \epsilon_{t+k}$. Kohlhas and Walther (2020) regressions are given by $x_{t+k} - F_t x_{t+k} = \alpha + \beta_{KW} x_t + \epsilon_{t+k}$. HAC-panel standard errors are reported in parentheses. The table also reports results using Hamilton(2017) and HP filter ($\lambda = 1600$) in the detrend columns to account for potential structural changes.

				No Detrending				Hamilton (2017) Detrending				HP Filter Detrending			
	FE (1Q)	FE (2Q)	FE (3Q)	FE (1Q)	FE (2Q)	FE (3Q)	FE (4Q)	FE (1Q)	FE (2Q)	FE (3Q)	FE (4Q)	FE (1Q)	FE (2Q)	FE (3Q)	FE (4Q)
Constant	-0.12	-0.24	-0.38	-0.10	-0.06	-0.03	0.06	-0.17	-0.30	-0.42	-0.55	-0.17	-0.29	-0.42	-0.56
	0.05	0.11	0.18	0.13	0.27	0.43	0.59	0.07	0.13	0.20	0.27	0.07	0.13	0.20	0.25
Forecast Revision	0.39	0.58	0.71												
	0.07	0.12	0.21												
Current Realization				-0.01	-0.05	-0.08	-0.13	0.01	0.01	0.01	-0.01	-0.07	-0.14	-0.22	-0.33
				0.03	0.05	0.08	0.11	0.02	0.04	0.06	0.09	0.04	0.08	0.13	0.18
R2	0.18	0.14	0.11	0.00	0.02	0.03	0.04	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.08
F-stat	27.48	22.85	11.75	0.25	0.79	1.00	1.32	0.09	0.07	0.02	0.01	2.96	2.97	2.83	3.42
N	92	92	92	91	90	89	88	91	90	89	88	91	90	89	88

FIGURE IA.B.1: RESPONSE OF US INTEREST RATE FORECASTS TO ROMER AND ROMER (2004) SHOCKS

The figure reports results from regressions of the form $x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma C_t + u_{t+h}$, where $x_{t+h} \in (i_{t+h}, \bar{\mathbb{E}}_{t+h} i_{t+h+k}, i_{t+h+k} - \bar{\mathbb{E}}_{t+h} i_{t+h+k})$, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are Romer and Romer (2004) monetary shocks, compiled by Wieland and Yang (2020). Expectations are measured as Treasury Bill forecasts from the Survey of Professional Forecasters. The sample consists of quarterly observations from Q3/1981 to Q4/2007.

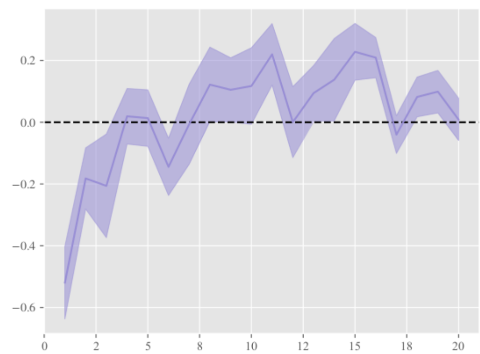


(a) US interest rate IRF to monetary shocks

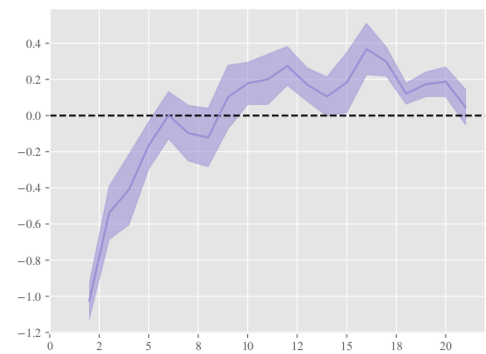
(b) US interest rate forecast errors IRF to monetary shocks

Figure IA.B.2: Kucinskas and Peters (2019) Composite Bias Coefficients for US Interest Rate Forecasts

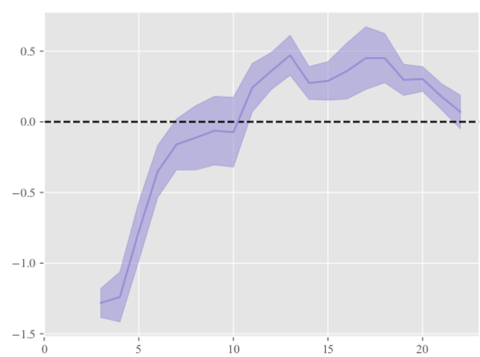
The figure reports composite bias coefficients for interest rate consensus (median) forecasts using Kucinskas and Peters (2019) method. Confidence intervals computed using Newey-West standard errors with max{4, 11} lags. Negative (positive) coefficients suggest under(over)-reaction. The IRF of forecast errors are based on the following regression, estimated via local projection $E_t x_t - F_{t-1} x_t = -b_0 - \sum_{l=1}^{\infty} sgn(\alpha_l) b_l \epsilon_{t-l} + \epsilon_t$, where $b_l = sgn(\alpha_l)(a_l - \alpha_l)$ are the bias coefficients, $x_t = \sum_{l=0}^{+\infty} \alpha_l \epsilon_{t-l}$, and $\mathbb{E}_t x_{t+1} = b_0 + \sum_{l=0}^{+\infty} a_{l+1} \epsilon_{t-l}$. Expectations are measured as Treasury Bill forecasts from the Survey of Professional Forecasters. The sample consists of quarterly observations from the survey of professional forecasters from Q1/1985 to Q4/2007.



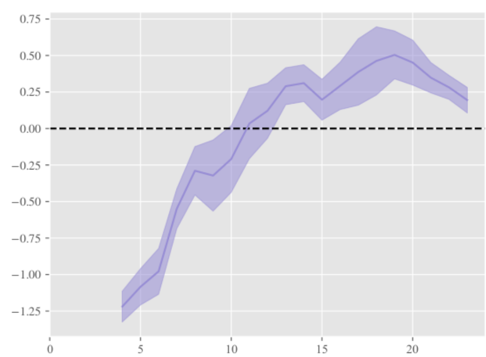
(a) One quarter-ahead interest rate forecasts



(b) Two quarter-ahead interest rate forecasts



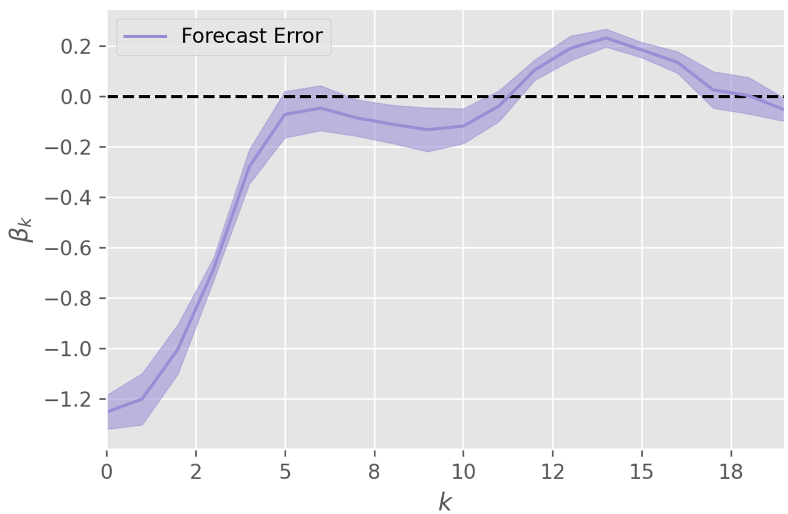
(c) Three quarter-ahead interest rate forecasts



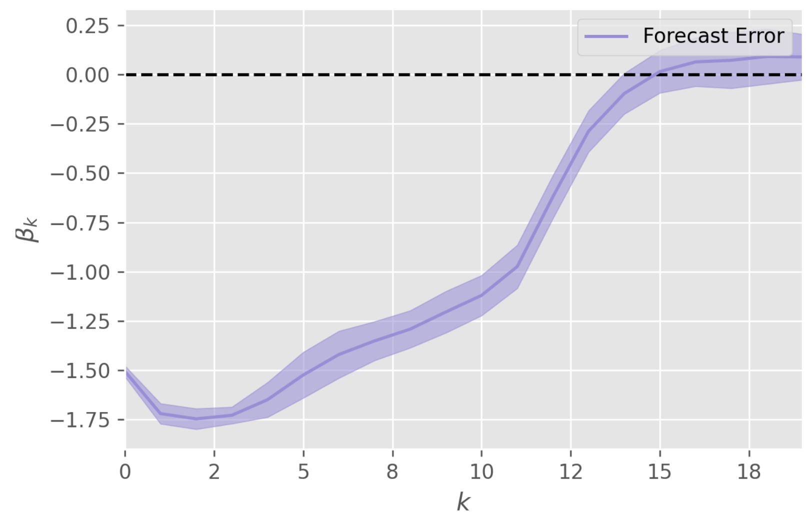
(d) Four quarter-ahead interest rate forecasts

FIGURE IA.B.3: KUCINSKAS AND PETERS (2019) COMPOSITE BIAS COEFFICIENTS FOR FOREIGN INTEREST RATE FORECASTS

The figure reports composite bias coefficients for interest rate consensus (median) forecasts using Kucinskas and Peters (2019) method. Confidence intervals computed using Newey-West standard errors with max{4, 11} lags. Negative (positive) coefficients suggest under(over)-reaction. The IRF of forecast errors are based on the following regression, estimated via local projection $E_t x_t - F_{t-1} x_t = -b_0 - \sum_{l=1}^{\infty} sgn(\alpha_l) b_l \epsilon_{t-l} + \epsilon_t$, where $b_l = sgn(\alpha_l)(a_l - \alpha_l)$ are the bias coefficients, $x_t = \sum_{l=0}^{+\infty} \alpha_l \epsilon_{t-l}$, and $\mathbb{E}_t x_{t+1} = b_0 + \sum_{l=0}^{+\infty} a_{l+1} \epsilon_{t-l}$. Expectations are measured forecasts from Consensus Economics. The sample consists of quarterly observations from the survey of professional forecasters from 1985 to 2007.



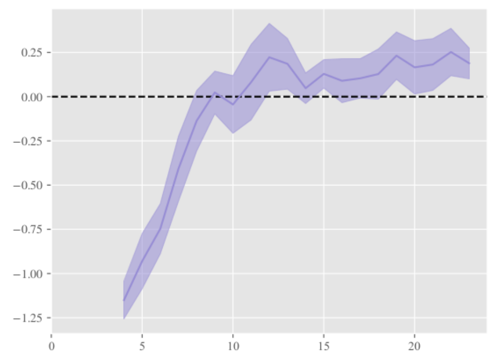
(a) One quarter-ahead interest rate forecasts



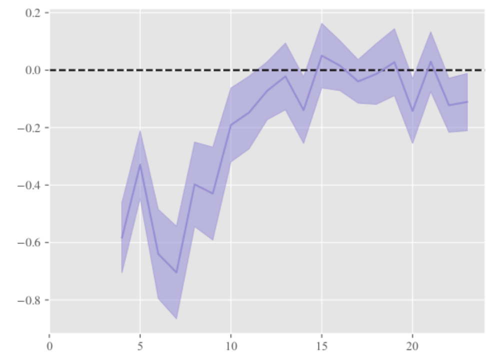
(b) Four quarter-ahead interest rate forecasts

FIGURE IA.B.4: KUCINSKAS AND PETERS (2019) COMPOSITE BIAS COEFFICIENTS FOR INTEREST RATE FORECASTS

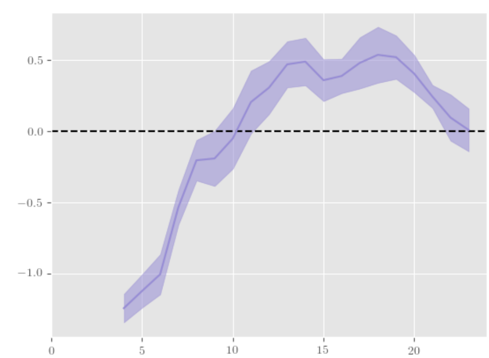
The figure reports composite bias coefficients for interest rate consensus (median) forecasts using Kucinskas and Peters (2019) method. Confidence intervals computed using Newey-West standard errors with max{4, 11} lags. Negative (positive) coefficients suggest under(over)-reaction. The IRF of forecast errors are based on the following regression, estimated via local projection $E_t x_t - F_{t-1} x_t = -b_0 - \sum_{l=1}^{\infty} sgn(\alpha_l) b_l \epsilon_{t-l} + \epsilon_t$, where $b_l = sgn(\alpha_l)(a_l - \alpha_l)$ are the bias coefficients, $x_t = \sum_{l=0}^{+\infty} \alpha_l \epsilon_{t-l}$, and $\mathbb{E}_t x_{t+1} = b_0 + \sum_{l=0}^{+\infty} a_{l+1} \epsilon_{t-l}$. Expectations are measured as Treasury Bill, Treasury Bond, Inflation, and Unemployment from the Survey of Professional Forecasters. The sample consists of quarterly observations from the survey of professional forecasters Q1/1969 to Q4/2007.



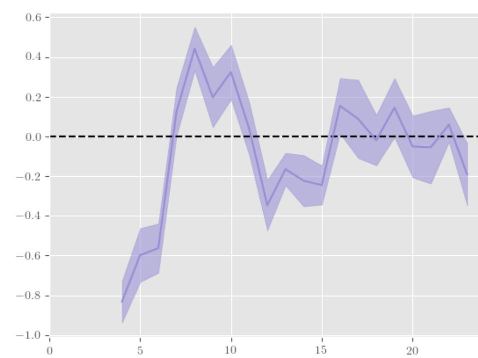
(a) Four quarter-ahead inflation forecasts



(b) Four quarter-ahead inflation forecasts



(c) Four-quarter-ahead T-Bill forecast



(d) Four quarter-ahead Treasury bond forecasts

TABLE IA.B.2: CONSENSUS VERSUS INDIVIDUAL FORECAST ERRORS

The table reports regression results following the approach of Angeletos, Huo and Sastry (2020), to analyze the underreaction and overreaction of consensus and individual expectations. Regressions of the form $x_{t+k} - \mathbb{E}_{i,t}x_{t+k} = \beta_{\text{Revision}}(\bar{\mathbb{E}}_t x_{t+k} - \bar{\mathbb{E}}_{t-k}x_{t+k}) + \beta_{\Delta_i\text{Revision}}\left[(\mathbb{E}_{i,t}x_{t+k} - \mathbb{E}_{i,t-k}x_{t+k}) - (\mathbb{E}_t x_{t+k} - \mathbb{E}_{t-k}x_{t+k})\right] + \epsilon_{i,t+k}$, where $\bar{\mathbb{E}}$ captures the average forecast across all forecasters, and $\mathbb{E}_i$ captures the forecast of forecaster i. Positive coefficients in the regressions correspond with *underreaction* to news and negative coefficients correspond with *overreaction* to news. Standard errors are two-way clustered by forecaster and time period. The sample consists of quarterly observations between 1969 to 2007 from the Survey of Professional Forecasters.

	Unemployment			Inflation			Treasury Bill		
	1Q	2Q	3Q	1Q	2Q	3Q	1Q	2Q	3Q
Revision	0.405	0.614	0.600	0.654	1.099	1.220	0.223	0.469	0.831
	(0.156)	(0.186)	(0.240)	(0.361)	(0.473)	(0.472)	(0.162)	(0.168)	(0.253)
Δ_iRevision	-0.226	-0.282	-0.272	-0.457	-0.429	-0.412	-0.269	-0.185	-0.144
	(0.033)	(0.044)	(0.051)	(0.035)	(0.045)	(0.055)	(0.048)	(0.070)	(0.061)
R^2	0.099	0.115	0.063	0.155	0.146	0.136	0.079	0.082	0.124
N	4331	4269	3966	4197	4153	3856	2544	2491	2393

IA.B.1 UIP and Consensus Exchange Rate Expectations

The forward premium puzzle has been an established fact in the academic literature dating back to Hansen and Hodrick (1980) and Fama (1984). One way to observe the puzzle is via regressions of the form

$$\lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1} \tag{28}$$

where $\lambda_{j,t+1}$ are the excess returns of borrowing at short-term interest rates in country j and lending at US short-term interest rates in dollars, i^d_j is the interest rate differential (the US short-term interest rate minus the foreign short-term interest rate), and β is the coefficient of interest. The UIP condition implies that $\beta = 0$, while empirical work has consistently reported estimates of $\beta > 0$.

While the UIP condition appears to fail spectacularly in the data, consensus (average) forecasts of currencies across market participants appear to align much more closely with UIP. That is, when we run the regression in Equation (28) replacing the independent variable with $\bar{\mathbb{E}}_t \lambda_{j,t+1}$, where $\bar{\mathbb{E}}$ captures the average expectation reported in forecasts, we find a coefficient β that is much closer to zero.[33]

Figure IA.B.5 plots the average beta from estimating Equation (28) for each country in our sample, using quarterly forecasted and realized excess returns as the dependent variables, and interest rate differentials implied by 3-month forward rates as the independent variables. Betas for individual countries are weighted by the total number of observations that we have for the country in our sample. The figure reports average betas for all countries together. The figure also plots 95% confidence intervals, computed using HAC-panel standard errors. The sample is from August 1986 through December 2007.

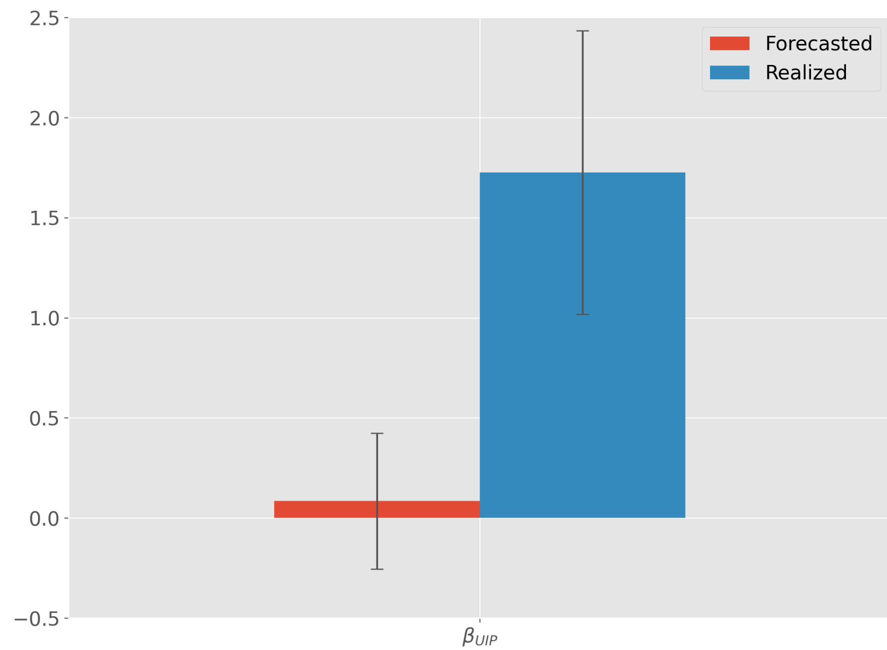
The figure reveals the well-known failure of UIP; the average coefficient for excess returns is 1.73. The beta for forecasted excess returns is 0.08, and is statistically indistinguishable from zero. That is, market participants report forecasts of excess returns that, on average, closely align with UIP, despite the fact that the UIP condition strongly fails in the data.

These results are important for two reasons. First, they suggest that incorrect beliefs may play an important role in explaining the exchange rate puzzles of interest to us. If the failure of UIP were entirely driven by risk premia, we might expect survey-based expectations to capture these risk premia; we find that they do not. Second, they provide us with useful stylized facts to consider in formulating an explanation for exchange rate behavior.

[33] The conclusions we draw from these regressions are similar to those found in Froot and Frankel (1989) for a sample of five currencies in the 1970s and 1980s. We extend the results to an additional set of currencies and a longer and later sample period, and find consistent evidence.

FIGURE IA.B.5: UIP REGRESSIONS USING REALIZED AND EXPECTED CURRENCY RETURNS

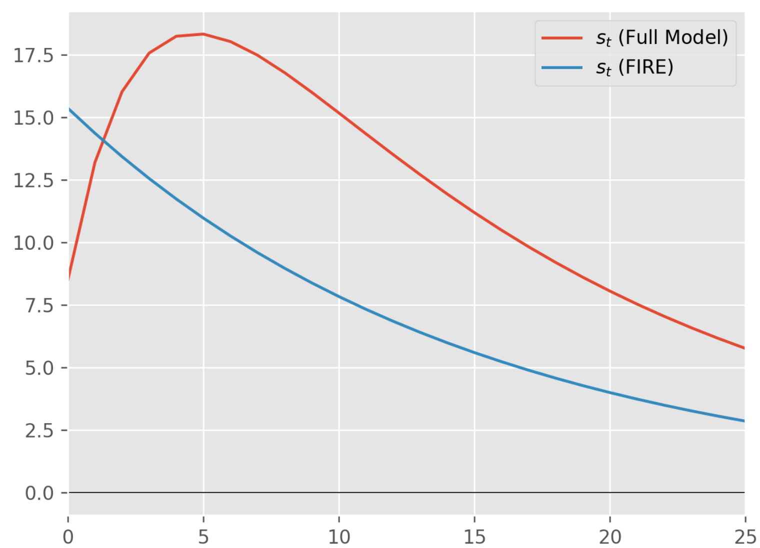
The figure presents regression coefficients from two sets of panel regressions: (a) $\lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1}$ and $\bar{\mathbb{E}}_t \lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1}$, where $\lambda_{j,t+1}$ are the excess returns from borrowing in currency j and lending in USD, $\bar{\mathbb{E}}_t \lambda_{j,t+1}$ is the expected excess return measured using consensus forecasts of the period $t+1$ exchange rate, and $i^d_{j,t}$ is the short-term interest rate differential between the US and country j at period t. The sample is from August 1986 through December 2007.



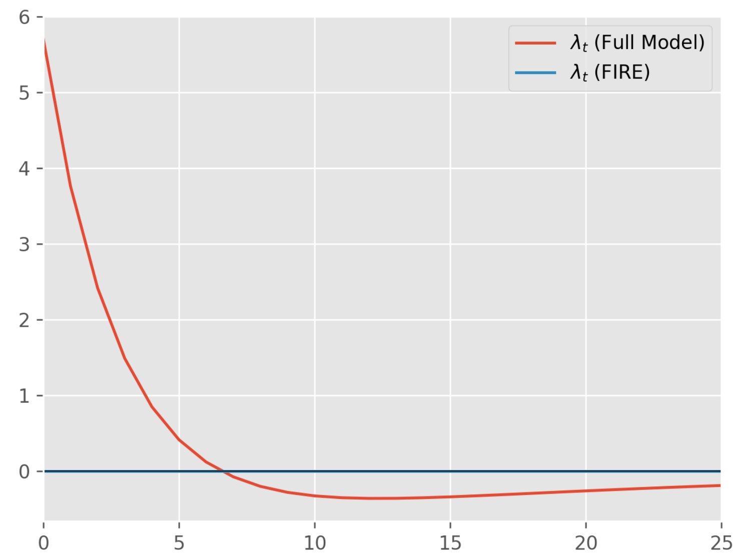
IA.C Exchange Rate Puzzles with Different Frictions

FIGURE IA.C.1: DELAYED OVERSHOOTING

The figure reports model's delayed overshooting and UIP deviations. We plot model's exchange rate and excess return IRFs after a one standard deviation shock to the fundamental process ξ_t.



(a) Exchange Rate



(b) Excess Return

FIGURE IA.C.2: PREDICTABILITY REVERSAL

The figure reports model's UIP regression for different k-period ahead horizons, including different frictions in the model. We simulate the estimated model 5,000 times for 144 periods. For each simulation and k-period ahead horizon, we estimate the following regression $\lambda_{t+k} = \alpha_k + \beta_k i_t^d + \epsilon_{t+k}$, where λ_{t+k} is the excess return between period $t+k-1$ and $t+k$, and i_t^d is the interest rate differential at period t. We report the average regression coefficient of all simulations.

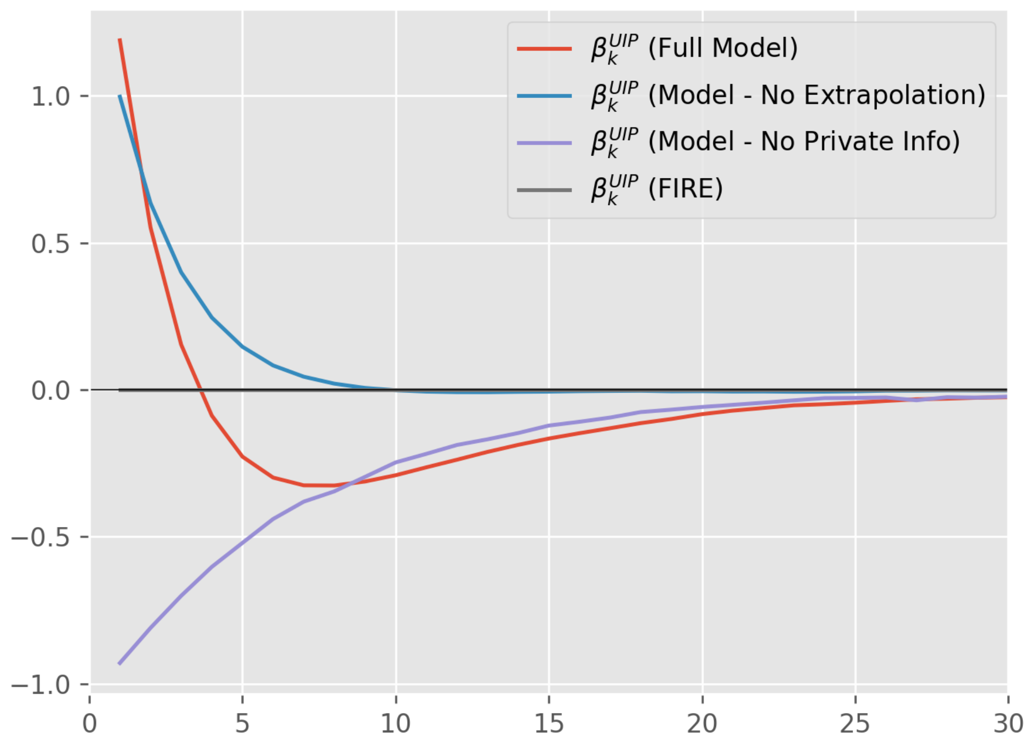


FIGURE IA.C.3: TIME-SERIES MOMENTUM AND REVERSAL

The figure plots autocorrelations of currency excess returns in the model, including different frictions. The k-period autocorrelation is calculated by simulating the estimated model 5,000 times for 144 periods, and taking an average autocorrelation of currency excess returns with k-period lagged excess returns in each simulated sample.

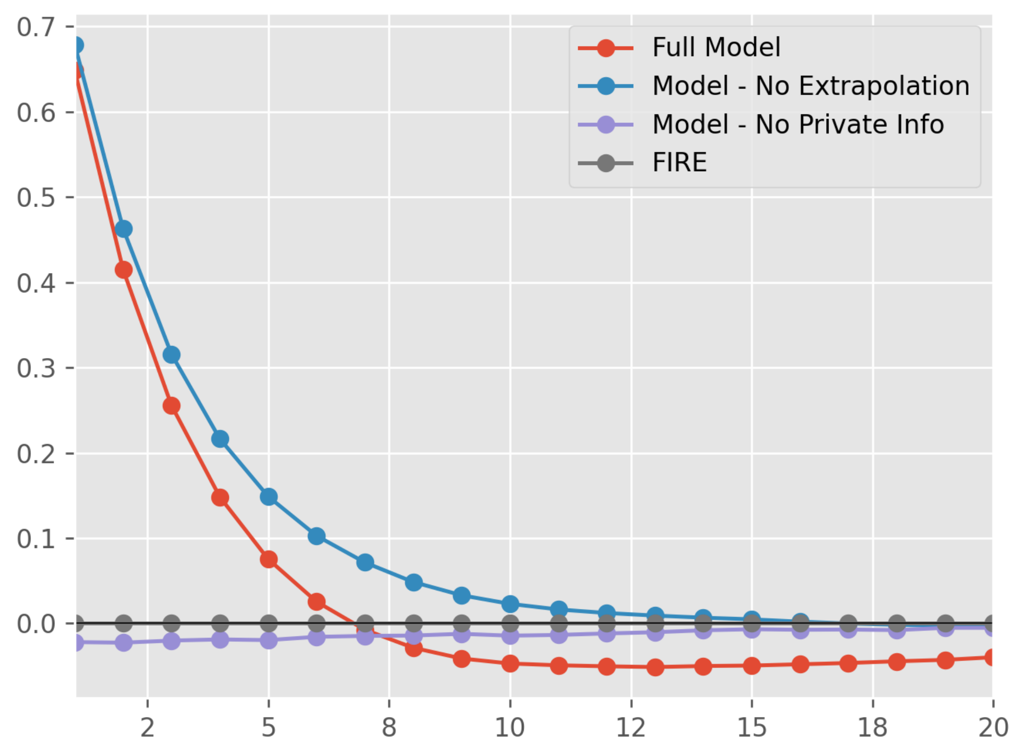


Figure IA.C.4: The Downward-Sloping Term Structure of UIP Violations

The figure plots the model-implied regression coefficients from regressing the returns to borrowing in n-period maturity foreign bonds and investing in n-period maturity home country bonds on the interest rate differential (the home currency interest rate minus the foreign country interest rate), for different values of n. The coefficients are computed by simulating the model 5,000 times for 144 periods. The figure plots regression coefficients for the full estimated model, as well as for versions of the model with different frictions.